Biosciences - Maths fundamentals
September 2023

Instructions

This form will be scanned, so we ask you to strictly adhere to the rules below:

- To tick a box, fill it in black (■) by using black ball point pen;
- To correct, erase the box with white corrector ; **Do not redraw it**;
- Do not write anything in the header or margins of the pages;
- The symbol ♣ indicates that the number of correct answers is indeterminate (0, 1, 2,. . .). Its absence means that the question has aone single correct answer.

Multiple-choice questions have a null expected value: right answer = 1 point; no answer = 0 point; wrong answer to a question with n proposals = $-\frac{1}{n-1}$ points.

All documents allowed.

Identity

Using your student card, fill in the fields below and encode your student number on the right side.

First and last names:

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These questions are based on the FIMI (initial training at INSA) maths program. You are not expected to be able to answer all the questions. This questionnaire will be used to assess the needs for a short introducory course in September, as well as for anticipating which maths notions need to be covered during the maths classes during the semester.

Question 1 ♣ (FIMI1 Tools for functions) Let p be the polynomial defined by $p(x) = ax - x^3$, for $x \in \mathbb{R}$. If $a < 0$, then p

- ☐ has three real roots
- ■ has only one real root
- ☐ is bounded
- ■ is strictly decreasing

Question 2 (FIMI1 Tools for functions) Take a look at Figure 1. Which equations correspond best to each panel?

- ☐ (A): $y = \frac{1}{x}$; (B): $y = -1.0 + x - 3x^2 + x^3$; (C): $y = \frac{1}{1+x}$
- ☐ (A): $y = \frac{1}{x}$; (B): $y = 1.0 + x - 3x^2 + x^3$; (C): $y = \ln(x)$
- ■ (A): $y = \exp(-0.5x)$; (B): $y = -1.0 + x - 3x^2 + x^3$; (C): $y = \ln(x)$
- ☐ (A): $y = \exp(-0.5x)$; (B): $y = 1.0 + x - 3x^2 + x^3$; (C): $y = \frac{1}{1+x}$

Question 3 ♣ (FIMI1 Tools for functions) Let p be the polynomial defined by $p(x) = ax - x^3$, for $x \in \mathbb{R}$. The point $x = 0$

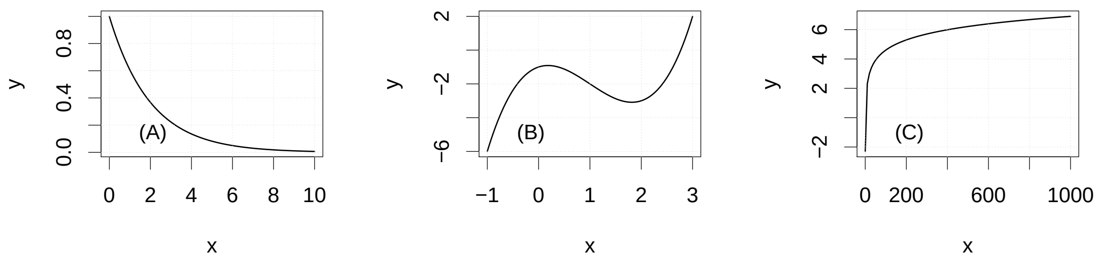

Figure 1

- ☐ is a local maximum of p
- ■ is an inflection point of p
- ■ is a root of p
- ☐ is a local minimun of p

Question 4 ♣ (FIMI2 Limits and continuity) Check the correct statements.

- ☐ The function $f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$ is continuous
- ■ $\lim_{x\to 0^+} \frac{\sqrt{x}+2}{x} = +\infty$
- ☐ $\lim_{x\to +\infty} \frac{\sqrt{x}+2}{x} = +\infty$
- ■ The function $f(x) = |x|$ is continuous

Question 5 ♣ (FIMI3 Derivatives) Check the correct statements.

- ■ $\frac{d}{dx}x^a = ax^{a-1}$ for all $a \in \mathbb{R}$
- ☐ $\frac{d}{dx}2^x = 2^x$
- ■ If two funtions f and g admit a derivative, $\frac{d}{dx}[f(g(x))] = \frac{d}{dy}f(y)|_{y=g(x)}\frac{d}{dx}g(x)$
- ☐ The function f, $f(x) = \sqrt{x}$, admits a derivative at $x = 0$

Question 6 (FIMI3 Derivatives) Let f, g two functions such that $f(x) = \ln(x)$ and $g(x) = \frac{1}{1+x^2}$. The derivative of the function h defined by $h(x) = f(g(x))$ is

- ☐ $\frac{2x}{1+x^2}$
- ☐ $-2x$
- ■ $\frac{-2x}{1+x^2}$
- ☐ $\frac{-2x}{(1+x^2)^3}$

Question 7 ♣ (FIMI4 Differential equations) Let a and b and x_0 be real numbers with $a > 0$. The solution of the differential equation

$$\frac{dx}{dt} = -ax + b$$

with initial condition $x(0) = x_0$ satisfies

- ☐ $x(t) = x_0e^{-at} + \frac{b}{a}$
- ■ $x(t) = x_0e^{-at} + \frac{b}{a}(1 - e^{-at})$

- ■ $\lim_{t\to\infty} x(t) = \frac{b}{a}$
- □ The equation does not have a solution

Question 8 ♣ (FIMI4 Differential equations) Let f be a continuous and bounded function on $\mathbb{R}$. The differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = f(t) + x^3$$

- □ is of order 3
- □ is linear
- ■ is non-autonomous
- ■ can be re-expressed as the system

$$\frac{dy}{dt} = -y + f(t) + x^3,$$

$$\frac{dx}{dt} = y.$$

Question 9 ♣ (FIMI4 Differential equations) Let x_0, r and K be positive real numbers with $x_0 < K$. The solution x of the differential equation

$$\frac{dx}{dt} = rx(1 - x/K)$$

with initial condition $x(0) = x_0$ possesses the following properties (check the correct ones):

- ■ t is an inflection point if $x(t) = K/2$
- □ x reaches a maximum of $x = rK/2$
- □ x is unbounded
- ■ x is bounded by K

Question 10 ♣ (FIMI6 Polynomials) Let p be the polynomial defined by

$$p(x) = (x^2 + 1)(x + 1)^2.$$

The polynomial p

- ■ possesses non-real roots
- □ possesses exactly two roots
- ■ possesses a double root at -1
- □ possesses a double root at $i = \sqrt{-1}$

Small o notation. Let f and g be two functions. We say that f is negligible before g, denoting $f(x) = o(g(x))$, if

$$\lim \frac{f(x)}{g(x)} = 0.$$

Usually the limit is taken at 0, or at infinity. For example, near $x = 0$, x^3 is negligible before x^2, and we write $x^3 = o(x^2)$. The same is true for any term x^n, with $n > 3$.

Question 11 (FIMI7 Taylor expansion) Let f be a function such that $f(x) = \sqrt{1 + x^2}$ (Figure 2). In the neighbourhood of $x = 0$,

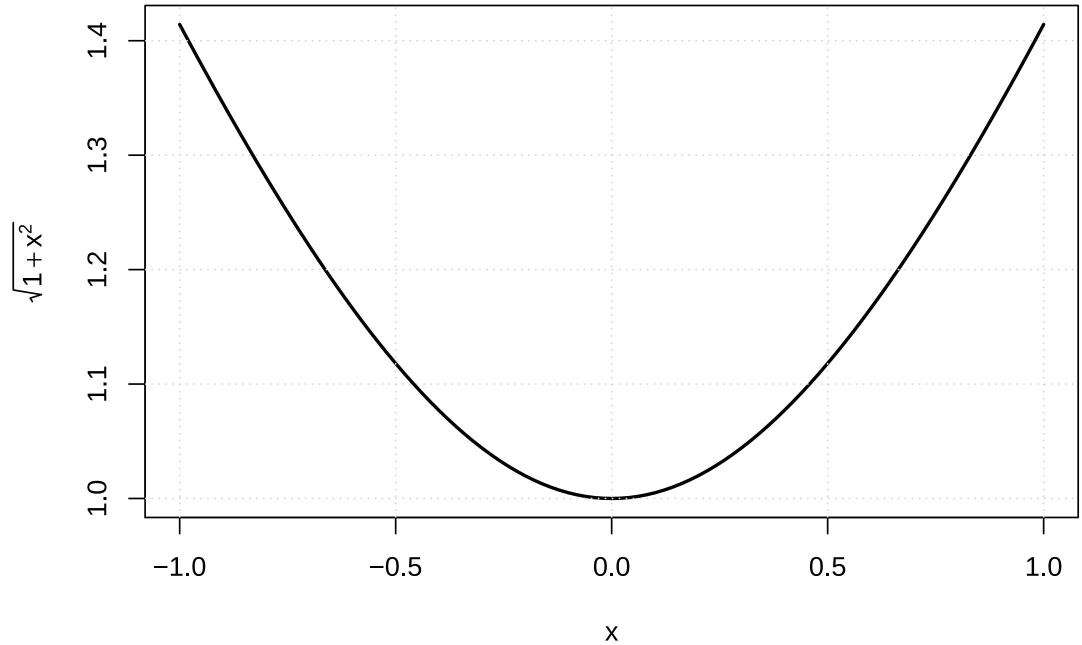

Figure 2. $f(x) = \sqrt{1+x^2}$**.**

- ☐ f admits no derivative
- ☐ $f(x) = 1 + \frac{x}{2} + o(x)$
- ■ $f(x) = 1 + \frac{x^2}{2} + o(x^2)$
- ☐ $f(x) = 1 + \frac{x}{2} + \frac{x^2}{2} + o(x^2)$

Question 12 ♣ **(FIMI8 Integration)** Let f be a function such that

$$f(x) = \begin{cases} c_0, & x \in [0,1) \\ c_1, & x \in [1,2] \end{cases},$$

and let g be a continuous function on the interval $[0,2]$. Check the correct statements.

- ■ $\int_0^2 f(x)dx = 0$ if and only if $c_0 = -c_1$
- ☐ $\int_0^2 f(x)g(x)dx = \int_0^2 f(x)dx \int_0^2 g(x)dx$
- ■ $\int_0^2 f(x) + g(x)dx = \int_0^2 f(x)dx + \int_0^2 g(x)dx$
- ■ $\int_0^2 f(x)g(x)dx = c_0 \int_0^1 g(x)dx + c_1 \int_1^2 g(x)dx$

Question 13 ♣ **(FIMI9 Primitives)** Check the correct statements.

- ■ $\int e^{-3x}x^2dx = \frac{-e^{-3x}}{3}x^2 + \int \frac{2}{3}e^{-3x}xdx$
- ■ $f(x) = \sqrt{x}$ admits the primitive $\frac{2}{3}x^{3/2}$
- ■ $\int_{-\pi/2}^{\pi/2} \sin(x)dx = 0$
- ■ $\int_1^{\infty} \frac{1}{x^2}dx = 1$

Question 14 ♣ (FIMI10 Linear systems) Consider the following system of linear equations:

$$\begin{aligned} 3x - 2y + z &= 1, \\ x + 2y - z &= 2, \\ -2x + y &= 3. \end{aligned}$$

Which of the following systems are equivalent to the one above?

■

$$\begin{aligned} z &= \frac{31}{4}, \\ x &= \frac{3}{4}, \\ y &= \frac{9}{2}. \end{aligned}$$

■

$$\begin{aligned} 3x - 2y \quad &+z = 1, \\ 4x \quad &= 3, \\ -2x + y \quad &= 3. \end{aligned}$$

□

$$\begin{aligned} 3x - 2y + z &= 1, \\ x + 2y - z &= 2, \end{aligned}$$

□

$$\begin{aligned} z &= 1, \\ 4x &= 3, \\ 2y &= 9. \end{aligned}$$

Question 15 ♣ (FIMI14 Sequences) Let u_0 and a be real numbers, and let $\{u_n\}_{n\geq 0}$ be the sequence defined by the recurrence relation

$$u_{n+1} = au_n, \quad n \geq 0.$$

The sequence $\{u_n\}_{n\geq 0}$ satisfies

- □ $u_{n+1} = a^n u_0$
- □ If $a < 0$, then u_n diverges to $\pm\infty$
- ■ $\lim_{n\to\infty} u_n = 0$ if and only if $|a| < 1$
- ■ $u_n = a^n u_0$

Question 16 ♣ (FIMI15 Determinants) Let A be the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ a & b & 0 \\ c & d & 0 \end{pmatrix}.$$

- □ $\det A = bc - ad$
- □ $\det A = 0$ for any a, b, c, d
- ■ $\det A = \det A^t$ (^{t} is the matrix transpose)
- ■ $\det A = ad - bc$

Question 17 ♣ (FIMI16 Factorisation) Let A, B and C be the three matrices

$$A = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}.$$

Which of these matrices admit an eigenvalue decomposition?

- □ B only

- ☐ A and B
- ☐ A and C
- ■ B and C

Question 18 **(FIMI16 Factorisation)** Let A, B and C be the three matrices

$$A = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}.$$

What are the eigenvalues of the matrices A, B and C?

- ■ A: -2, -2; B: -1, 1; C: 1, 1
- ☐ A: -2, -2; B: 1, 1; C: 2, 2
- ☐ A: -2, 0; B: -1, 1; C: 0, 2
- ☐ A: -2, 0; B: 1, 1; C: -1, 1

Question 19 **(FIMI13 Matrices)** Find the two-by-two matrix A such that for any vector $(x_1, x_2)^t$, $Ax = (x_2, x_1)^t$.

- ☐ $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$
- ■ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- ☐ $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$
- ☐ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Question 20 **(FIMI13 Matrices)** Let A be the matrix

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

The matrix A corresponds to

- ☐ A translation
- ☐ A reflexion
- ☐ A projection
- ■ A rotation

Question 21 ♣ **(FIMI20 Calculus)** Let f and g be functions such that $f(x, y) = \sin(xy)$ and $g(x, y) = 1 + e^{2x+y}$.

- ■ The gradient ∇f defined by $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})^t$ is $(y\cos(xy), x\cos(xy))^t$
- ■ $\frac{\partial^2 g}{\partial x^2} = -y^2 \sin(xy)$
- ☐ $\frac{\partial^2 g}{\partial x \partial y} = e^{2x+y}$
- ■ The first order expansion of g around $(x, y) = (0, 0)$ is $2 + 2x + y$
- ☐ $\frac{\partial f}{\partial x} = \sin(y)$

Question 22 **(FIMI21 Curves and surfaces)** Consider the curve defined by the equation $x - x^3 = y$, (Figure 3). The line tangent to the curve at point $x = 1/2$ is

- ☐ $y = \frac{1}{2}x + \frac{1}{2}$
- ☐ $y = \frac{1}{4}x - \frac{1}{4}$
- ■ $y = \frac{1}{4}x + \frac{1}{4}$
- ☐ $y = -\frac{1}{2}x + \frac{1}{2}$

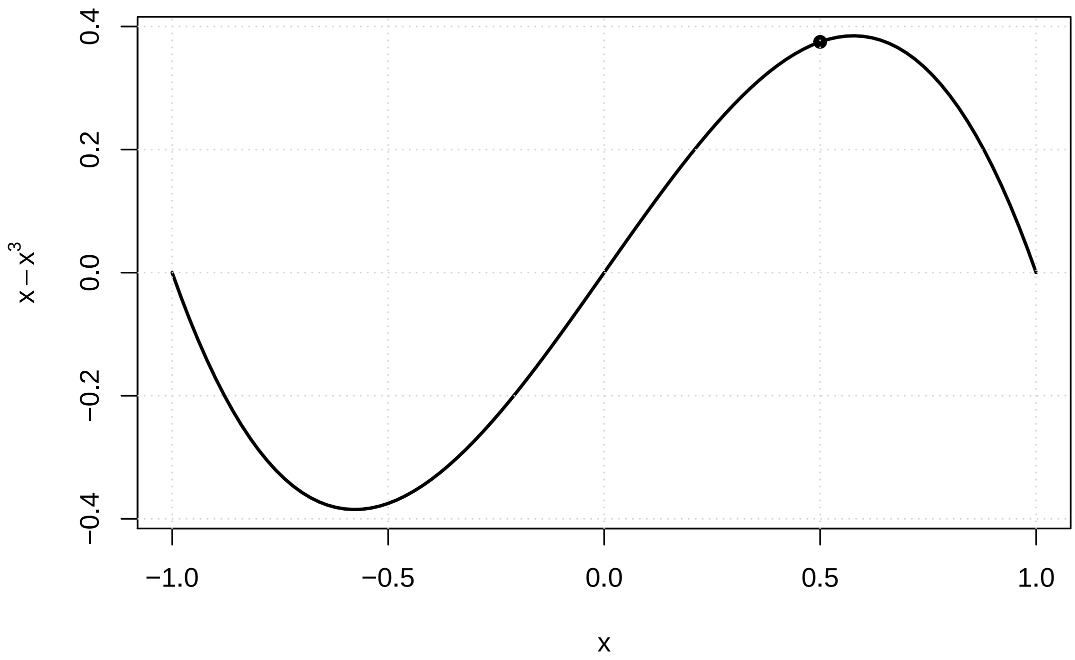

Figure 3. $x - x^3 = y$.