INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Contraintes – Torseur de Cohésion – Résistance statique

Arbre secondaire Boîte Vitesses

Équipe pédagogique CONAN

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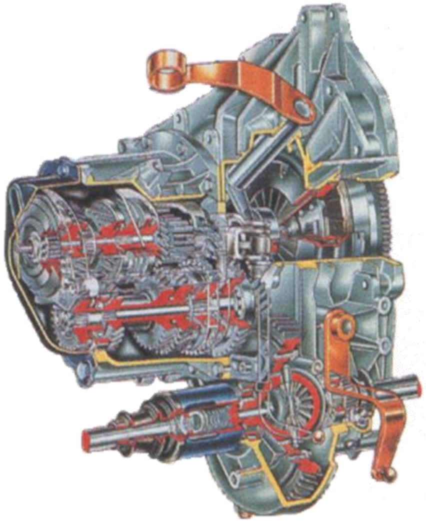
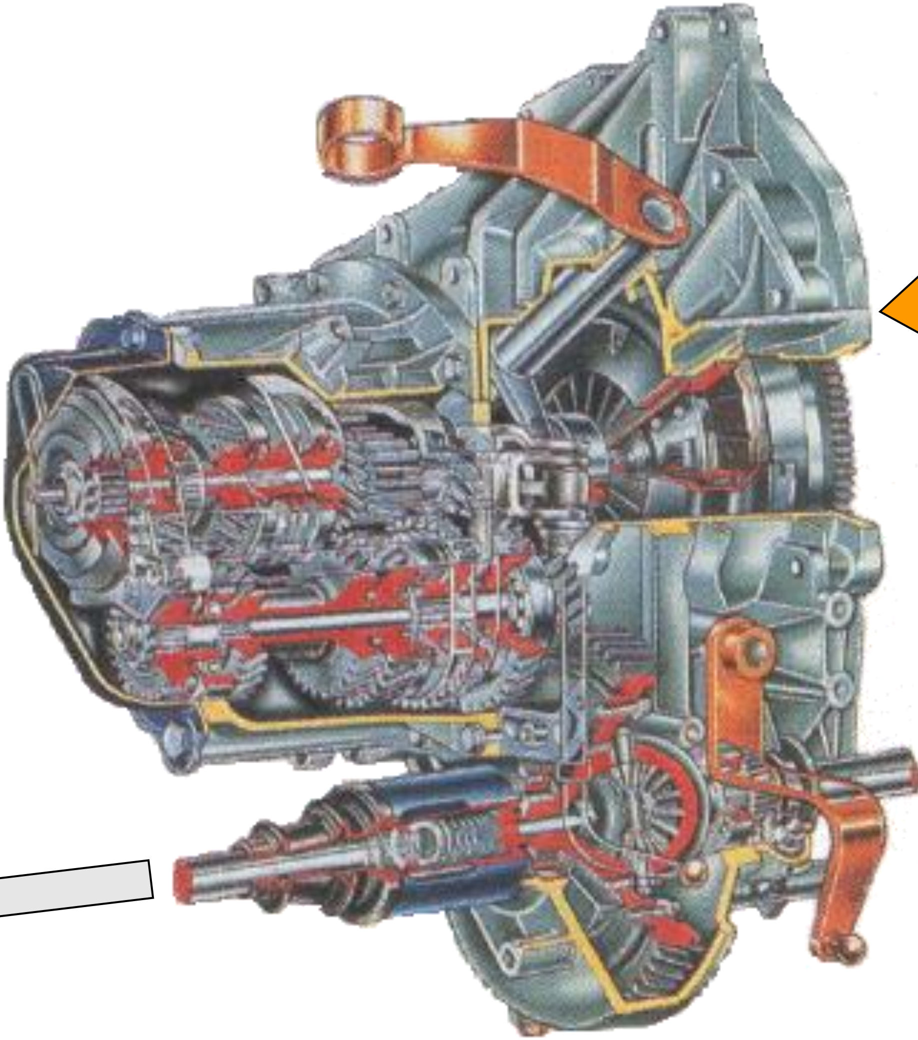
Objectifs :

- Calcul des efforts extérieurs 3D sur une pièce isolée : l’arbre secondaire.
- Calcul du torseur des efforts intérieurs (torseur de cohésion),
- Tracés des diagrammes,
- Identification de la section critique (où le dimensionnement doit se faire)
- Calculs des contraintes en différents points d’une section, contrainte équivalente
- Critères de résistance statique.



Boite manuelle

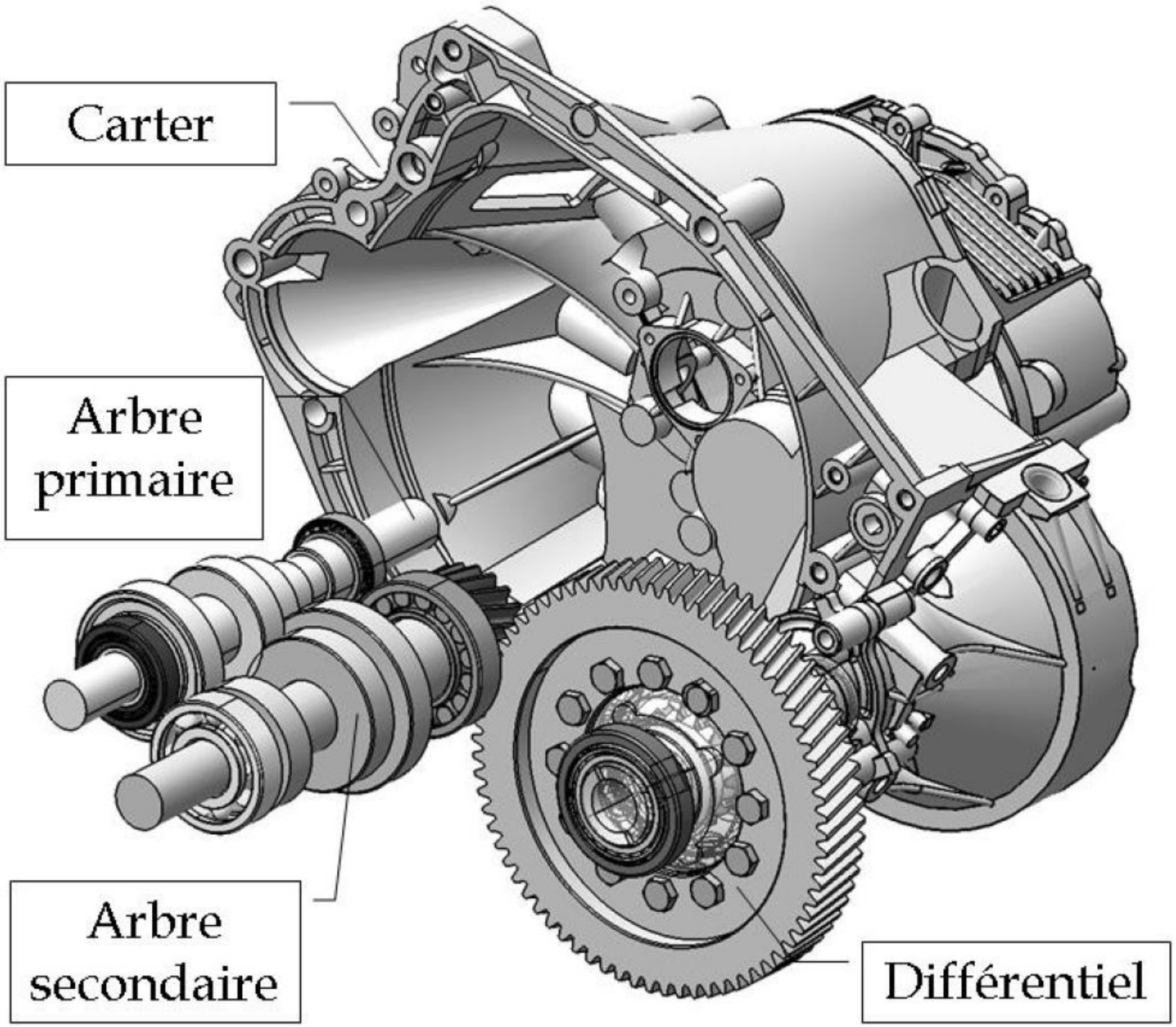


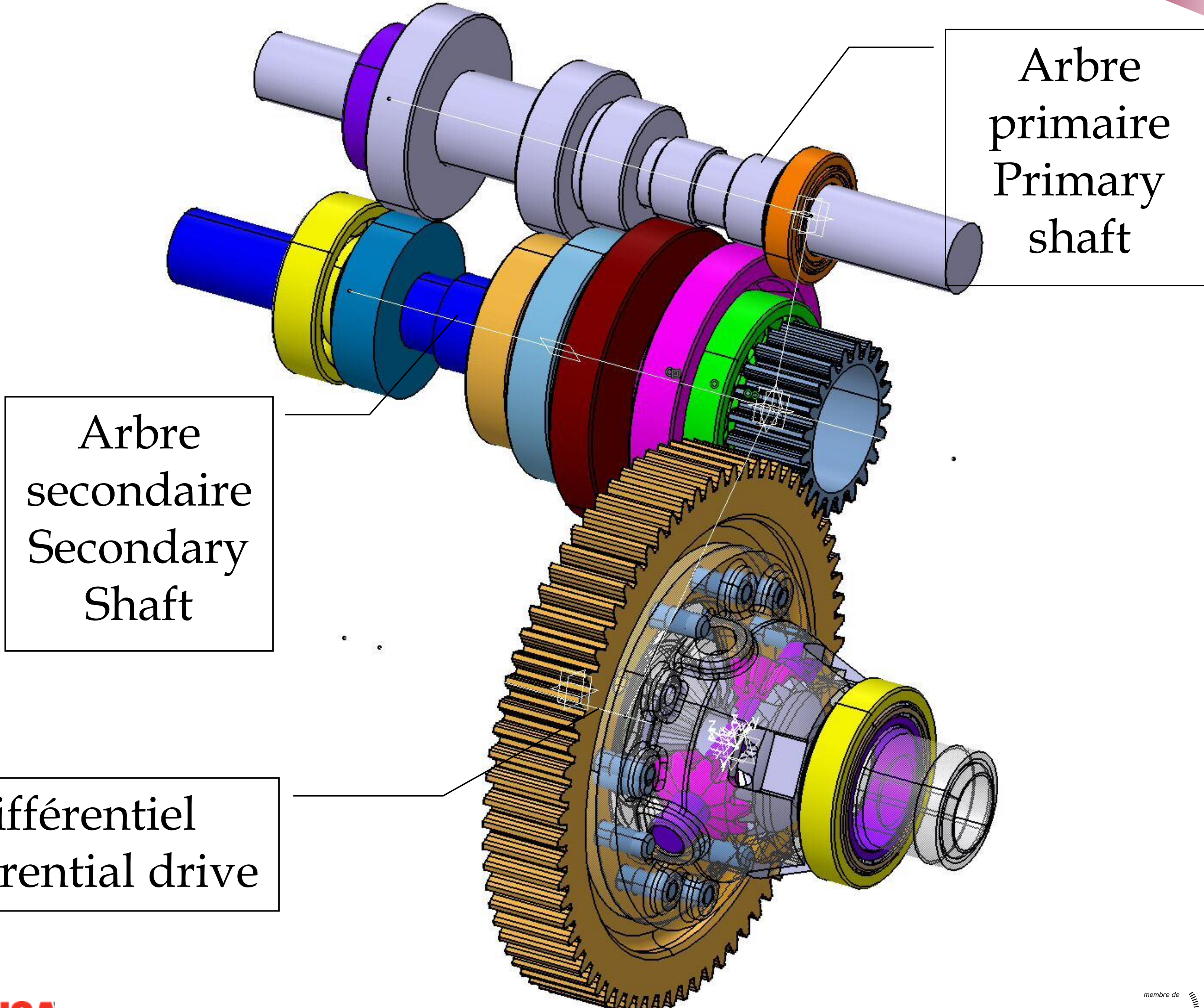
Entrée : Moteur + Embrayage

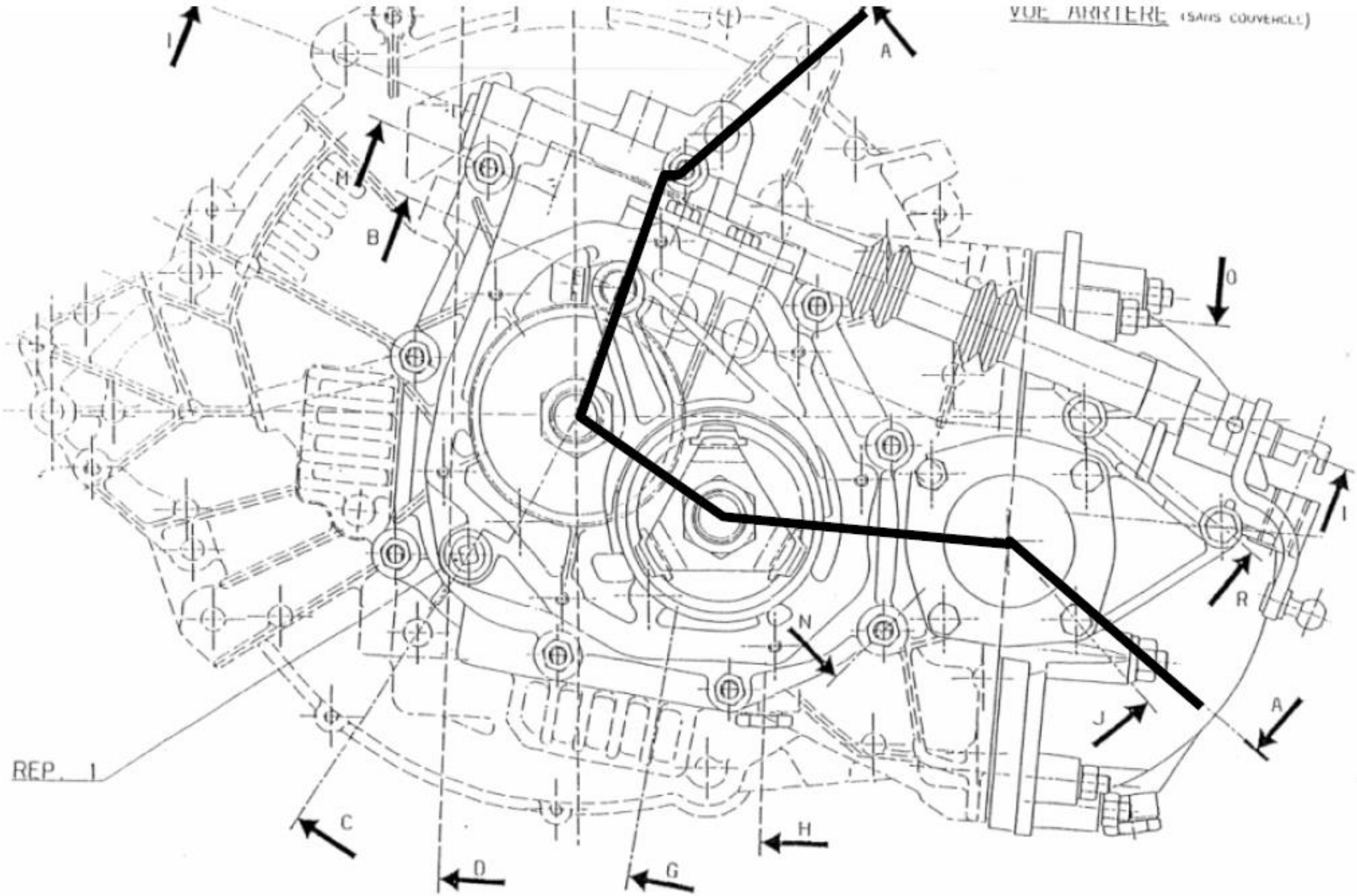
Sorties : Roues

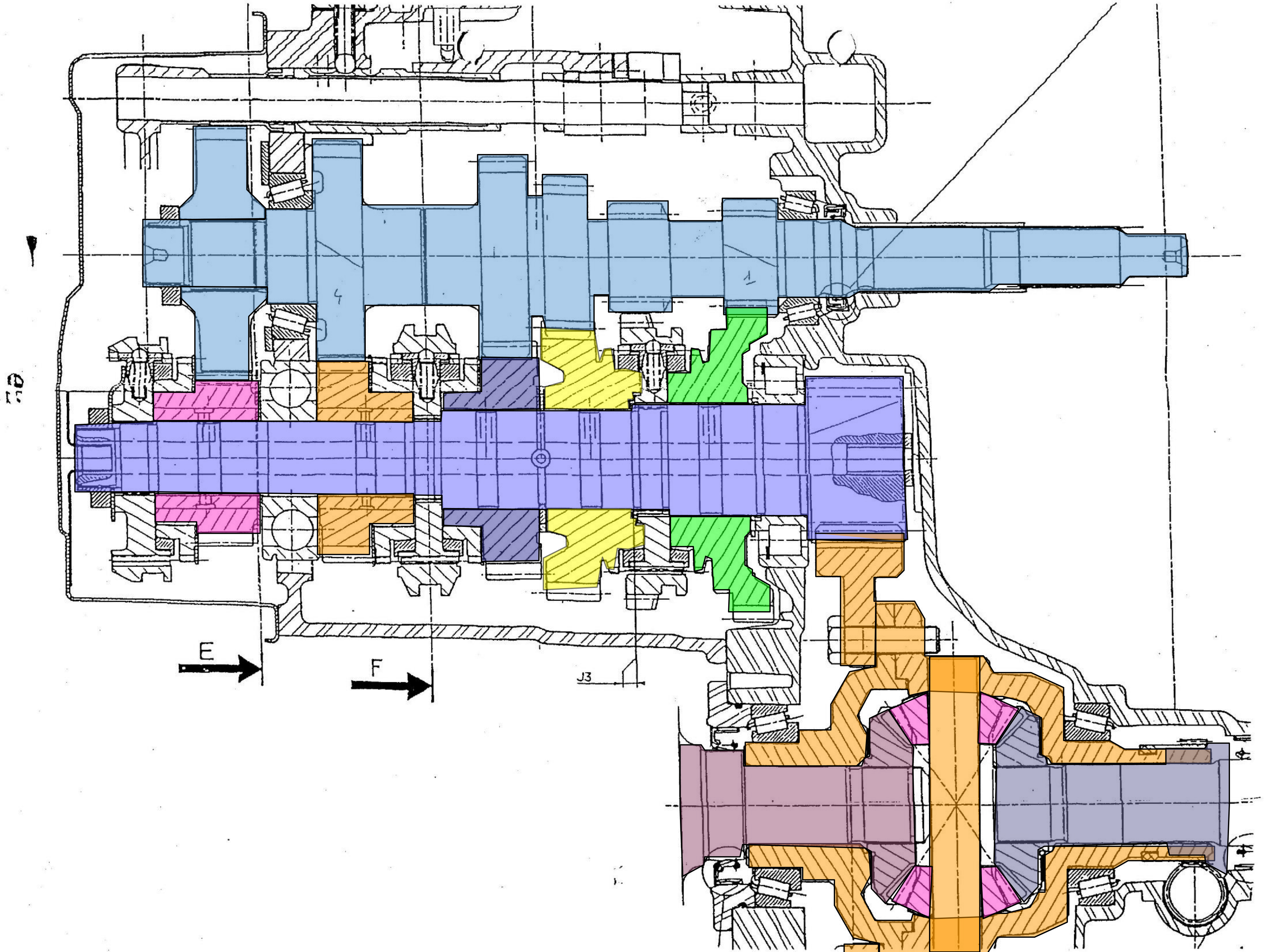
Source PSA

Boite manuelle



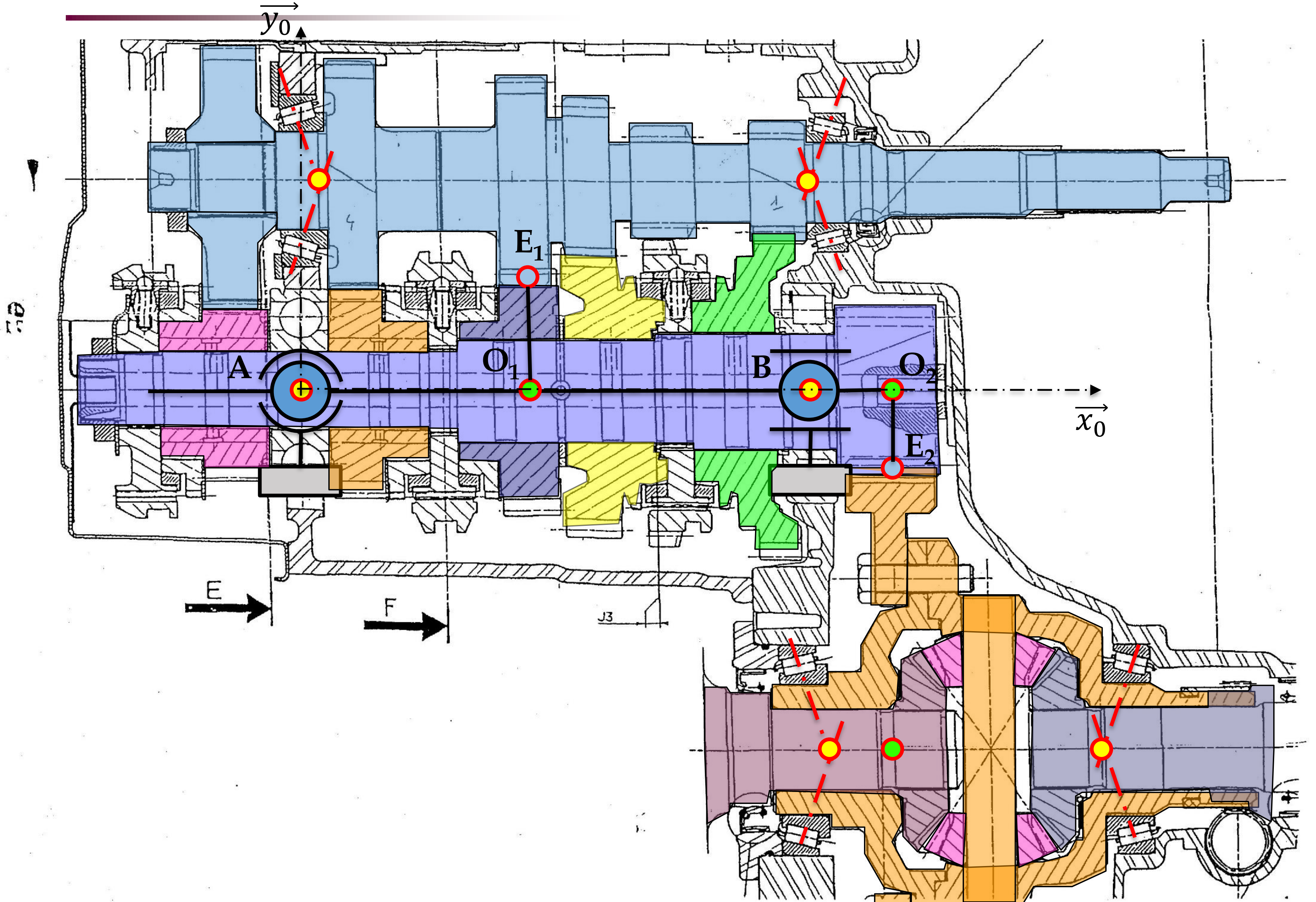

Arbre primaire
Primary shaft
Arbre secondaire
Secondary Shaft
Différentiel
Differential drive

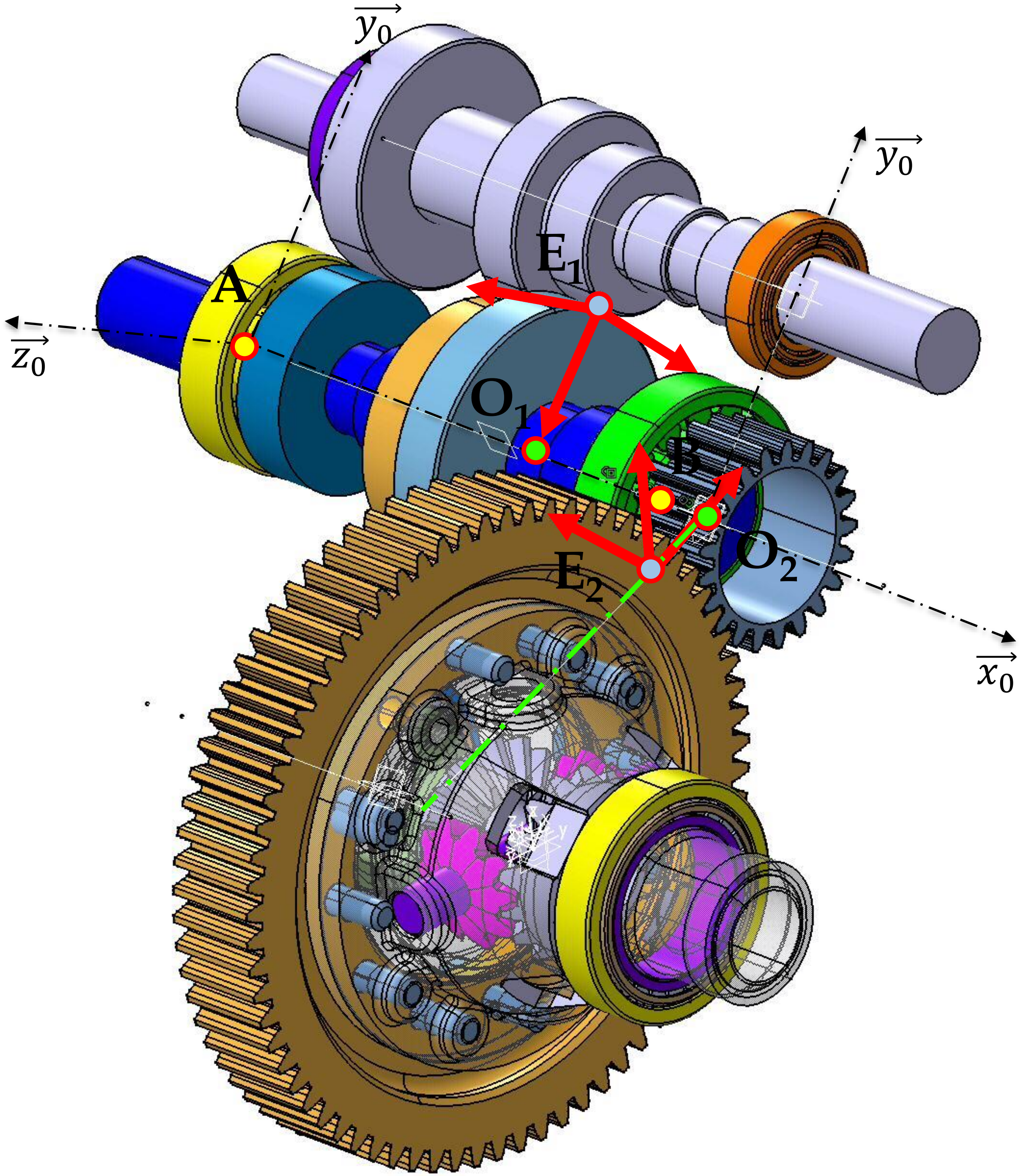

VUE ARRIERE (SANS COUVERCLE)
A
M
B
O
I
R
N
J
A
REP. 1
C
D
G
H



E
F
J3

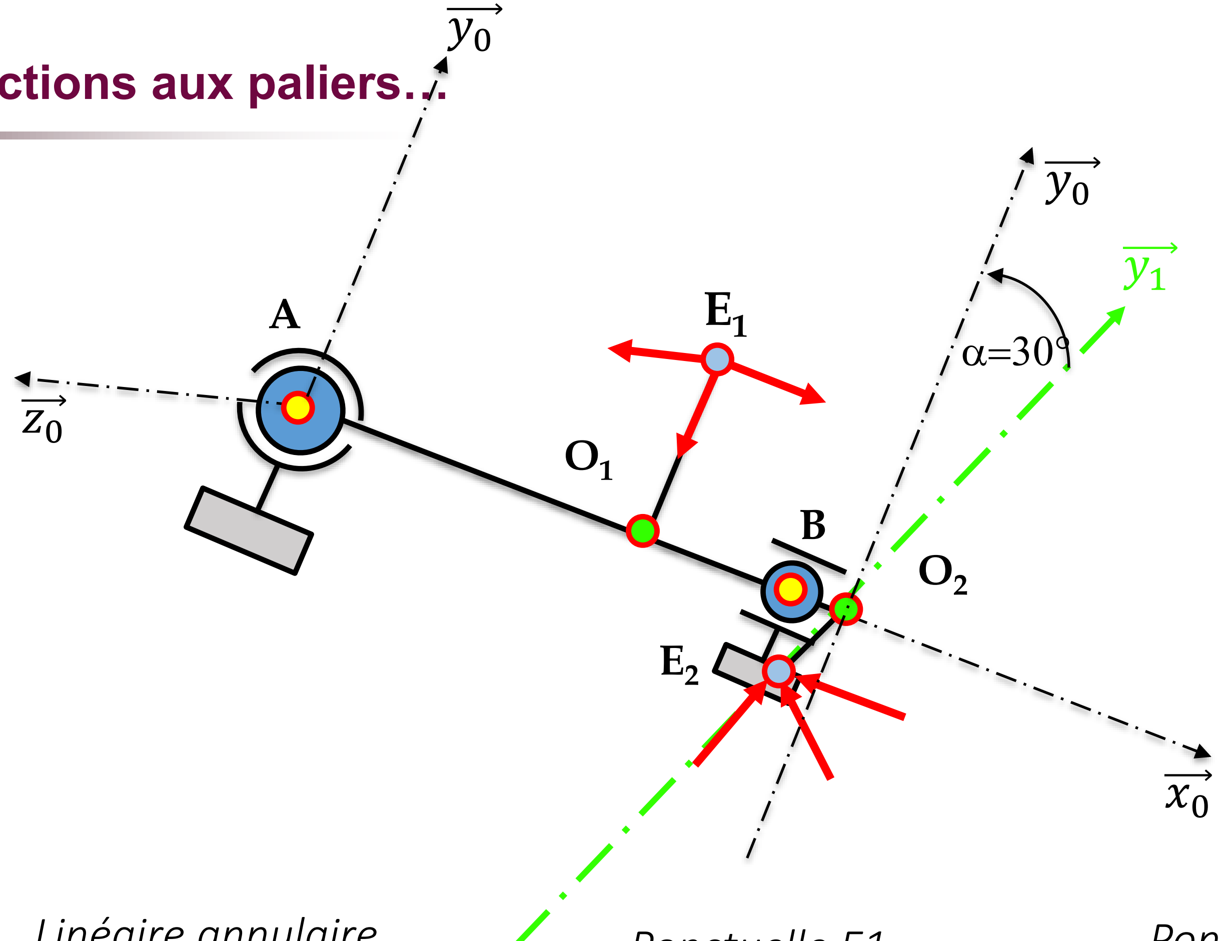
Schématisation: points d'intérêt, axes….





$\overrightarrow{y_0}$
$\overrightarrow{y_0}$
E1
A
$\overrightarrow{z_0}$
O1
B
O2
E2
$\overrightarrow{x_0}$

Calcul des réactions aux paliers…



Rotule en A (Rlt à billes)

$$\overrightarrow{F_A} = \left|\begin{matrix} X_A \\ Y_A \\ Z_A \end{matrix}\right._0$$

$$\overrightarrow{M_A} = \vec{0}$$

Linéaire annulaire (Rlt rlx cyl)

$$\overrightarrow{F_B} = \left|\begin{matrix} 0 \\ Y_B \\ Z_B \end{matrix}\right._0$$

$$\overrightarrow{M_B} = \vec{0}$$

Ponctuelle E1 (Pignon)

$$\overrightarrow{F_1} = \left|\begin{matrix} X_1 = 4330\text{N} \\ Y_1 = -2690\text{N} \\ Z_1 = 7370\text{N} \end{matrix}\right._0$$

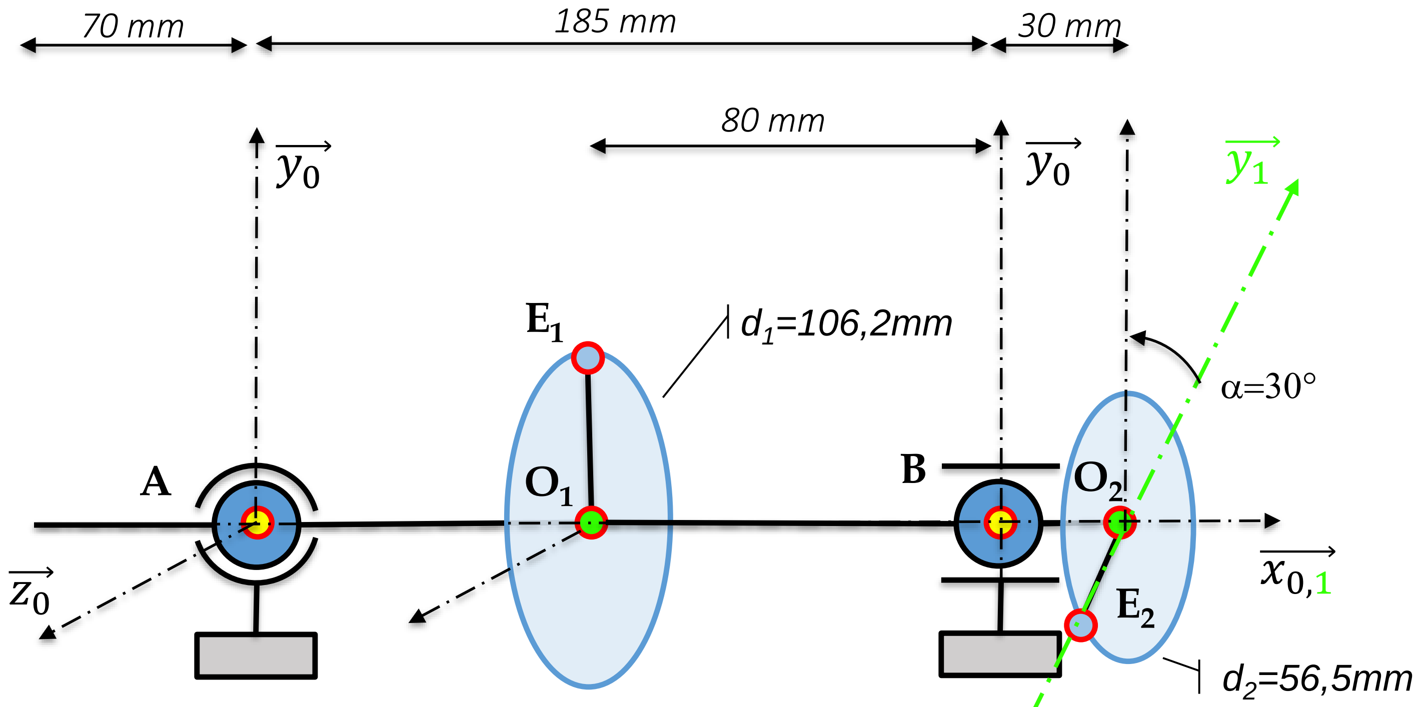
$$\overrightarrow{M_1} = \vec{0}$$

Ponctuelle E2 (Pignon Diff)

$$\overrightarrow{F_2} = \left|\begin{matrix} X_2 = -7060N \\ Y_2 = 5820N \\ Z_2 = 13855N \end{matrix}\right._1$$

$$\overrightarrow{M_2} = \vec{0}$$

Calcul des réactions aux paliers…



$$\overrightarrow{F_A} = \left|\begin{matrix} X_A \\ Y_A \\ Z_A \end{matrix}\right._0 \qquad \overrightarrow{M_A} = \vec{0}$$

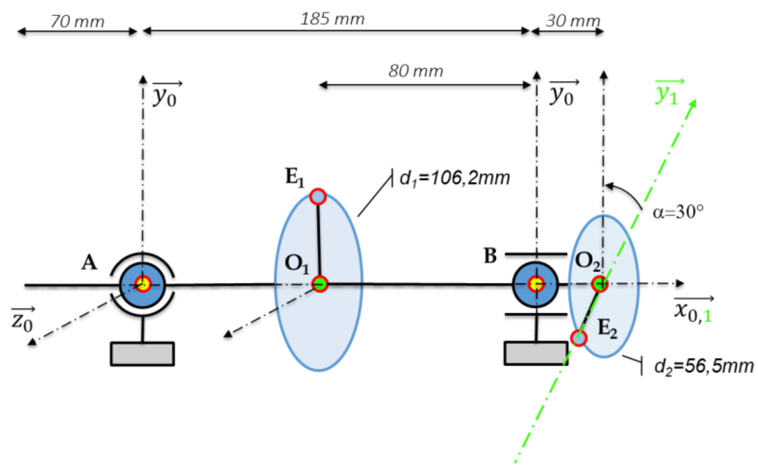
$$\overrightarrow{F_1} = \left|\begin{matrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{matrix}\right._0 \qquad \overrightarrow{M_1} = \vec{0}$$

$$\overrightarrow{F_B} = \left|\begin{matrix} 0 \\ Y_B \\ Z_B \end{matrix}\right._0 \qquad \overrightarrow{M_B} = \vec{0}$$

$$\overrightarrow{F_2} = \left|\begin{matrix} X_2 = -7060N \\ Y_2 = 5820N \\ Z_2 = 13855N \end{matrix}\right._1 \qquad \overrightarrow{M_2} = \vec{0}$$

$$\overrightarrow{F_2} = \left|\begin{matrix} -7060N \\ Y_2 \cos\alpha + Z_2 \sin\alpha = 11968N \\ -Y_2 \sin\alpha + Z_2 \cos\alpha = 9089N \end{matrix}\right.$$

Calcul des réactions aux paliers…



$$\vec{F_1} = \left|\begin{matrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{matrix}\right._0 \qquad \vec{F_2} = \left|\begin{matrix} X_2 = -7060N \\ Y_2 = 5820N \\ Z_2 = 13855N \end{matrix}\right._1$$

$$\vec{M_1} = \vec{0} \qquad \vec{M_2} = \vec{0}$$

$$\left|\begin{matrix} X_A + X_1 + X_2 = 0 \\ Y_A + Y_1 + Y_B + Y_2\cos\alpha + Z_2\sin\alpha = 0 \\ Z_A + Z_1 + Z_B - Y_2\sin\alpha + Z_2\cos\alpha = 0 \end{matrix}\right._0$$

$$\sum \overrightarrow{M_{ext}(A)} = \vec{0} = \overrightarrow{AE_1} \wedge \vec{F_1} + \overrightarrow{AB} \wedge \vec{F_B} + \overrightarrow{AE_2} \wedge \vec{F_2}$$

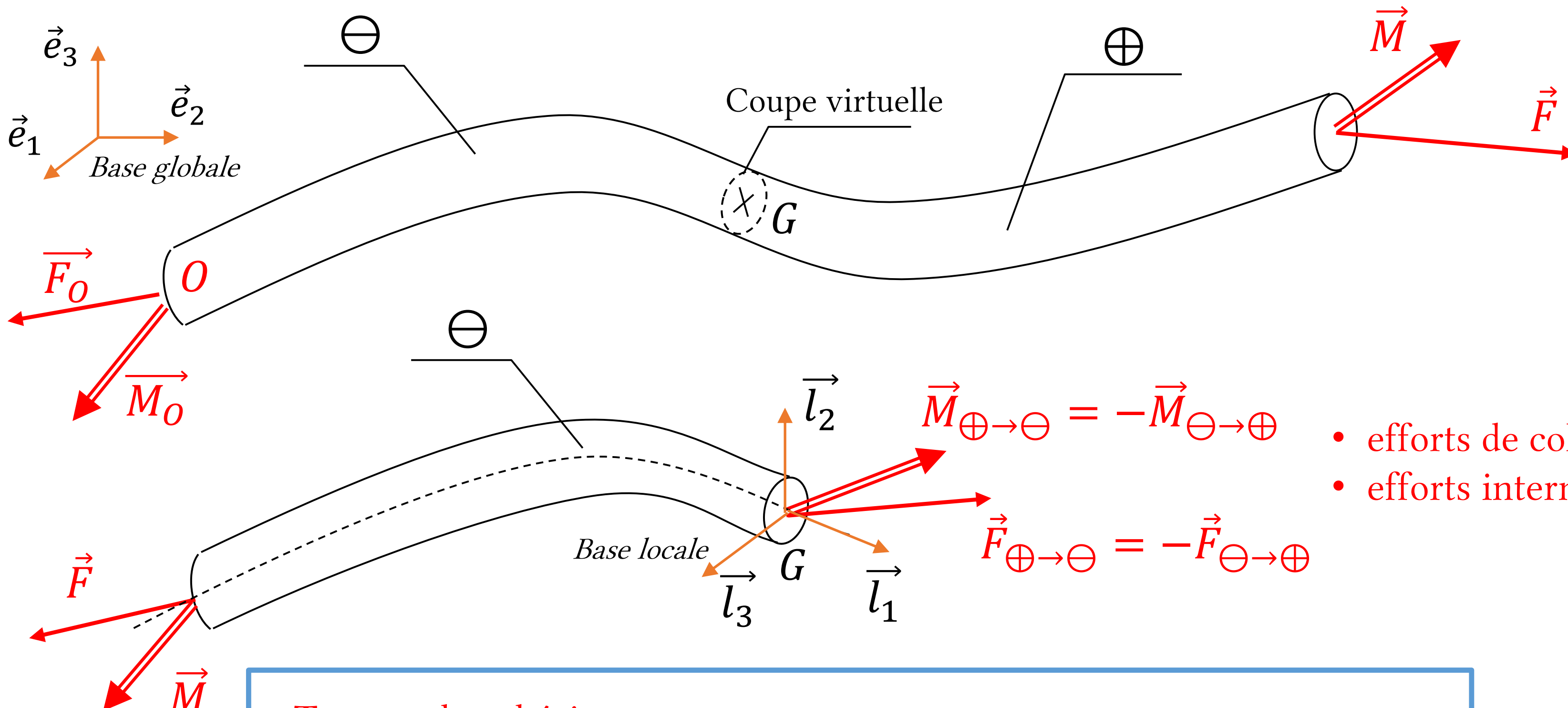
$$\vec{0} = \left|\begin{matrix} 0.105 \\ 0.0531 \\ 0 \end{matrix}\right._0 \wedge \left|\begin{matrix} X_1 \\ Y_1 \\ Z_1 \end{matrix}\right._0 + \left|\begin{matrix} 0.185 \\ 0 \\ 0 \end{matrix}\right._0 \wedge \left|\begin{matrix} 0 \\ Y_B \\ Z_B \end{matrix}\right._0 + \left|\begin{matrix} 0.215 \\ -0.02825\cos\alpha \\ 0.02825\sin\alpha \end{matrix}\right._0 \wedge \left|\begin{matrix} X_2 \\ Y_2\cos\alpha + Z_2\sin\alpha \\ -Y_2\sin\alpha + Z_2\cos\alpha \end{matrix}\right._0$$

$$\left|\begin{matrix} 0.0531 * Z_1 - 0.02825 * Z_2 = 0 \\ Y_B = \frac{1}{0.185}(0.0531 * X_1 - 0.105 * Y_1 - 0.215(Y_2\cos\alpha + Z_2\sin\alpha) - 0.02825 * X_2\cos\alpha) = -10205N \\ Z_B = \frac{1}{0.185}(-0.105 * Z_1 - 0.215(-Y_2\sin\alpha + Z_2\cos\alpha) + 0.02825 * X_2.\sin\alpha) = -15285N \end{matrix}\right._0$$

$$\vec{F_A} = \left|\begin{matrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174N \end{matrix}\right._0 \qquad \vec{F_B} = \left|\begin{matrix} X_B = 0N \\ Y_B = -10205N \\ Z_B = -15285N \end{matrix}\right._0$$

Rappel de cours: convention, efforts internes, Torseur de cohésion

On peut toujours couper un solide virtuellement (= de manière fictive) et regarder ce qui se passe à l'intérieur de la matière…
Dans le cas des poutres on calcule les **efforts de cohésion (ou efforts internes).**



- efforts de cohésion
- efforts internes

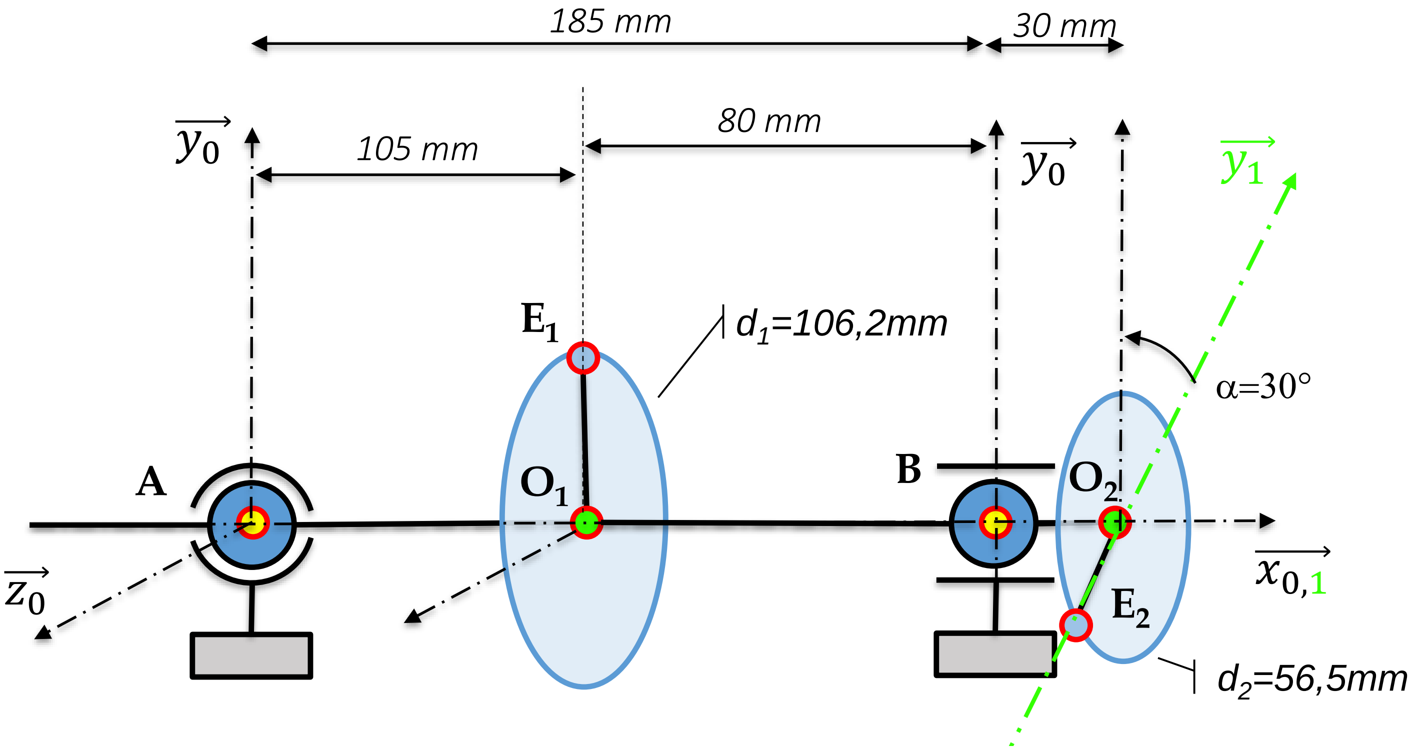
Torseur de cohésion

$$\{\mathcal{T}_{\oplus\to\ominus}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{[G,\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

Base propre des moments quadratiques

N_1 : effort normal
$T_{2,3}$: efforts tranchants
M_1 : moment de torsion
$M_{2,3}$: moments de flexion

Calcul du torseur de cohésion



$$\vec{F_A} = \left|\begin{matrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174N \end{matrix}\right._0 \qquad \vec{F_1} = \left|\begin{matrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{matrix}\right._0 \qquad \vec{F_B} = \left|\begin{matrix} X_B = 0N \\ Y_B = -10205N \\ Z_B = -15285N \end{matrix}\right._0 \qquad \vec{F_2} = \left|\begin{matrix} X_2 = -7060N \\ Y_2 = 11968N \\ Z_2 = 9089N \end{matrix}\right.$$

$$\vec{M_1} = \vec{0}$$

Sur AO_1, O_1B, BO_2

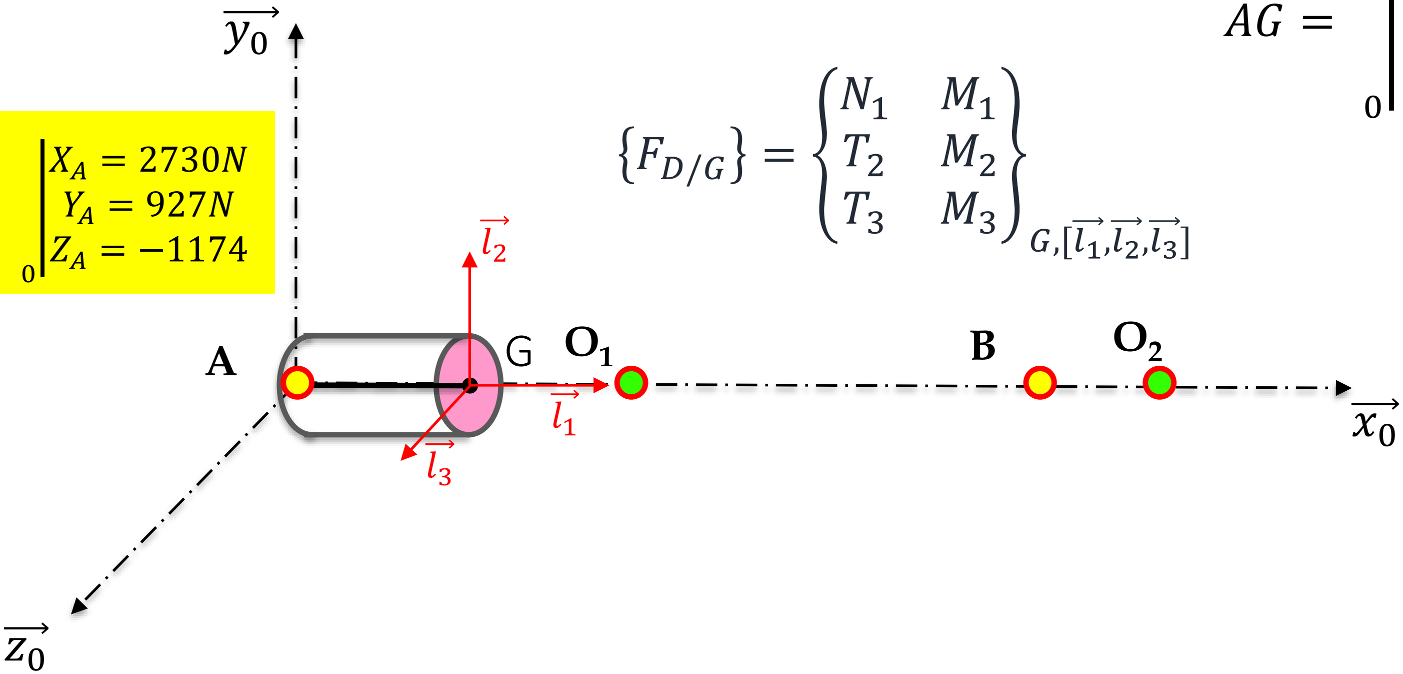
- On garde la partie gauche, PFS en G

$$\overrightarrow{F_A} = \begin{vmatrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{vmatrix}_0$$

$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

$$\overrightarrow{AG} = \begin{vmatrix} x_G \\ 0 \\ 0 \end{vmatrix}_0$$

$x_g \in [0\ 0{,}105]$



Calcul des efforts intérieurs, torseurs de cohésion

$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

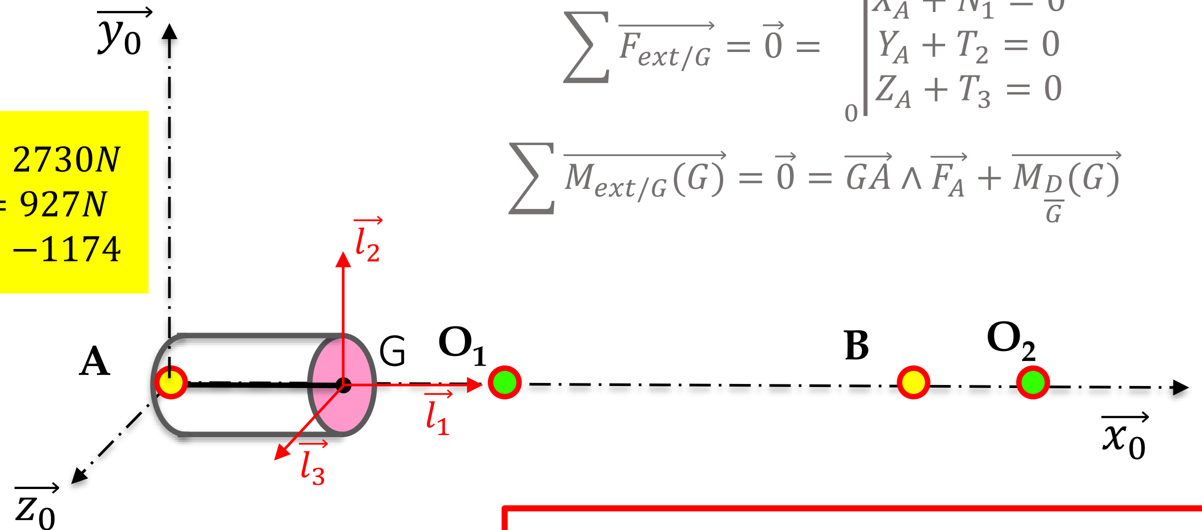
$$\sum \overrightarrow{F_{ext/G}} = \vec{0} = \left|\begin{matrix} X_A + N_1 = 0 \\ Y_A + T_2 = 0 \\ Z_A + T_3 = 0 \end{matrix}\right._0$$

$$\overrightarrow{AG} = \left|\begin{matrix} x_G \\ 0 \\ 0 \end{matrix}\right._0$$

$$\sum \overrightarrow{M_{ext/G}(G)} = \vec{0} = \overrightarrow{GA} \wedge \overrightarrow{F_A} + \overrightarrow{M_{\frac{D}{G}}(G)}$$

$x_g \in [0\ 0{,}105]$

$$\overrightarrow{F_A} = \left|\begin{matrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{matrix}\right._0$$



$$\overrightarrow{R_{D/G\_AO_1^-}} = \left|\begin{matrix} N_1 = -X_A = -2730N \\ T_2 = -Y_A = -927N \\ T_3 = -Z_A = 1174N \end{matrix}\right._0$$

$$\overrightarrow{M_{\frac{D}{G}}(G)} = \left|\begin{matrix} M_1 = 0 \\ M_2 = -x_G.Z_A = x_G.1174\ N.m \\ M_3 = x_G.Y_A = x_G.927\ N.m \end{matrix}\right._0$$

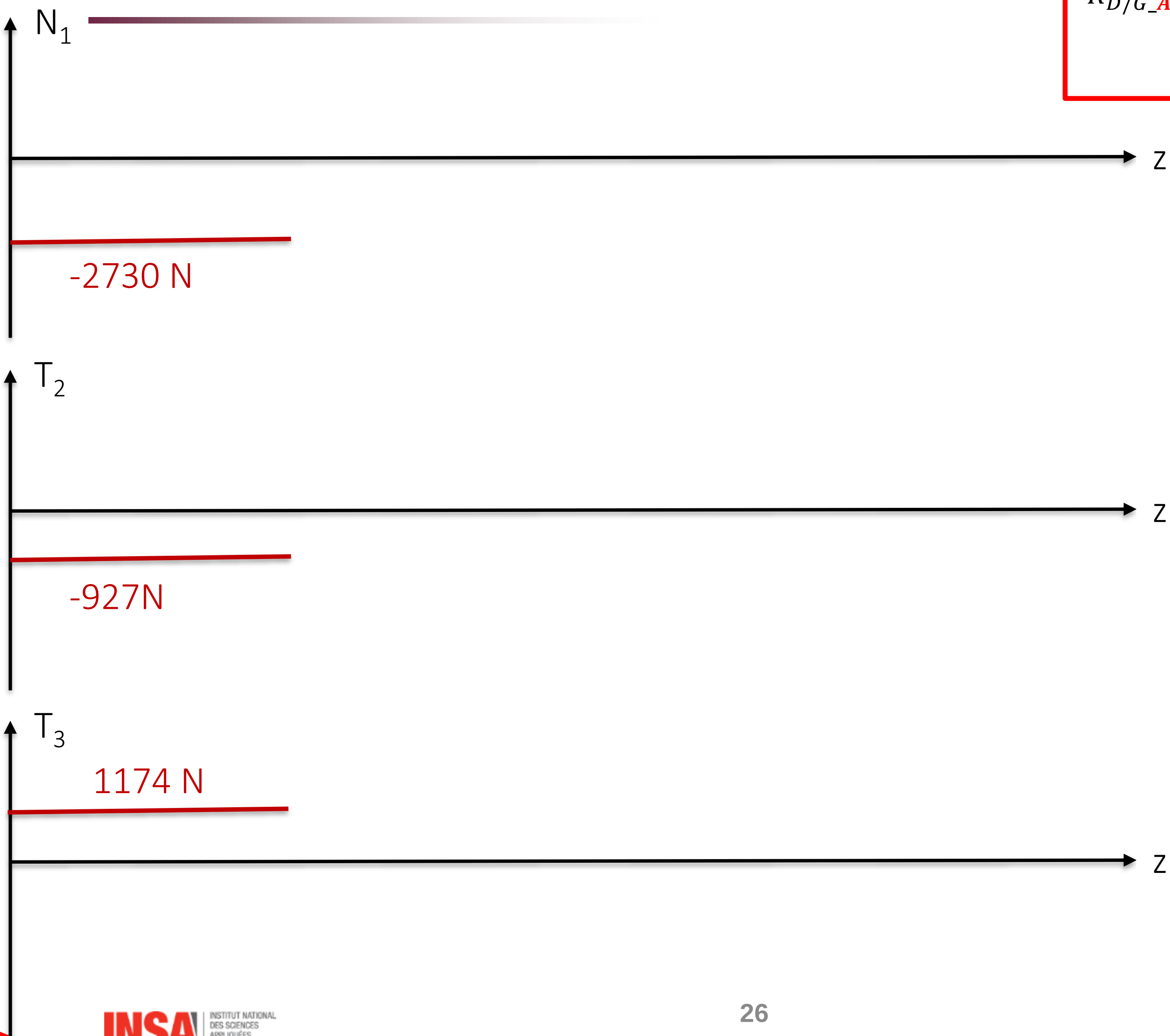
$$\sum \overrightarrow{M_{ext/G}(G)} = \vec{0} = \overrightarrow{GA} \wedge \overrightarrow{F_A} + \overrightarrow{M_{\frac{D}{G}}(G)}$$

$$\vec{0} = \vec{0} + \left|\begin{matrix} -x_G \\ 0 \\ 0 \end{matrix}\right._0 \wedge \left|\begin{matrix} X_A \\ Y_A \\ Z_A \end{matrix}\right._0 + \left|\begin{matrix} M_1 \\ M_2 \\ M_3 \end{matrix}\right._0$$

$$\overrightarrow{M_{\frac{D}{G}}(O_1^-)} = \left|\begin{matrix} M_1 = 0 \\ M_2 = 123\ N.m \\ M_3 = 97N.m \end{matrix}\right._0$$

Evolution du torseur de cohésion

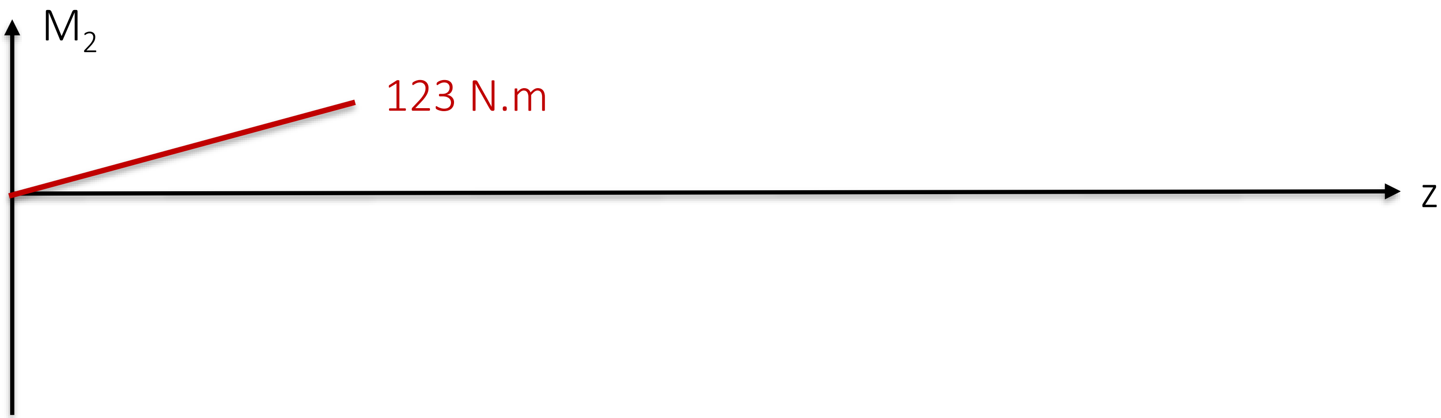
$$\overrightarrow{R_{D/G\_AO_1^-}} = \left|_{0} \begin{matrix} N_1 = -2730N \\ T_2 = -927N \\ T_3 = 1174N \end{matrix}\right.$$



Evolution du torseur de cohésion

$$\overrightarrow{M_{\frac{D}{G}}(G)} = \left|\begin{array}{l} M_1 = 0 \\ M_2 = -x_G.Z_A = x_G.1174\ N.m \\ M_3 = x_G.Y_A = x_G.927\ N.m \end{array}\right._0$$

$$\overrightarrow{M_{\frac{D}{G}}(\boldsymbol{O_1^-})} = \left|\begin{array}{l} M_1 = 0 \\ M_2 = 123\ N.m \\ M_3 = 97 N.m \end{array}\right._0$$



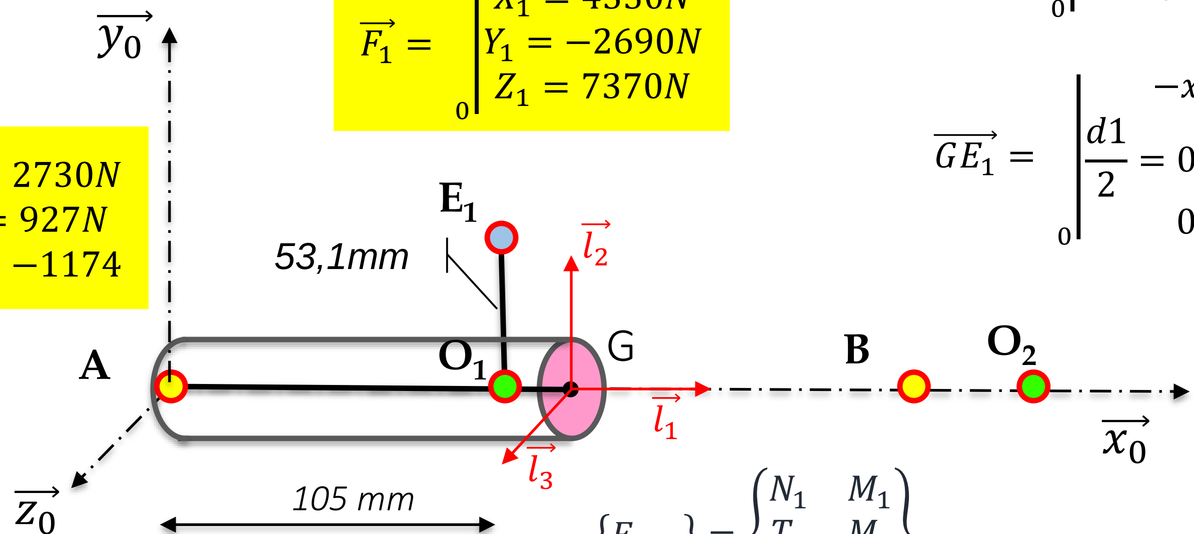
O_1B : Calcul des efforts intérieurs, torseurs de cohésion

$$\overrightarrow{AG} = \begin{vmatrix} 0{,}105 + x_G \\ 0 \\ 0 \end{vmatrix}_0 \qquad x_g \in [0\ 0{,}08]$$

$$\overrightarrow{F_1} = \begin{vmatrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{vmatrix}_0$$

$$\overrightarrow{F_A} = \begin{vmatrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{vmatrix}_0$$

$$\overrightarrow{GE_1} = \begin{vmatrix} -x_G \\ \frac{d1}{2} = 0.0531 \\ 0 \end{vmatrix}_0$$



$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

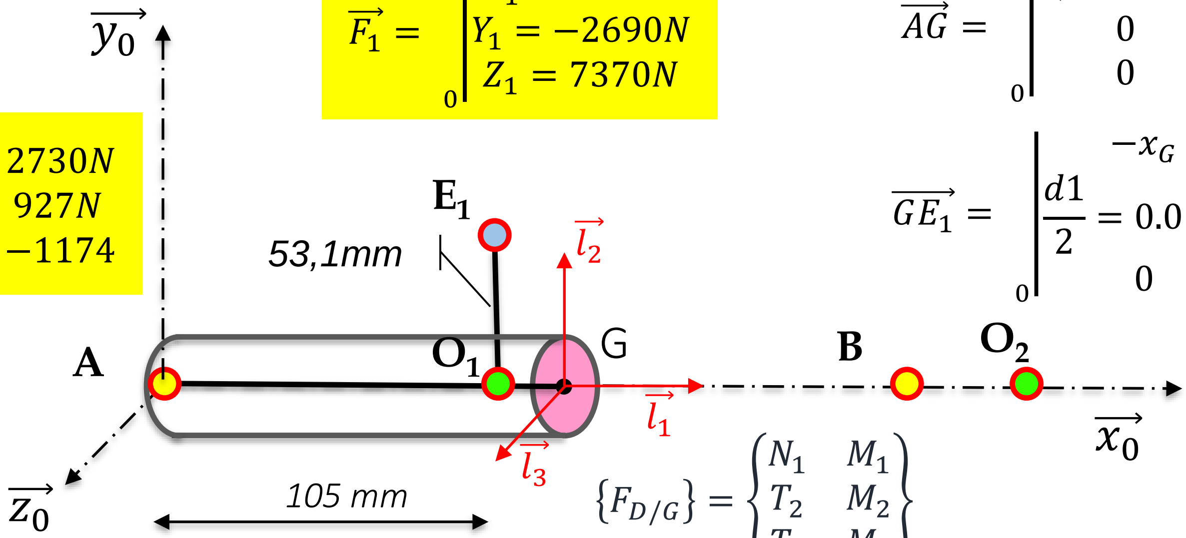
$$\sum \overrightarrow{F_{ext/G}} = \vec{0} = \begin{vmatrix} X_A + X_1 + N_1 = 0 \\ Y_A + Y_1 + T_2 = 0 \\ Z_A + Z_1 + T_3 = 0 \end{vmatrix}_0$$

$$\overrightarrow{R_{D/G\_O_1^+B^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 1763N \\ T_3 = -6196N \end{vmatrix}_0$$

Calcul des efforts intérieurs, torseurs de cohésion

$$\overrightarrow{F_1} = \begin{vmatrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{vmatrix}_0 \qquad \overrightarrow{AG} = \begin{vmatrix} 0{,}105 + x_G \\ 0 \\ 0 \end{vmatrix}_0 \qquad x_g \in [0\ 0{,}08]$$

$$\overrightarrow{F_A} = \begin{vmatrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{vmatrix}_0 \qquad \overrightarrow{GE_1} = \begin{vmatrix} -x_G \\ \dfrac{d1}{2} = 0.0531 \\ 0 \end{vmatrix}_0$$



$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

$$\sum \overrightarrow{M_{\underline{ext}}(G)} = \vec{0} = \overrightarrow{GA} \wedge \overrightarrow{F_A} + \overrightarrow{GE_1} \wedge \overrightarrow{F_1} + \overrightarrow{M_{\frac{D}{G}}(G)}$$

$$\begin{vmatrix} -0.105 - x_G \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 2730 \\ 927 \\ -1174 \end{vmatrix} + \begin{vmatrix} -x_G \\ 0.0531 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 4330 \\ -2690 \\ 7370 \end{vmatrix} + \begin{vmatrix} M_1 \\ M_2 \\ M_3 \end{vmatrix} = \vec{0}$$

$$\begin{vmatrix} 0 \\ -1174.(0.105 + x_G) \\ -927.(0.105 + x_G) \end{vmatrix} + \begin{vmatrix} 391 \\ 7370.x_G \\ 2690.x_G - 230 \end{vmatrix} + \begin{vmatrix} M_1 \\ M_2 \\ M_3 \end{vmatrix} = \vec{0}$$

$$\overrightarrow{M_{\frac{D}{G}}(G)} = \begin{vmatrix} M_1 = -391\ N.m \\ M_2 = -6196.x_G + 123\ N.m \\ M_3 = -1763x_G + 327\ N.m \end{vmatrix}_0$$

$x_G = 0$

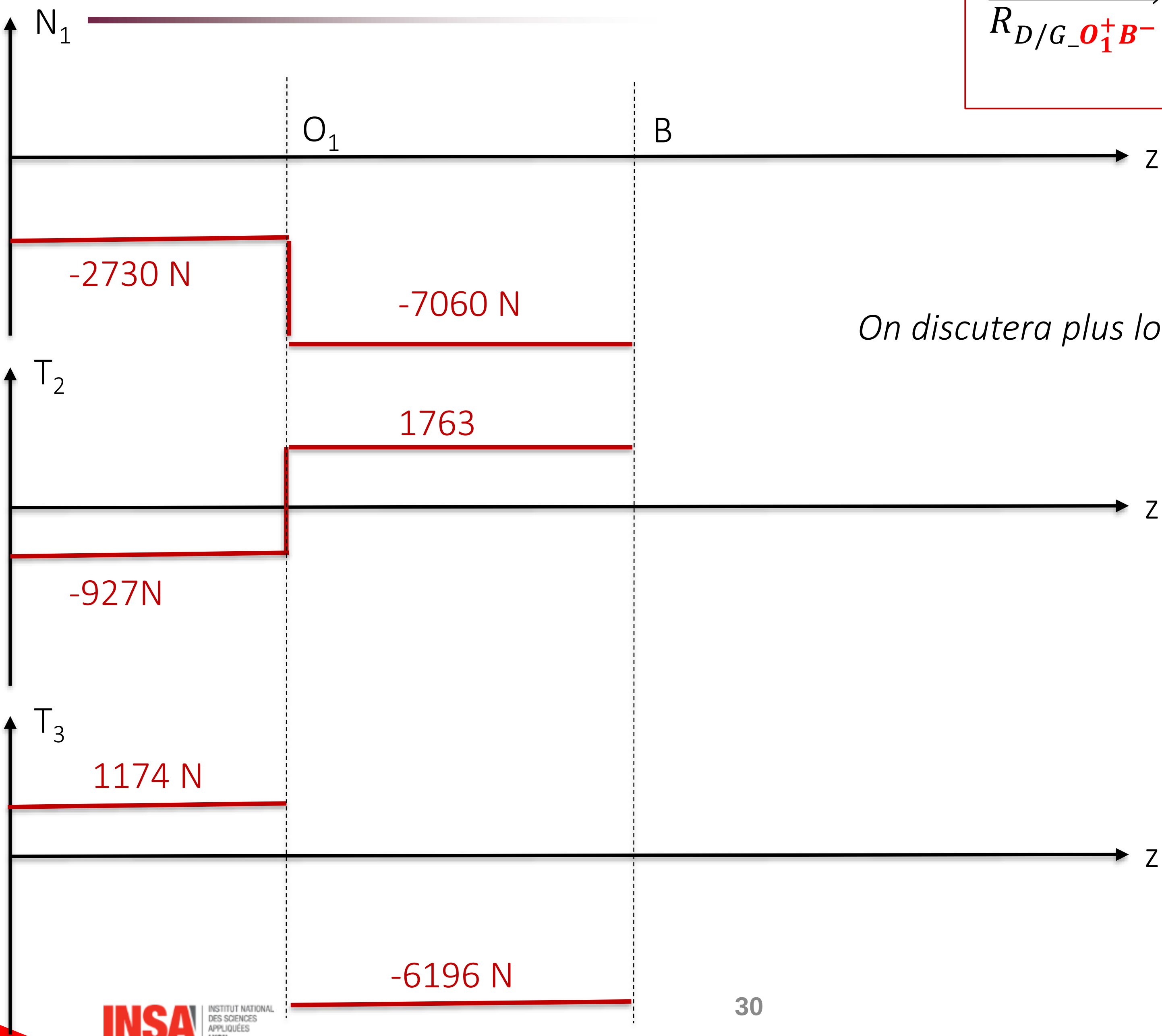
$$\overrightarrow{M_{\frac{D}{G}}(\boldsymbol{O_1^+})} = \begin{vmatrix} M_1 = -391N.m \\ M_2 = 123N.m \\ M_3 = 327N.m \end{vmatrix}_0$$

$x_G = 0{,}08$

$$\overrightarrow{M_{\frac{D}{G}}(\boldsymbol{B})} = \begin{vmatrix} M_1 = -391N.m \\ M_2 = -372\ N.m \\ M_3 = 186N.m \end{vmatrix}_0$$

Evolution du torseur de cohésion

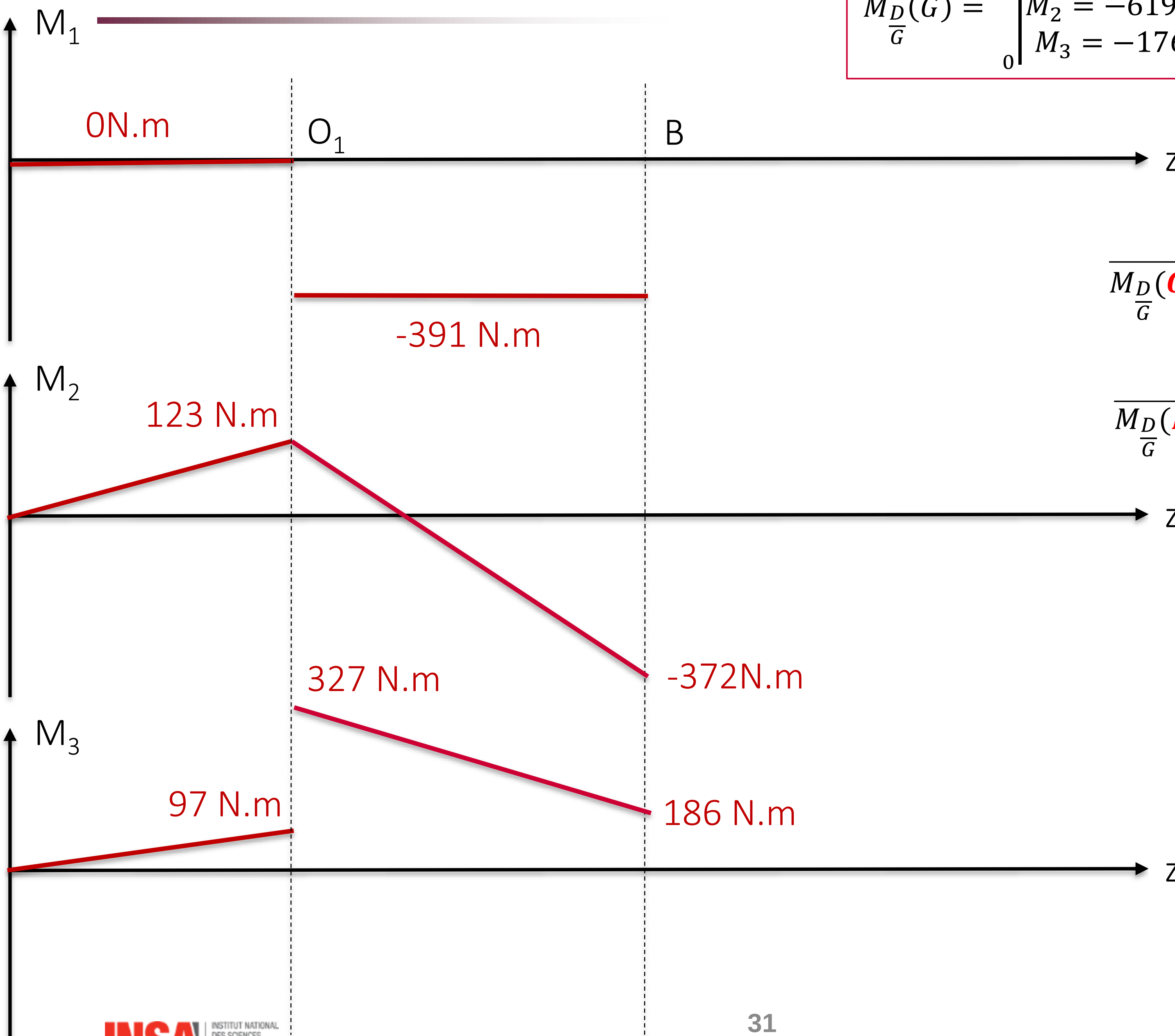
$$\overrightarrow{R_{D/G\_O_1^+B^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 1763N \\ T_3 = -6196N \end{vmatrix}_0$$



On discutera plus loin de la discontinuité

Evolution du torseur de cohésion

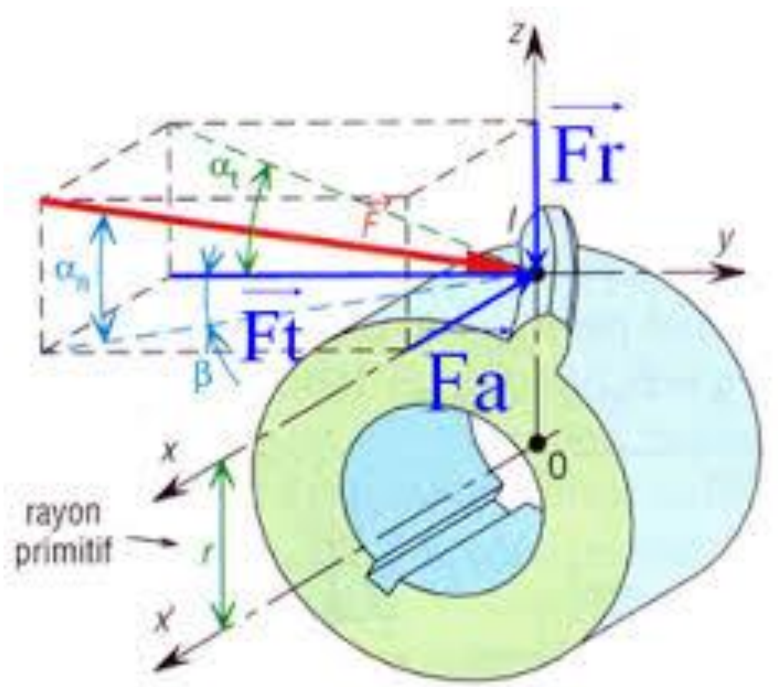
$$\overrightarrow{M_{D\overline{G}}(G)} = \left|\begin{aligned} M_1 &= -0.391\ N.m \\ M_2 &= -6196.x_G + 123\ N.m \\ M_3 &= -1763x_G + 327\ N.m \end{aligned}\right._0$$



$$\overrightarrow{M_{D\overline{G}}(O_1^+)} = \left|\begin{aligned} M_1 &= -391 N.m \\ M_2 &= 123 N.m \\ M_3 &= 327 N.m \end{aligned}\right._0$$

$$\overrightarrow{M_{D\overline{G}}(B)} = \left|\begin{aligned} M_1 &= -391 N.m \\ M_2 &= -372\ N.m \\ M_3 &= 186 N.m \end{aligned}\right._0$$

Discontinuité dans le torseur retour sur la modèle d'engrenage



https://pierreprovot.wordpress.com/les-engrenages/les-engrenages-droits-denture-hlicoidale/

✓Caractéristiques de l'engrenages

- R : Rayon primitif
- α :angle de pression
- β : angle d'hélice

✓Caractéristiques des efforts

- F : effort résultant à la denture
- Ft : effort tangentiel

$$F_t = F.\cos(\alpha).\cos(\beta)$$

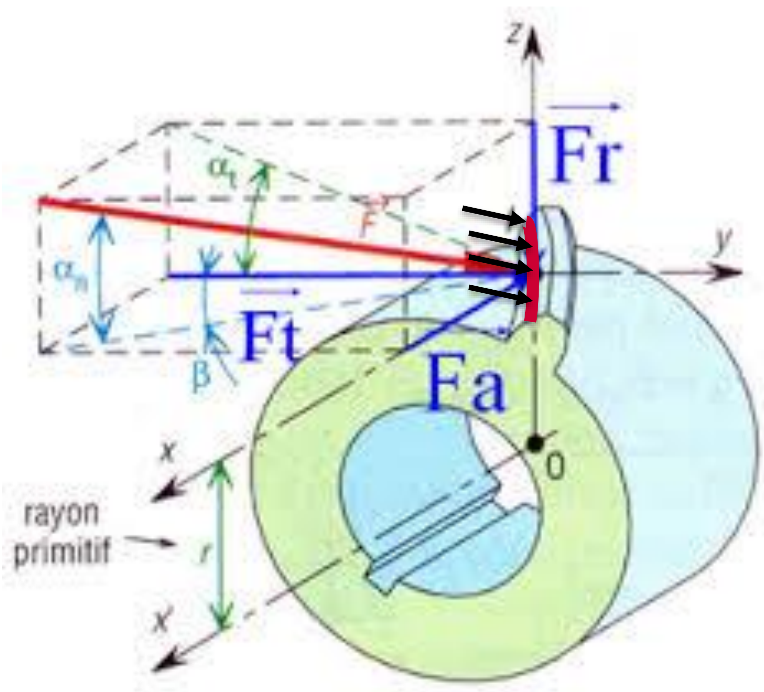
- Fr : effort radial

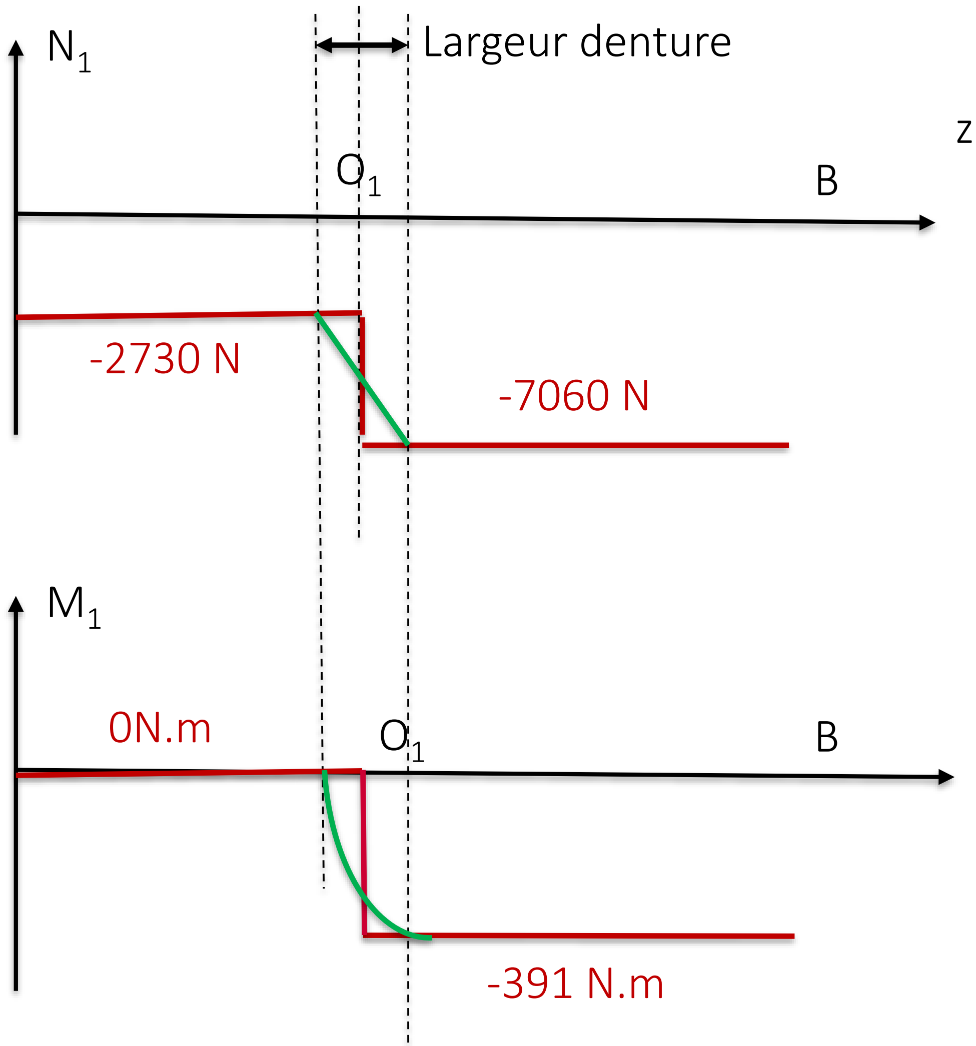
$$F_r = F.\sin(\alpha)$$

- Fa : effort axial

$$F_a = F.\cos(\alpha).\sin(\beta)$$

Discontinuité dans le torseur retour sur la modèle d'engrenage





Dans la réalité les efforts sont répartis le long de la denture et sur plusieurs dents

Rq : La forme réelle dépendra de la répartition des efforts le long de la denture (pas linéaire)

Approche 1

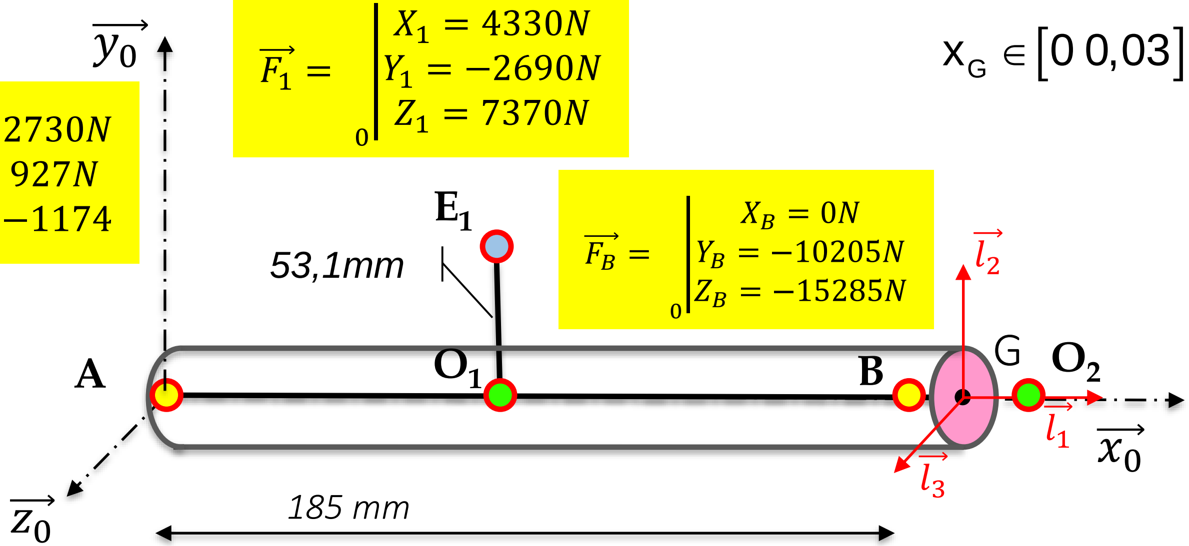
$$\overrightarrow{AG} = \begin{vmatrix} 0{,}185 + x_G \\ 0 \\ 0 \end{vmatrix}_0$$

$$x_G \in [0\ 0{,}03]$$

$$\overrightarrow{F_1} = \begin{vmatrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{vmatrix}_0$$

$$\overrightarrow{F_A} = \begin{vmatrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{vmatrix}_0$$

$$\overrightarrow{F_B} = \begin{vmatrix} X_B = 0N \\ Y_B = -10205N \\ Z_B = -15285N \end{vmatrix}_0$$

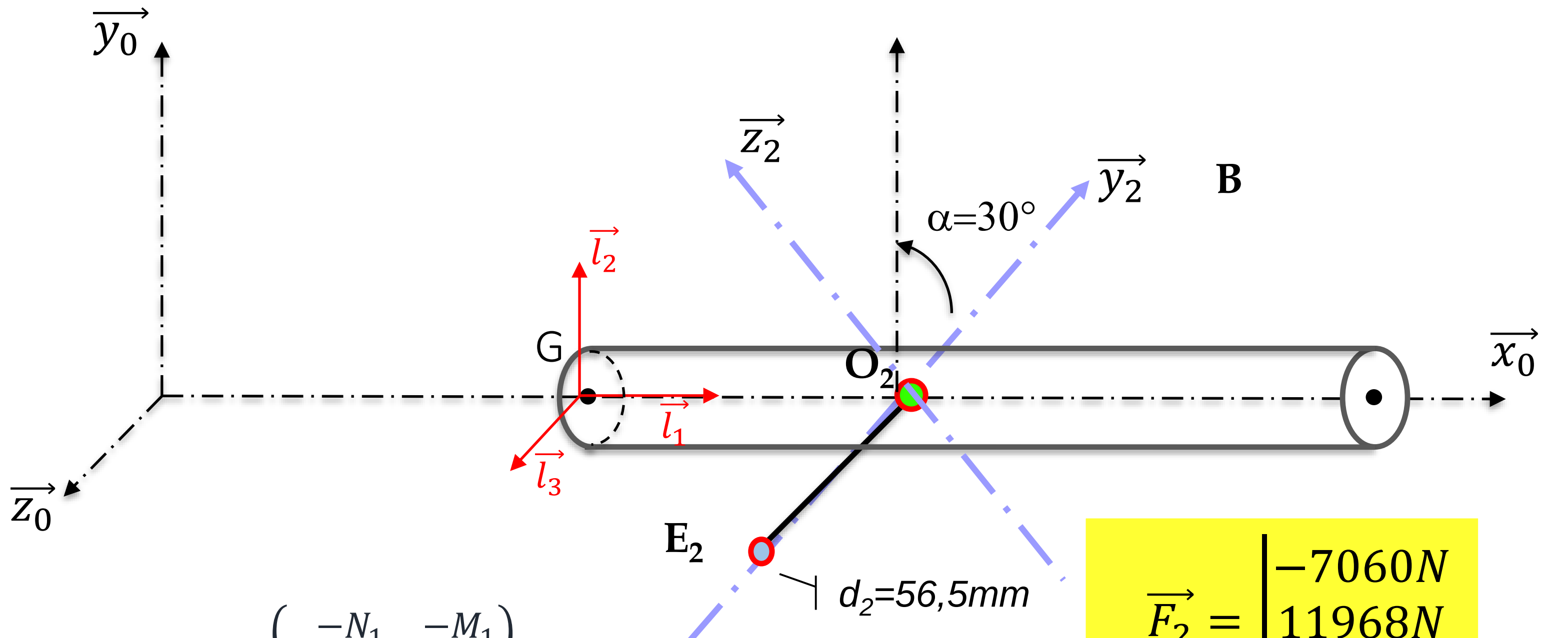


$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

Calcul des efforts intérieurs, torseurs de cohésion

<u>*Approche 2*</u>

$$\overrightarrow{GO_2} = \begin{vmatrix} x_G \\ 0 \\ 0 \end{vmatrix}_0 \qquad x_G \in [0\ 0{,}03]$$



$$\overrightarrow{F_2} = \begin{vmatrix} -7060N \\ 11968N \\ 9089N \end{vmatrix}$$

$$\{F_{G/D}\} = \begin{Bmatrix} -N_1 & -M_1 \\ -T_2 & -M_2 \\ -T_3 & -M_3 \end{Bmatrix}_{G,[\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

Approche 1

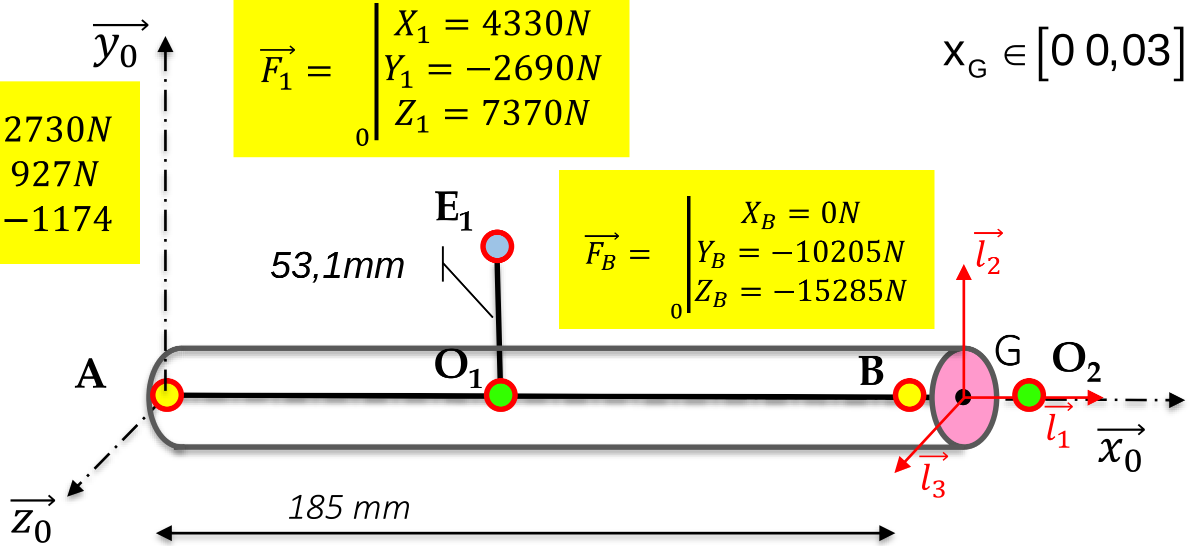
$$\overrightarrow{AG} = \begin{vmatrix} 0{,}185 + x_G \\ 0 \\ 0 \end{vmatrix}_0$$

$$x_G \in [0\ 0{,}03]$$

$$\overrightarrow{F_A} = \begin{vmatrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{vmatrix}_0$$

$$\overrightarrow{F_1} = \begin{vmatrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{vmatrix}_0$$

$$\overrightarrow{F_B} = \begin{vmatrix} X_B = 0N \\ Y_B = -10205N \\ Z_B = -15285N \end{vmatrix}_0$$



$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\overrightarrow{R_{\frac{D}{G_{o_1^+}}}} = \begin{vmatrix} N_1 = -X_A - X_1 - X_B \\ T_2 = -Y_A - Y_1 - Y_B \\ T_3 = -Z_A - Z_1 - Z_B \end{vmatrix}_0$$

⇒

$$\overrightarrow{R_{D/G\_B^+O_2^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 11968N \\ T_3 = 9089N \end{vmatrix}_0$$

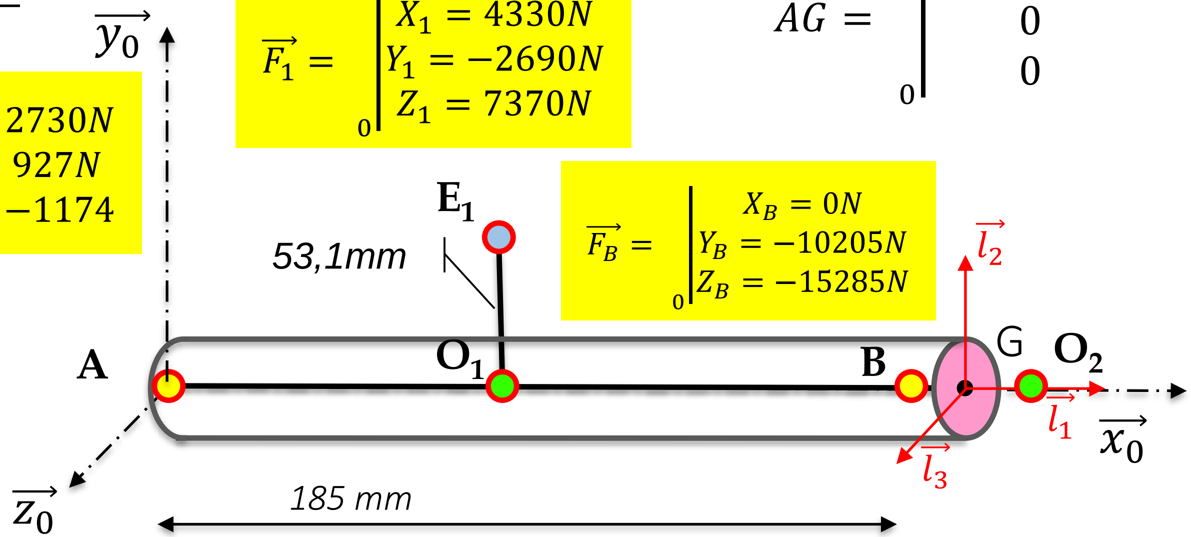
Calcul des efforts intérieurs, torseurs de cohésion

Approche 1

$$\overrightarrow{F_A} = \begin{vmatrix} X_A = 2730N \\ Y_A = 927N \\ Z_A = -1174 \end{vmatrix}_0 \qquad \overrightarrow{F_1} = \begin{vmatrix} X_1 = 4330N \\ Y_1 = -2690N \\ Z_1 = 7370N \end{vmatrix}_0$$

$$\overrightarrow{AG} = \begin{vmatrix} 0{,}185 + x_G \\ 0 \\ 0 \end{vmatrix}_0 \qquad \overrightarrow{GE_1} = \begin{vmatrix} -0{,}08 - x_G \\ 0.0531 \\ 0 \end{vmatrix}_0$$

$$\overrightarrow{F_B} = \begin{vmatrix} X_B = 0N \\ Y_B = -10205N \\ Z_B = -15285N \end{vmatrix}_0 \qquad \overrightarrow{GB} = \begin{vmatrix} -x_G \\ 0 \\ 0 \end{vmatrix}_0$$



$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\sum_G \overrightarrow{M_{ext}(G)} = \vec{0} = \overrightarrow{GA} \wedge \overrightarrow{F_A} + \overrightarrow{GE_1} \wedge \overrightarrow{F_1} + \overrightarrow{GB} \wedge \overrightarrow{F_B} + +\underset{G}{\overrightarrow{M_D(G)}}$$

$$\begin{vmatrix} -0.185 - x_G \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 2730 \\ 927 \\ -1174 \end{vmatrix} + \begin{vmatrix} -0.08 - x_G \\ 0.0531 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 4330 \\ -2690 \\ 7370 \end{vmatrix} + \begin{vmatrix} -x_G \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -10205 \\ -15285 \end{vmatrix} + \begin{vmatrix} M_1 \\ M_2 \\ M_3 \end{vmatrix} = \vec{0}$$

$$\begin{vmatrix} 0 \\ -1174.(0.185 + x_G) \\ -927.(0.185 + x_G) \end{vmatrix} + \begin{vmatrix} 391 \\ 7370.(0.08 + x_G) \\ 2690.(0.08 + x_G) - 230 \end{vmatrix} + \begin{vmatrix} 0 \\ -15285.x_G \\ +10205.x_G \end{vmatrix} + \begin{vmatrix} M_1 \\ M_2 \\ M_3 \end{vmatrix} = \vec{0}$$

$$\Rightarrow \quad \begin{vmatrix} M_1 = -391\,N.m \\ M_2 = 9089.x_G - 372 \\ M_3 = -11968x_G + 186 \end{vmatrix}$$

En B+

$$\begin{vmatrix} M_1 = -391\,N.m \\ M_2 = -372\,N.m \\ M_3 = 186\,N.m \end{vmatrix}$$

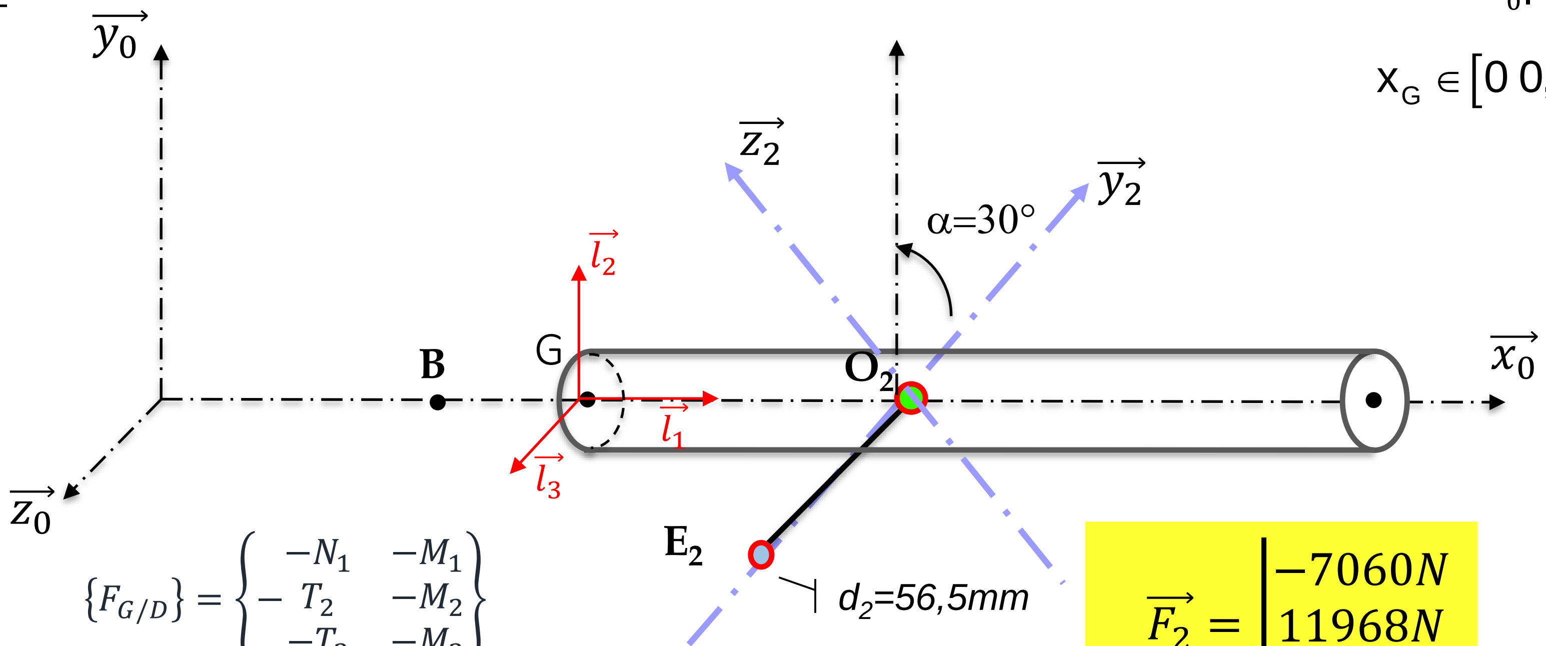
En O2-

$$\begin{vmatrix} M_1 = -391\,N.m \\ M_2 = -100\,N.m \\ M_3 = -173\,N.m \end{vmatrix}$$

Approche 2

$$\overrightarrow{GO_2} = \begin{vmatrix} x_G \\ 0 \\ 0 \end{vmatrix}_0$$

$x_G \in [0\ 0{,}03]$



$$\{F_{G/D}\} = \begin{Bmatrix} -N_1 & -M_1 \\ -T_2 & -M_2 \\ -T_3 & -M_3 \end{Bmatrix}_{G,[\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

$$\overrightarrow{F_2} = \begin{vmatrix} -7060N \\ 11968N \\ 9089N \end{vmatrix}$$

$$\begin{vmatrix} -N_1 - 7060 = 0 \\ -T_2 + 11968 = 0 \\ -T_3 9089 = 0 \end{vmatrix}_0$$

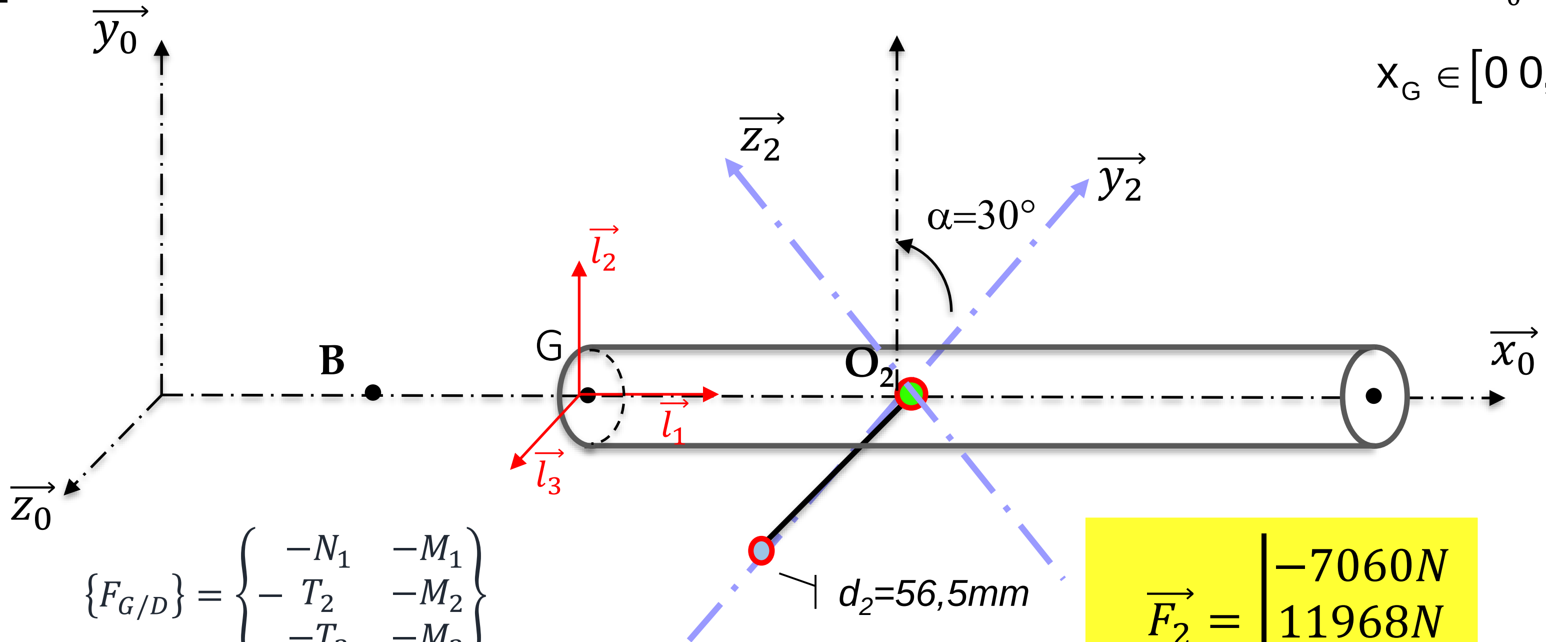
⇨

$$\overrightarrow{R_{D/G\_B^+O_2^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 11968N \\ T_3 = 9089N \end{vmatrix}_0$$

$$\overrightarrow{GO_2} = \begin{vmatrix} x_G \\ 0 \\ 0 \end{vmatrix}_0$$

$$x_G \in [0\ 0{,}03]$$

Approche 2

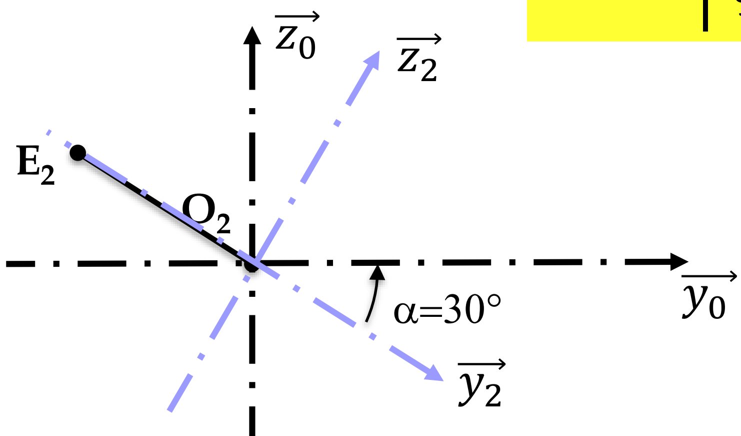


$$\overrightarrow{F_2} = \begin{vmatrix} -7060N \\ 11968N \\ 9089N \end{vmatrix}$$

$$\{F_{G/D}\} = \begin{Bmatrix} -N_1 & -M_1 \\ -T_2 & -M_2 \\ -T_3 & -M_3 \end{Bmatrix}_{G,[\overrightarrow{l_1},\overrightarrow{l_2},\overrightarrow{l_3}]}$$

$$\sum \overrightarrow{M_{\underline{ext}}(G)} = \vec{0} = \overrightarrow{GE_2} \wedge \overrightarrow{F_{E2}} + +\overrightarrow{M_{\frac{G}{D}}(G)}$$

$$\overrightarrow{G_2E_2} = \begin{vmatrix} x_G \\ -\frac{d_2}{2}\cos 30 = -0.0245 \\ \frac{d_2}{2}\sin 30 = 0.0141 \end{vmatrix}_0$$

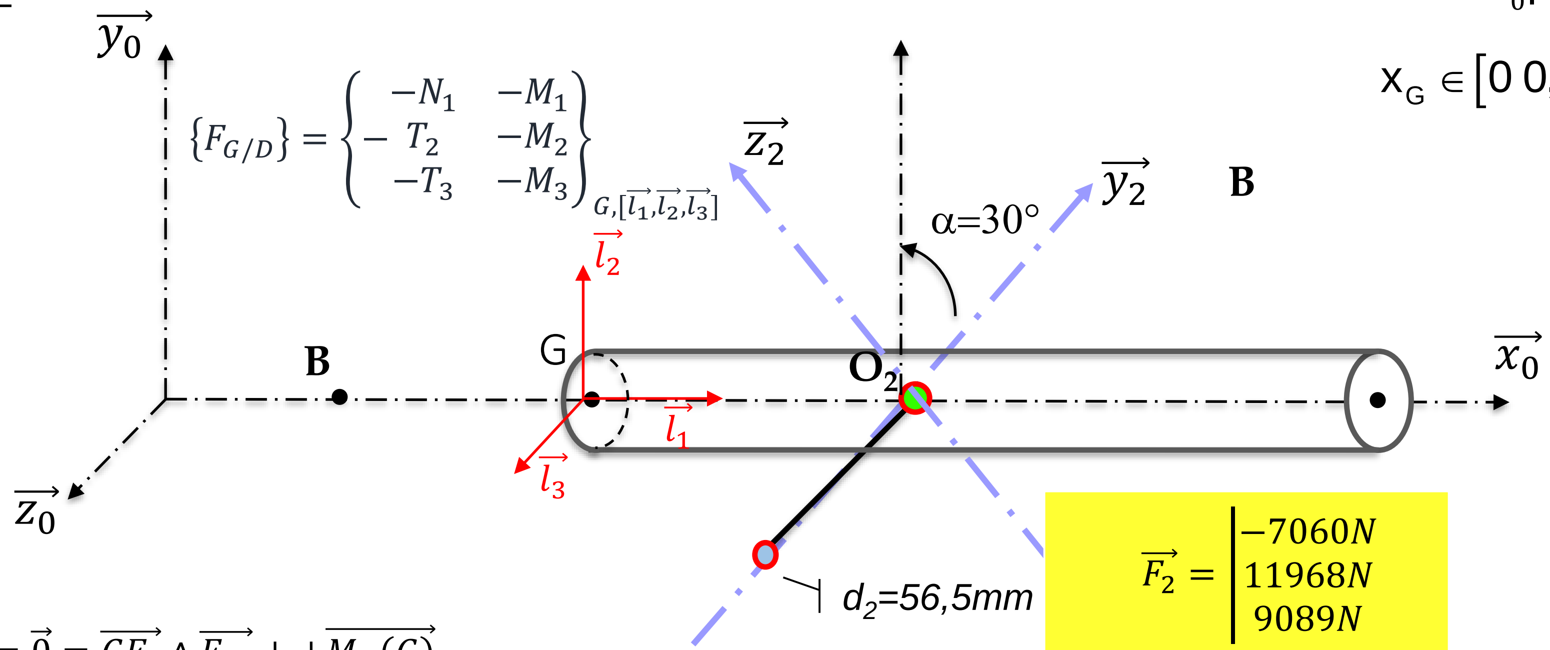


$$\overrightarrow{GO_2} = \begin{vmatrix} x_G \\ 0 \\ 0 \end{vmatrix}_0$$

$x_G \in [0\ 0{,}03]$

Approche 2

$$\{F_{G/D}\} = \begin{Bmatrix} -N_1 & -M_1 \\ -T_2 & -M_2 \\ -T_3 & -M_3 \end{Bmatrix}_{G,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$



$$\overrightarrow{F_2} = \begin{vmatrix} -7060N \\ 11968N \\ 9089N \end{vmatrix}$$

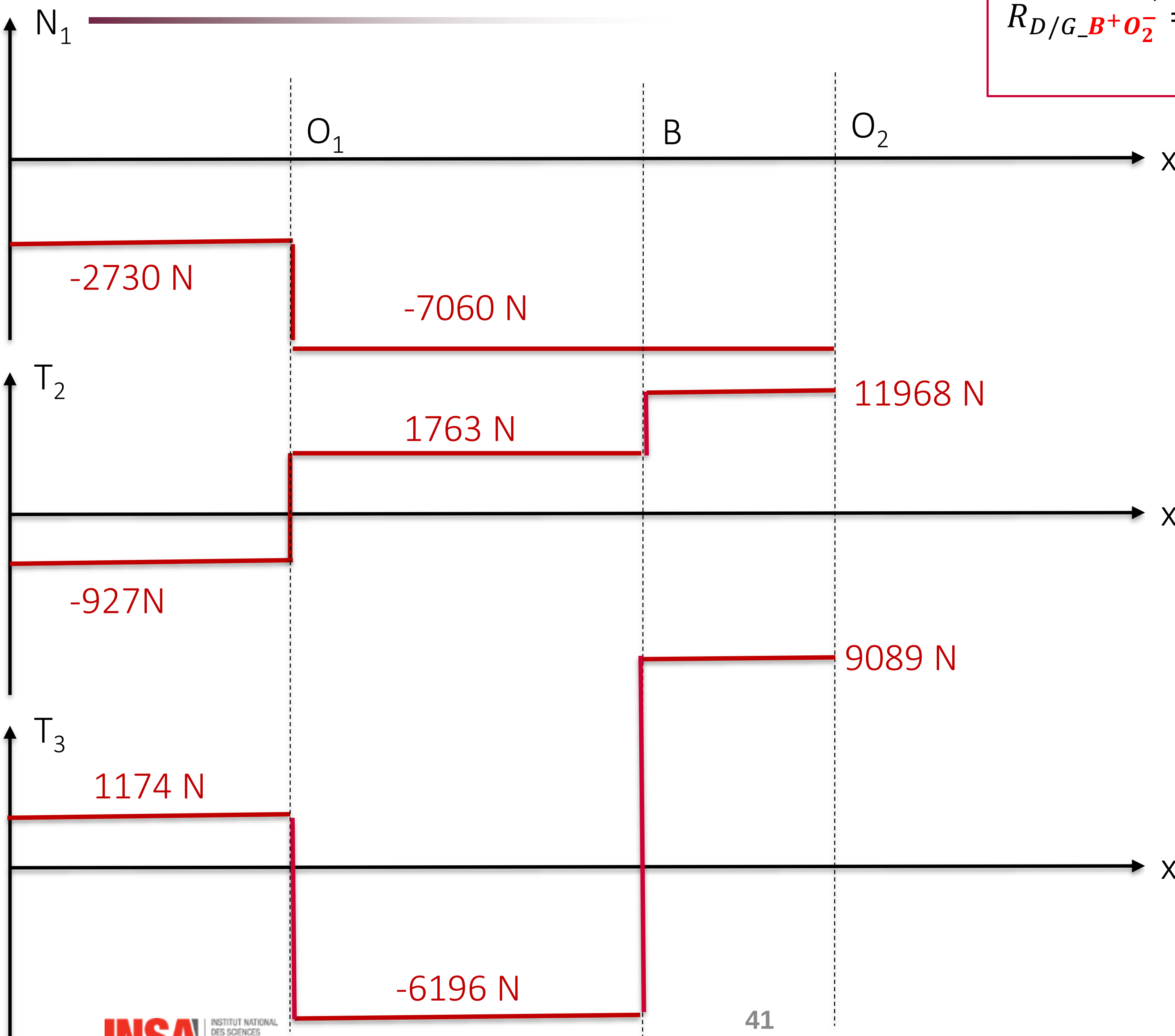
$$\sum \overrightarrow{M_{ext}(G)} = \vec{0} = \overrightarrow{GE_2} \wedge \overrightarrow{F_{E2}} + +\overrightarrow{M_{\frac{G}{D}}(G)}$$

$$\begin{vmatrix} x_G \\ -0.0245 \\ 0.0141 \end{vmatrix} \wedge \begin{vmatrix} -7060 \\ 11968 \\ 9089 \end{vmatrix} + \begin{vmatrix} -M_1 \\ -M_2 \\ -M_3 \end{vmatrix} = \vec{0} \Rightarrow \begin{vmatrix} M_1 = -391\,\text{N.m} \\ M_2 = 9089.x_G - 100 \\ M_3 = -11968x_G - 173 \end{vmatrix}$$

Attention : Le x_G n'est pas défini de la même manière que dans l'approche 1

Evolution du torseur de cohésion

$$\overrightarrow{R_{D/G\_B^+O_2^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 11968N \\ T_3 = 9089N \end{vmatrix}_0$$

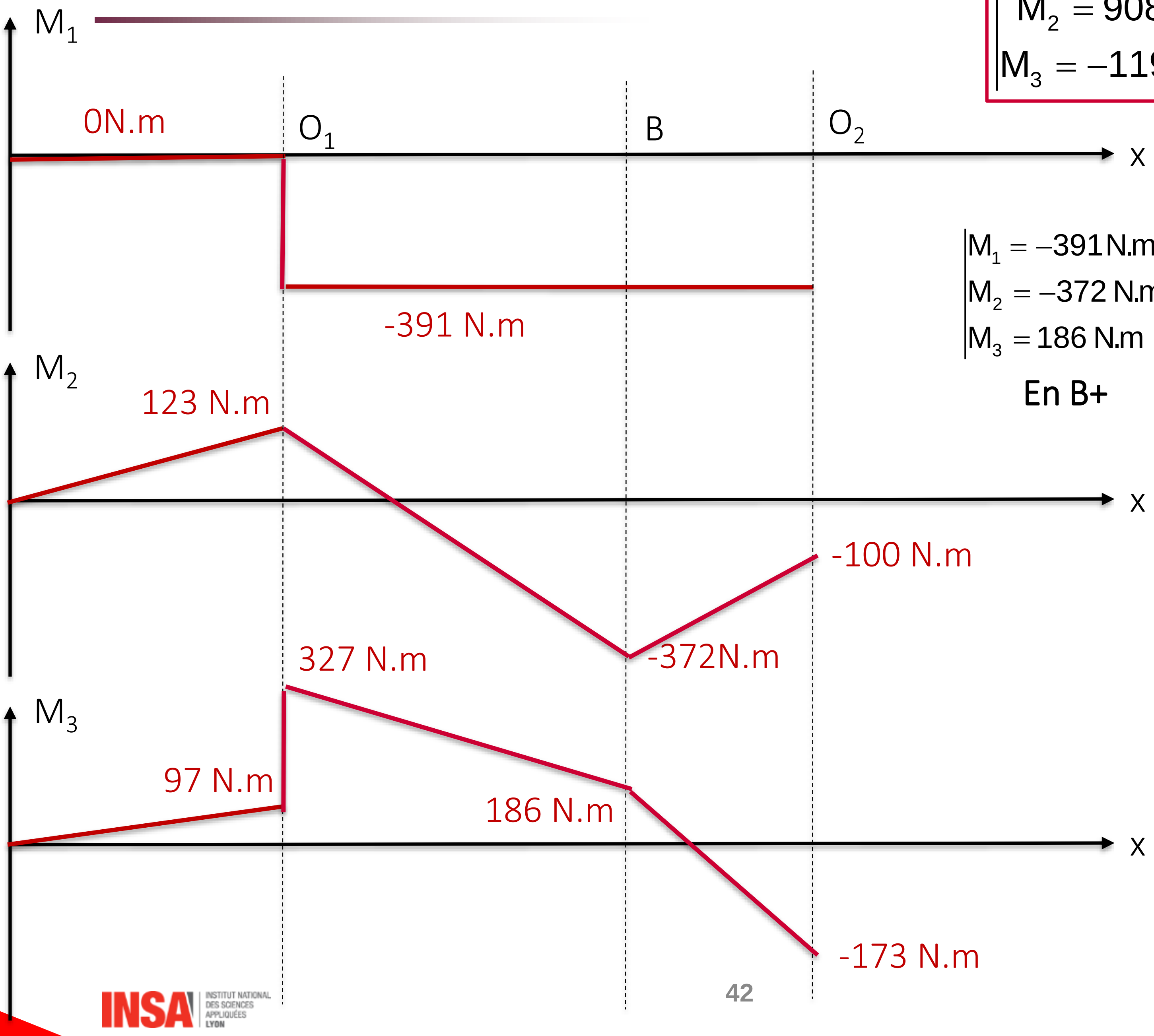


$$\overrightarrow{F_2} = \begin{vmatrix} -7060N \\ 11968N \\ 9089N \end{vmatrix}$$

Rq : On retrouve bien les efforts en E2

Evolution du torseur de cohésion

$$\begin{vmatrix} M_1 = -391\,N.m \\ M_2 = 9089.x_G - 372 \\ M_3 = -11968x_G + 186 \end{vmatrix}$$



$$\begin{vmatrix} M_1 = -391\,N.m \\ M_2 = -372\,N.m \\ M_3 = 186\,N.m \end{vmatrix}$$

En B+

$$\begin{vmatrix} M_1 = -391\,N.m \\ M_2 = -100\,N.m \\ M_3 = -173\,N.m \end{vmatrix}$$

En O2-

Rappel : relations torseur de cohésion/contraintes

Démarche conception

$$\sigma_{11} = \frac{N}{S} + \frac{M_2}{I_{22}} x_3 - \frac{M_3}{I_{33}} x_2 \qquad \sigma_{12} = \frac{T_2}{S} - \frac{M_1}{I_{11}} x_3 \qquad \sigma_{13} = \frac{T_3}{S} + \frac{M_1}{I_{11}} x_2$$

$$\begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{[G,\vec{l_1},\vec{l_2},\vec{l_3}]} \leftrightarrows \sigma_{11}, \sigma_{12}, \sigma_{13}$$

$$N_1 = \int_S \sigma_{11} dS \qquad M_1 = \int_S x_2\sigma_{13} - x_3\sigma_{12}\, dS$$

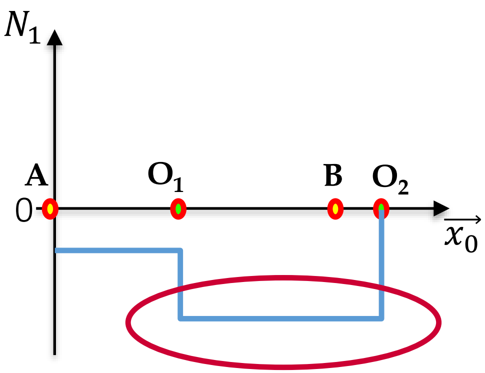
$$T_2 = \int_S \sigma_{12} dS \qquad M_2 = \int_S x_3\sigma_{11}\, dS$$

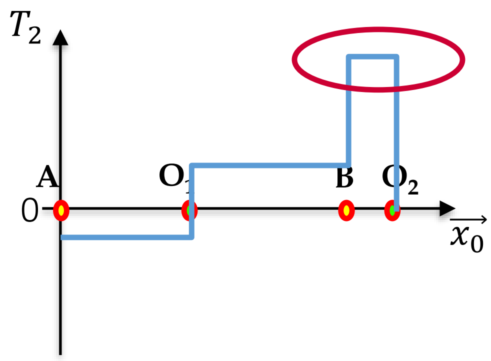
$$T_3 = \int_S \sigma_{13} dS \qquad M_3 = -\int_S x_2\sigma_{11}\, dS$$

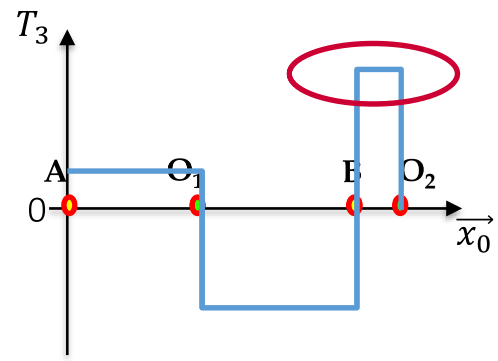
Section critique

$$\{F_{D/G}\} = \begin{Bmatrix} N_1 & M_1 \\ T_2 & M_2 \\ T_3 & M_3 \end{Bmatrix}_{G,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\overrightarrow{R_{D/G\_AO_1^-}} = \begin{vmatrix} N_1 = -2730N \\ T_2 = -927N \\ T_3 = 1174N \end{vmatrix}_0 \qquad \overrightarrow{R_{D/G\_O_1^+B^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 1763N \\ T_3 = -6196N \end{vmatrix}_0 \qquad \overrightarrow{R_{D/G\_B^+O_2^-}} = \begin{vmatrix} N_1 = -7060N \\ T_2 = 11968N \\ T_3 = 9089N \end{vmatrix}_0$$

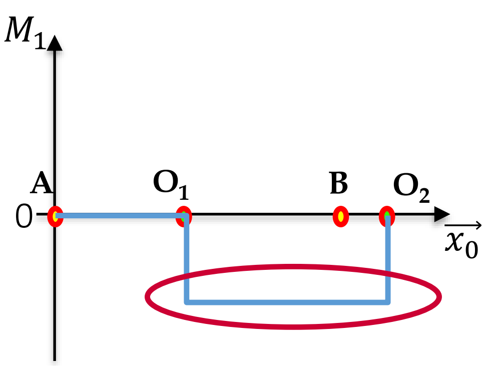


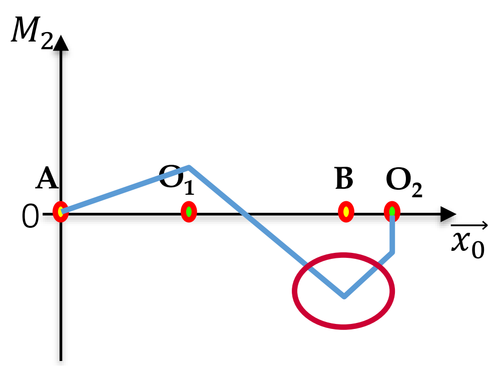


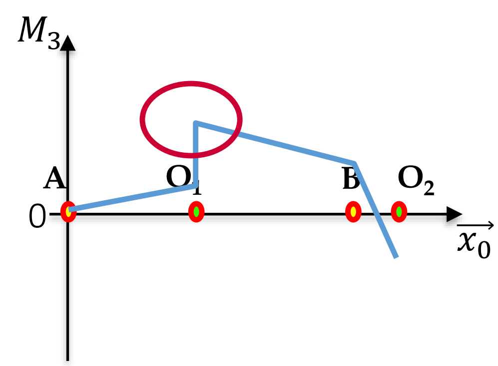


$$\overrightarrow{M_{D/G}(O_1^-)} = \begin{vmatrix} M_1 = 0 \\ M_2 = 123\ N.m \\ M_3 = 97N.m \end{vmatrix}_0 \qquad \overrightarrow{M_{D/G}(O_1^+)} = \begin{vmatrix} M_1 = -391N.m \\ M_2 = 123N.m \\ M_3 = 327N.m \end{vmatrix}_0$$

$$\overrightarrow{M_{D/G}(B)} = \begin{vmatrix} M_1 = -391N.m \\ M_2 = -372\ N.m \\ M_3 = 186N.m \end{vmatrix}_0 \qquad \overrightarrow{M_{D/G}(O_2^-)} = \begin{vmatrix} M_1 = -391N.m \\ M_2 = -100N.m \\ M_3 = -173N.m \end{vmatrix}_0$$







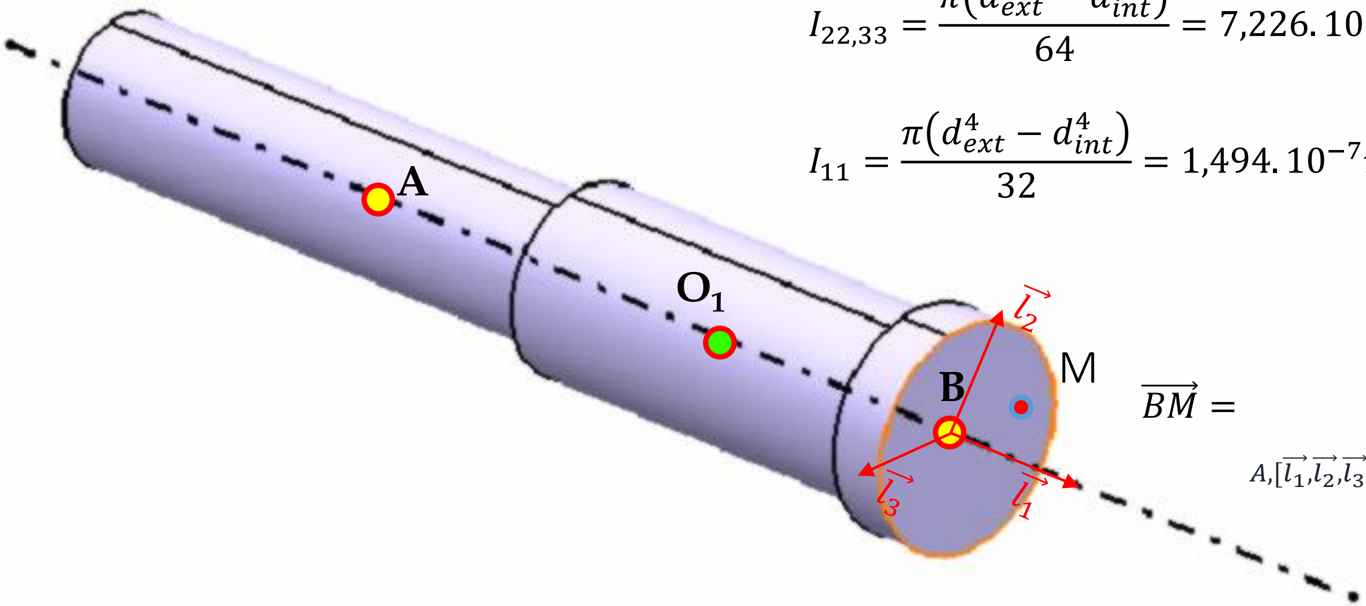
Contraintes dans la section critique B+

$$\{F_{D/G}\} = \begin{Bmatrix} N_1 = -7060N & M_1 = -391N.m \\ T_2 = 11968N & M_2 = -372N.m \\ T_3 = 9089N & M_3 = 186N.m \end{Bmatrix}_{B^+,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$S = \frac{\pi\left(d_{ext}^2 - d_{int}^2\right)}{4} = \frac{\pi(0.035^2 - 0.013^2)}{4} = 8{,}294.10^{-4}m^2$$

$$I_{22,33} = \frac{\pi\left(d_{ext}^4 - d_{int}^4\right)}{64} = 7{,}226.10^{-8}m^4$$

$$I_{11} = \frac{\pi\left(d_{ext}^4 - d_{int}^4\right)}{32} = 1{,}494.10^{-7}m^4$$



$$\overrightarrow{BM} = \begin{vmatrix} 0 \\ x_2 \\ x_3 \end{vmatrix}_{A,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\sigma_{11} = \frac{N}{S} + \frac{M_2}{I_{22}}x_3 - \frac{M_3}{I_{33}}x_2 = \frac{-7060}{8{,}294.10^{-4}} - \frac{372}{7{,}226.10^{-8}}x_3 - \frac{186}{7{,}226.10^{-8}}x_2$$

$$\sigma_{12} = \frac{T_2}{S} - \frac{M_1}{I_{11}}x_3 = \frac{11968}{8{,}294.10^{-4}} + \frac{391}{1{,}494.10^{-7}}x_3$$

$$\sigma_{13} = \frac{T_3}{S} + \frac{M_1}{I_{11}}x_2 = \frac{9089}{8{,}294.10^{-4}} - \frac{391}{1{,}494.10^{-7}}x_2$$

Contraintes dans la section critique B+

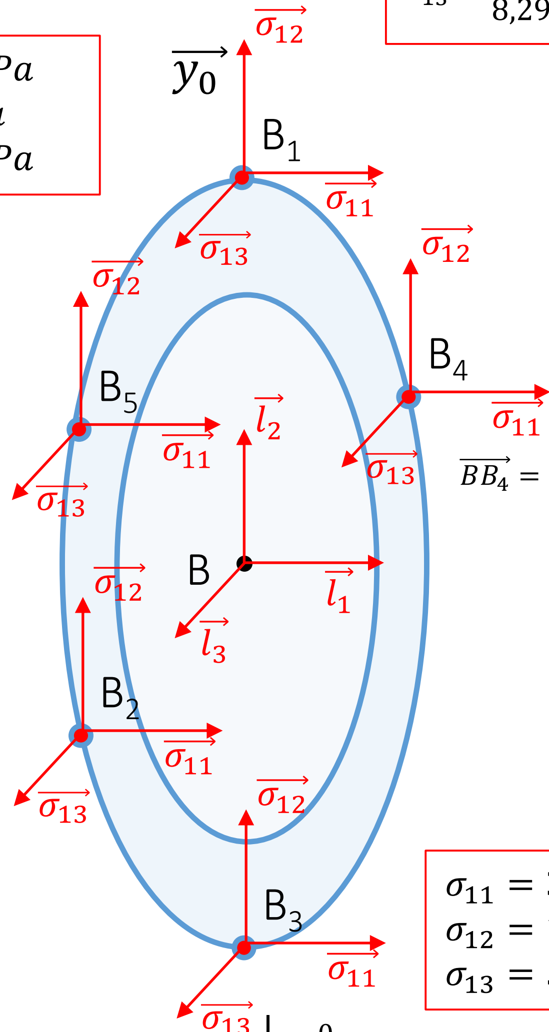
$$\sigma_{11} = \frac{-7060}{8{,}294.10^{-4}} - \frac{372}{7{,}226.10^{-8}}x_3 - \frac{186}{7{,}226.10^{-8}}x_2$$
$$\sigma_{12} = \frac{11968}{8{,}294.10^{-4}} + \frac{391}{1{,}494.10^{-7}}x_3$$
$$\sigma_{13} = \frac{9089}{8{,}294.10^{-4}} - \frac{391}{1{,}494.10^{-7}}x_2$$

$$\overrightarrow{BB_3} = \begin{vmatrix} 0 \\ d_{ext}/2 \\ 0 \end{vmatrix}_{A,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\sigma_{11} = -5{,}36.10^7 Pa$$
$$\sigma_{12} = 1{,}44.10^7 Pa$$
$$\sigma_{13} = -3{,}48.10^7 Pa$$

$$\sigma_{11} = 8{,}16.10^7 Pa$$
$$\sigma_{12} = -3{,}14.10^7 Pa$$
$$\sigma_{13} = 1{,}1.10^7 Pa$$

$$\sigma_{11} = 1.04.10^8 Pa$$
$$\sigma_{12} = 4{,}68.10^7 Pa$$
$$\sigma_{13} = -2{,}14.10^7 Pa$$



$$\overrightarrow{BB_4} = \begin{vmatrix} 0 \\ 0 \\ -d_{ext}/2 \end{vmatrix}_{A,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\overrightarrow{BB_3} = \begin{vmatrix} 0 \\ d_{ext}/(2\sqrt{2}) \\ d_{ext}/(2\sqrt{2}) \end{vmatrix}_{A,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

$$\sigma_{11} = -9{,}86.10^7 Pa$$
$$\sigma_{12} = 6{,}02.10^7 Pa$$
$$\sigma_{13} = 1{,}1.10^7 Pa$$

$$\sigma_{11} = 3{,}65.10^7 Pa$$
$$\sigma_{12} = 1{,}44.10^7 Pa$$
$$\sigma_{13} = 5{,}68.10^7 Pa$$

$$\overrightarrow{BB_2} = \begin{vmatrix} 0 \\ 0 \\ d_{ext}/2 \end{vmatrix}_{A,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

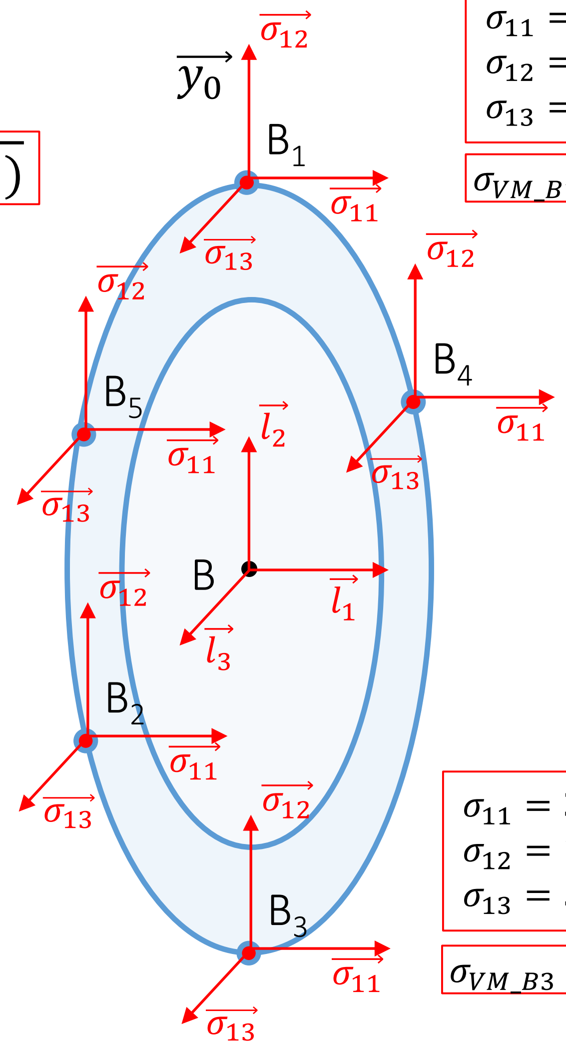
$$\overrightarrow{BB_3} = \begin{vmatrix} 0 \\ -d_{ext}/2 \\ 0 \end{vmatrix}_{A,[\vec{l_1},\vec{l_2},\vec{l_3}]}$$

Contraintes dans la section critique B+, contrainte équivalente

$$\sigma_{VM} = \sqrt{\sigma^2 + 3\tau^2}$$

$\tau = \sqrt{{\sigma_{12}}^2 + {\sigma_{13}}^2}$ $\quad \sigma = \sigma_{11}$

$$\sigma_{VM} = \sqrt{{\sigma_{11}}^2 + 3({\sigma_{12}}^2 + {\sigma_{13}}^2)}$$



$\sigma_{11} = -5{,}36.10^7 Pa$
$\sigma_{12} = 1{,}44.10^7 Pa$
$\sigma_{13} = -3{,}48.10^7 Pa$

$\sigma_{VM\_B1} = 8{,}44.10^7 Pa = 84{,}4 MPa$

$\sigma_{11} = 8{,}16.10^7 Pa$
$\sigma_{12} = -3{,}14.10^7 Pa$
$\sigma_{13} = 1{,}1.10^7 Pa$

$\sigma_{VM\_B4} = 9{,}99.10^7 Pa = 99{,}9 MPa$

$\sigma_{11} = 1.04.10^8 Pa$
$\sigma_{12} = 4{,}68.10^7 Pa$
$\sigma_{13} = -2{,}14.10^7 Pa$

$\sigma_{VM\_B5} = 1.37.10^8 Pa = 137 MPa$

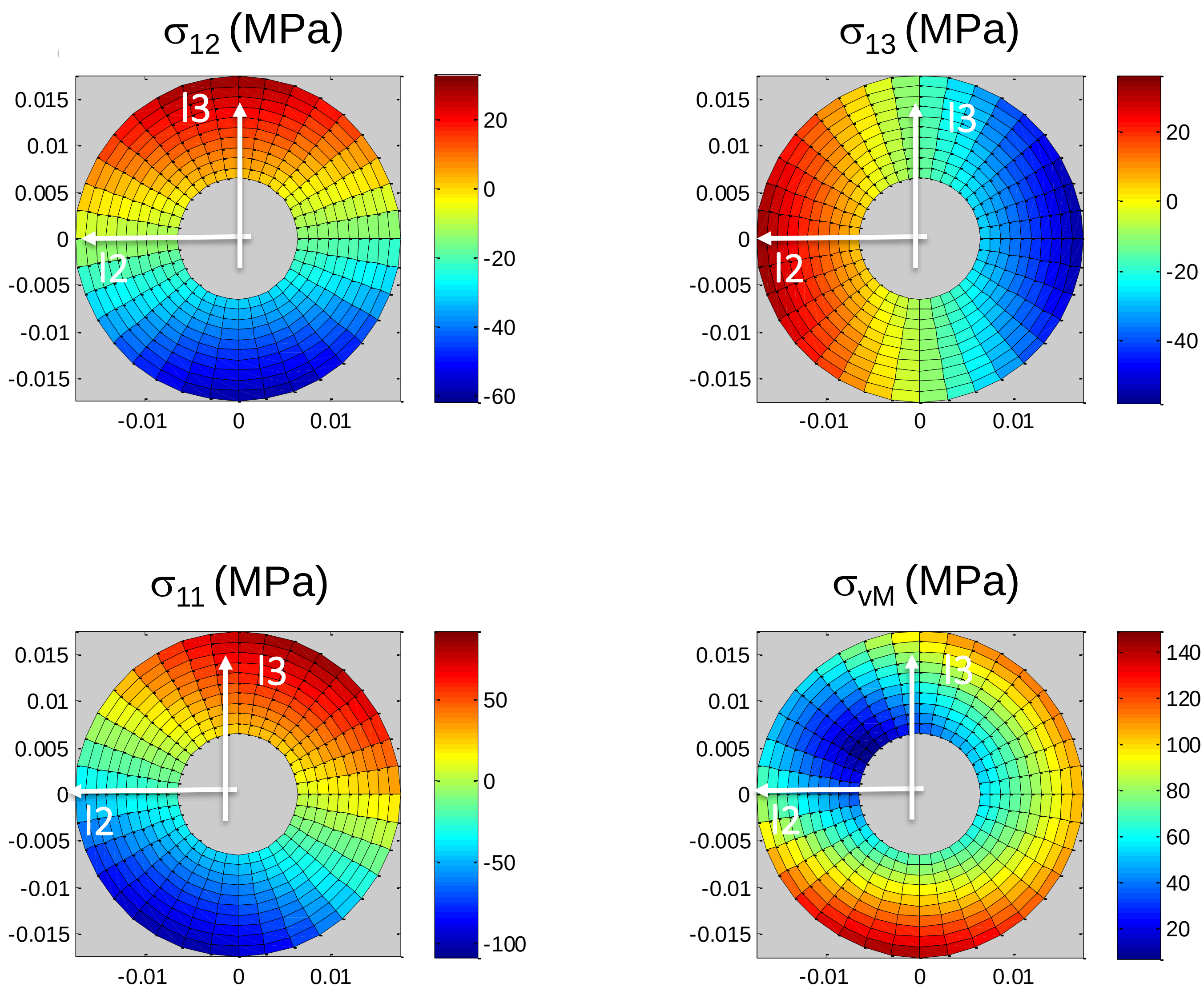
$\sigma_{11} = -9{,}86.10^7 Pa$
$\sigma_{12} = 6{,}02.10^7 Pa$
$\sigma_{13} = 1{,}1.10^7 Pa$

$\sigma_{VM\_B2} = 1{,}45.10^8 Pa = 145 MPa$

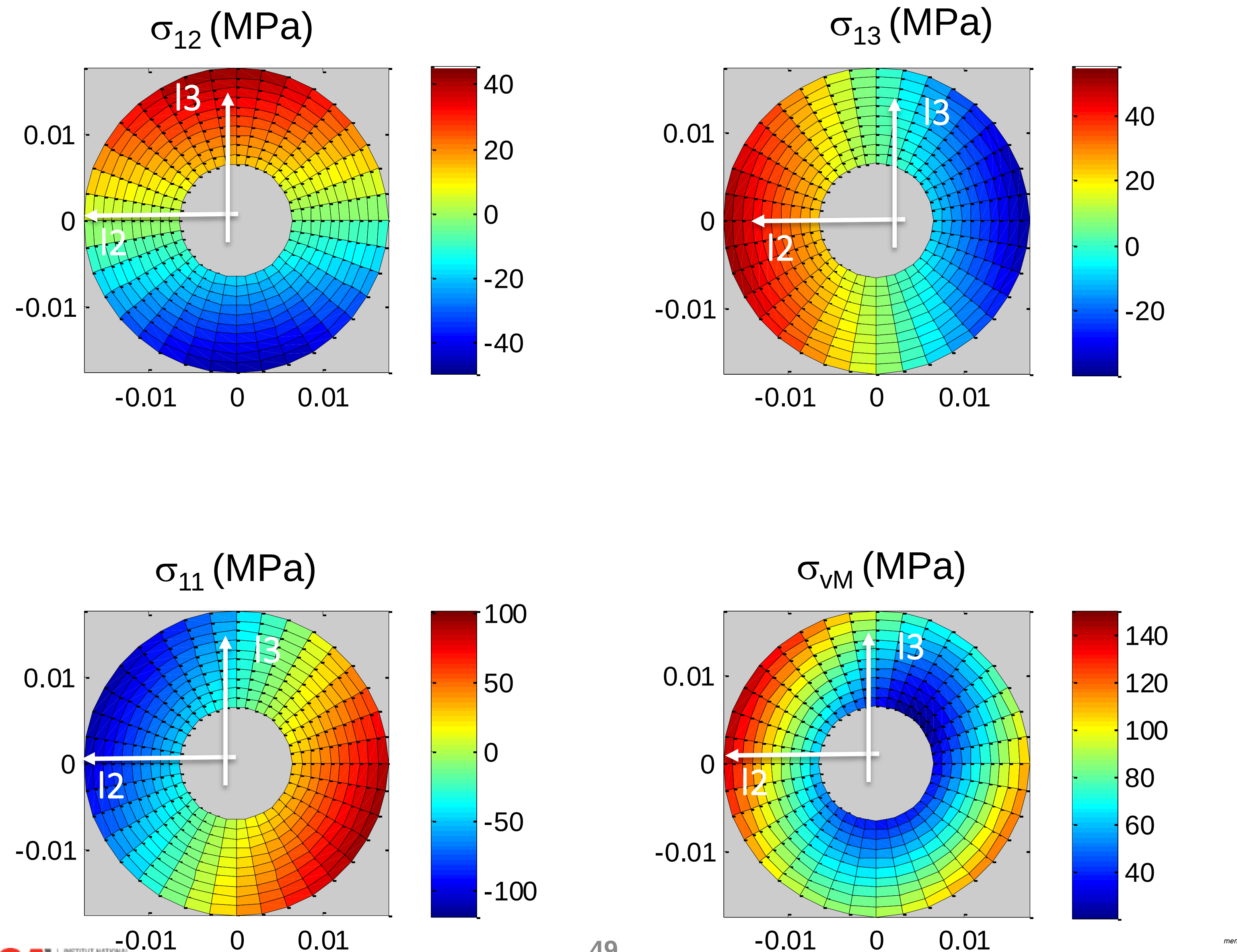
$\sigma_{11} = 3{,}65.10^7 Pa$
$\sigma_{12} = 1{,}44.10^7 Pa$
$\sigma_{13} = 5{,}68.10^7 Pa$

$\sigma_{VM\_B3} = 1{,}08.10^8 Pa = 108 MPa$

Contraintes dans la section B+



Contraintes section O1



Evolution des contraintes….