ECO-Design and Topology optimization

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LABORATORY OF TOPOLOGY OPTIMIZATION AND MULTIPHYSICS ANALYSIS

UNICAMP

Objectives

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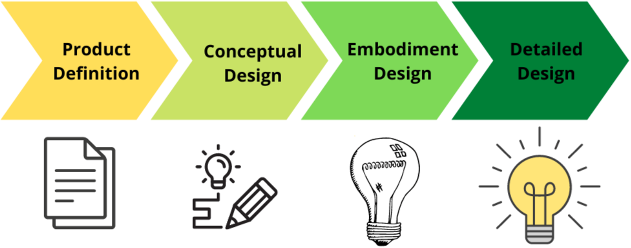
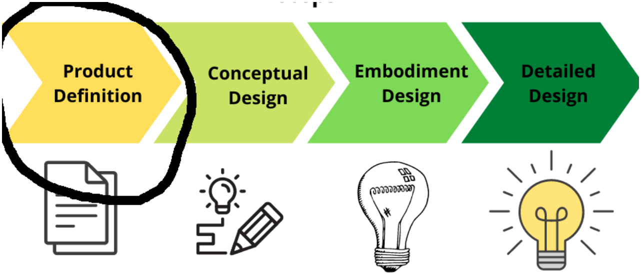
- To introduce the Conceptual Design Tools;
- Chose a simple but representative case study;
- Show the software usage for calculation of multi material truss structures mechanical eco-design parameter.
- Truss Topology Optimization or Gradient base Design of Steel-Timber Structures for Environmental Objectives;
- Project definitions.

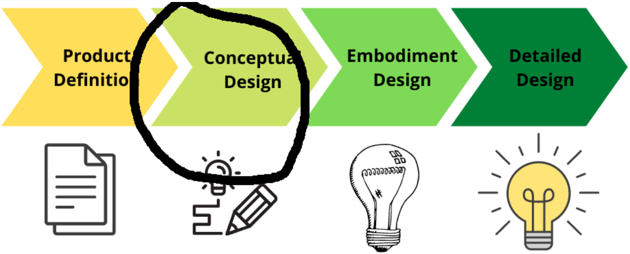
What is the conceptual mechanical/Structural Design?

A product design process is the sequence of steps a engineer employs to formulate, design, and manufacture a product or structure.



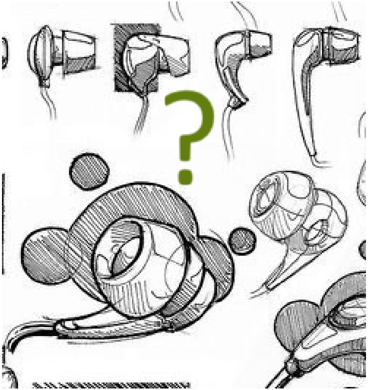


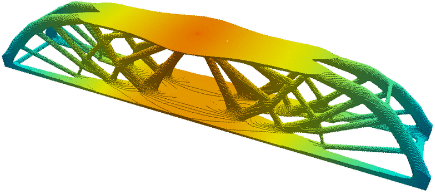
- A written request with a problem statement - Clients, Marketing ...;
- All Product requirements must be clarify in detail;
- It ensures that the engineers and designers understand the requirement of the product;

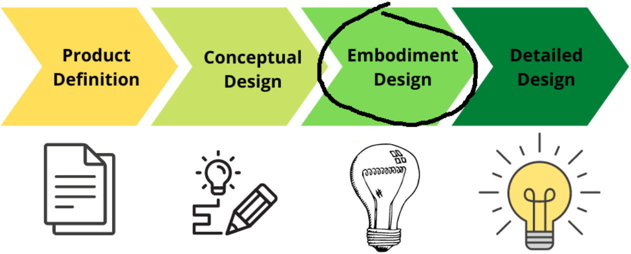


- Clarify the problem;
- Search externally;
- Search internally;
- Explore systematically;
- Reflection and choosing the concept;

In this case, some non intuitive solutions or innovative concepts may not be found. Topology Optimization is an option to find non intuitive solutions.

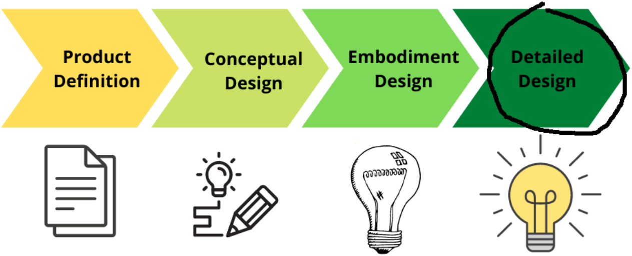


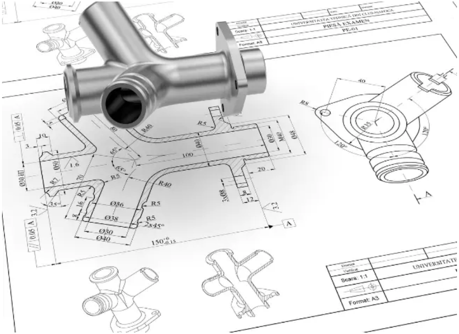




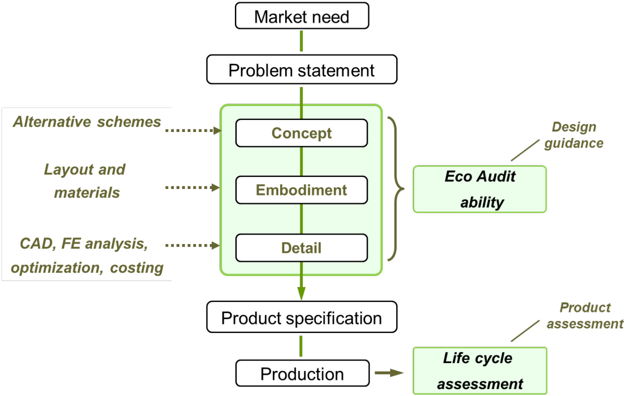
- Product architecture;
- Design configuration (Topology Optimization);

- Parametric design

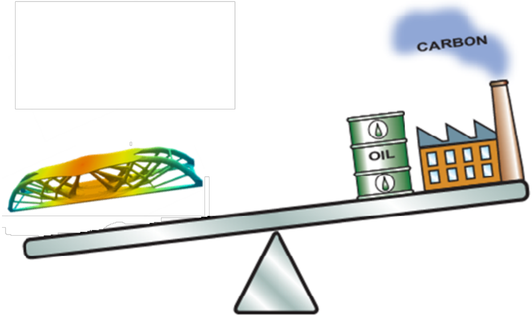

Product Definition
Conceptual Design
Embodiment Design
Detailed Design



The ECO-Design requirements can be considered in the phase of Conceptual - Embodiment - Detailed Design.



The goal is coupled the topological optimization and/or numerical simulation techniques with ECO-Design requirements in the Conceptual Design phase.



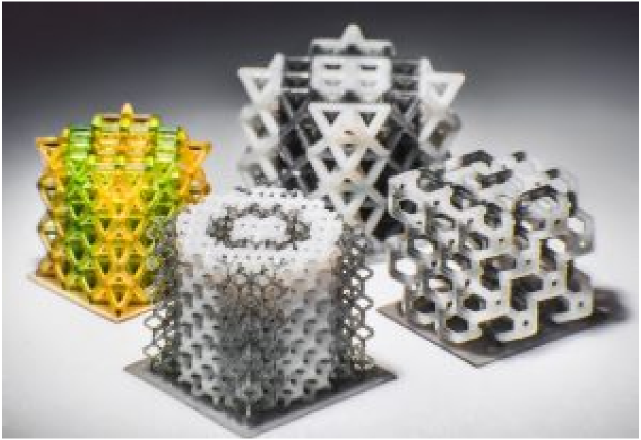
Chosen ECO-design indicator > CO_2 carbon footprint

Problems Definitions > Simple but representative structure

To introduce this method, we consider the case of multi-material structures, for example a Truss build with steel mixed with parts processed using wood, as shown in figures

Figure: Metal Timber Structures

Metamaterials Design - Multi-material micro-lattice polymeric structures fabricated using 3D printing



The chosen problems: Timber-Steel Truss

Properties taken for timber and steel in the current work and the ratio of each property taken as steel/timber (e.g. E_{steel}/E_{timber}).

Property		Steel	Timber	Property ratio
E	(GPa)	200	11	18.2
σ_y	(MPa)	±345	$-8.6/+6.6$	40.1/52.3
ρ	(kg/m^3)	7870	570	13.8
ECC	($kg_{CO_2e}/kg_{material}$)	1.45	0.42	3.5

Truss Topology Optimization of Steel-Timber Structures for Embodied Carbon Objectives. Ho Yin Ernest Ching Master of Science - MIT 2020

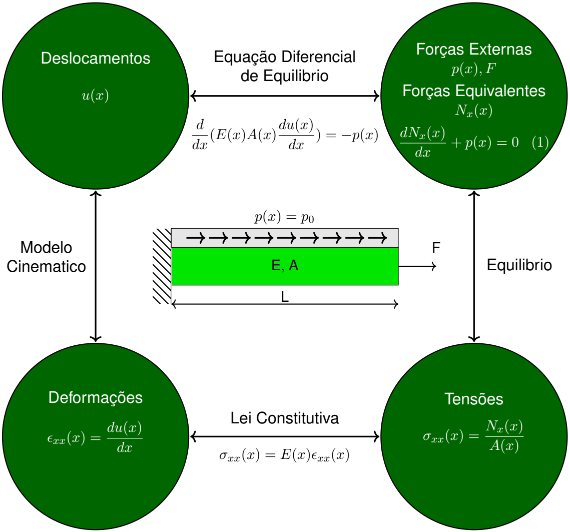
Truss modeling and solution using finite element method

Formulation of static equilibrium equation of 2D truss using the finite element method. The state variables for each element can be calculate: volume, displacements, forces, stress.

The principal steps that will be remember are:

- Equilibrium equation;
- Discretization and approximation of the solution using Finite Element Method;
- Assemblage of the global system and Local to global transformation.
- Solve the discrete equilibrium equation to find nodal displacements.
- Post processing to evaluate the stresses, forces,weights, CO_2 embodied,etc

Equilibrium Differential equation for bar problems



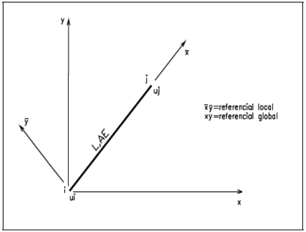
Summary - bar problem

Find $u(x)$ such that:

$$\begin{cases} EA\dfrac{d^2u(x)}{dx^2} + p(x) = 0 & \text{in } \Omega \\ \text{sujeito a:} & \\ u = \bar{u} & \text{in } \Gamma_1 \\ \dfrac{\partial u}{\partial x} = \dfrac{\bar{F}}{EA} & \text{in } \Gamma_2 \end{cases}$$

Finite Element Discretization - 2D Truss problem

The local and global coordinate system are $(\bar{x}, \bar{y})$ and (x, y).



The displacement vector u is:

$$u = \{ \; u_1, v_1, u_2, v_2 \; \}$$

Using $cos\theta = \lambda$ e $sen\theta = \mu$, the elemental bar matrix can be written as:

$$[K] = \frac{AE}{L} \begin{bmatrix} \lambda^2 & & & \text{Sim.} \\ \lambda\mu & \mu^2 & & \\ -\lambda^2 & -\lambda\mu & \lambda^2 & \\ -\lambda\mu & -\mu^2 & \lambda\mu & \mu^2 \end{bmatrix} \quad (1)$$

and the force vector for one element:

$$F = \{ \ F_{ix}, F_{iy}, F_{jx}, F_{jy} \ \}$$

with all forces in the global (x, y) .
After Assemblage procedure, we can find the stiffness matrix and load vector for truss as shown bellow:

$$[Kg]\{ug\} = \{Fg\} \quad (2)$$

Show the software usage for calculation of multi material truss structures mechanical eco-design parameter.

Tutorial Presentation

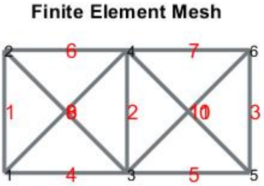
Matlab script

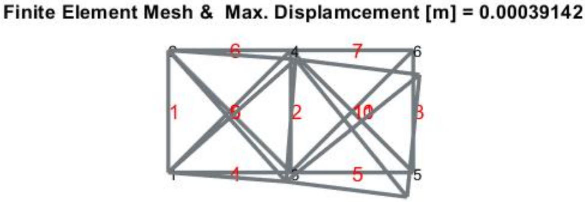
Implementation and testes of a simple truss using a Matlab script. Code Directory» ECO-TOP-2.m

The matlab implementation can be divided in the following tasks:

- Data base: node coordinates, elemental incidences, geometric and material properties
- Boundary conditions and load cases.
- Elemental properties: stiffness matrices and load vectors.
- Assemblage, boundary conditions ans solve.
- Post processing.

One script has been developed, to simulated the static equilibrium conditions, considering a cantilever boundary conditions, as represented in Figure (2). Two mesh generators have been developed. The first one for a X reinforced truss and, the second mesh generator all bar are connected with all nodes.





Multi material Truss - Mechanical and Ecological variables

Implementation of a typical truss 2D case, and evaluate local/global indicator:

strain energies/compliance,

Embodied carbon Coefficients of the structure and

stress for each bars.

Compare the results of a steel truss with a timber/steel truss and timber trusses.

The second part of this implementation are related with the pos-processing of variables.

- $\mathbf{u_g}$ the nodal displacements;
- σ_{xx} the elemental stress;
- $C = \mathbf{F_g^T} \mathbf{u_g}$ the structure Compliance ;
- $Ce = \mathbf{u_e}^T \mathbf{K_e} \mathbf{u_e}$ the elemental compliance;
- $\dfrac{dC}{dA_e}$ the sensitivity number;
- $We = \rho_e A_e L_e$ the elemental weight;
- $Wt = \sum We$ the weight of the structure;
- ECC the material Embodied Carbon coefficient;
- $ECCe = \rho_e A_e L_e ECC$ the elemental Embodied Carbon Coefficient.

In figure (3) we can observe one example with two materials and in figure (26) we can see the displacements and bar stress for a simple supported beam case.

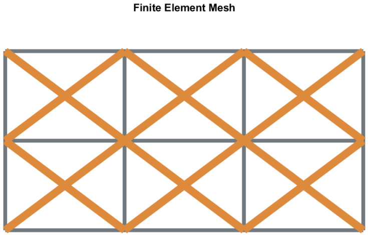
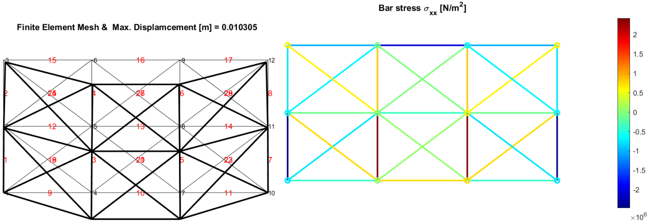


Figure: Steel(gray)/Timber(brown) truss



Truss Topology Optimization or Gradient base Design of Steel-Timber Structures

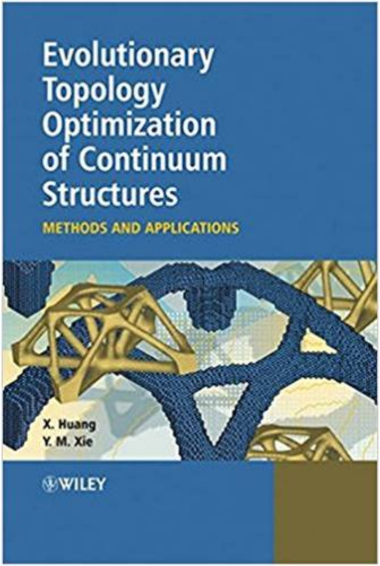
Evolutionary Compliance Optimization: An heuristic Method

- We perform a single-material truss topology optimization for minimum compliance with weight and box constraints.
- The limits A_{min} and A_{max} cross section area variables are defined by the user.
- Formulation and sensitivity analysis, algorithm, implementation and tests are performed.

BESO heuristic Method - characteristics

- Method was inspired in the nature evolution: Bones, plants, etc
- It can be applied for different Criteria: Stress, Compliance, Natural frequencies, Eco-design functions, etc
- This version of the Evolutionary method have been proposed by Xie, Stevens and Huang.
- The method is intuitive and simple to implement.
- The version allow add and remove Material, changing the cross section Area of the bars.
- The material must be gradually removed.
- The variations are made by constant discrete values of the cross section Area A_i.
- The method is based on the gradient analysis.

Basic reference



In the discrete form, the problem of compliance C minimization with weight constraint g_w, can be written as:

$$
\begin{array}{ll}
Minimize & C = \mathbf{F_g}^T \mathbf{u_g} \\
Subjectto & \mathbf{K_g}(A_e)\mathbf{u_g} = \mathbf{F_g} \\
 & g_w = \sum_{e=1}^{nel} A_e L_e \rho_e \leq W_{lim} \\
 & A_{min} \leq A_e \leq A_{max}
\end{array}
\tag{3}
$$

where,

- C is the Compliance (objective function);
- $\mathbf{F_g}$ is the nodal force vector;
- $\mathbf{u_g}$ is the nodal displacement vector;
- $\mathbf{K_g}(A_e)$ is the global stiffness matrix ;
- A_e is the elemental cross section area;
- L_e is the elemental length;
- ρ_e is the material density of the element e;
- W_{lim} is the limit value of the total weight of the truss;
- A_{min} and A_{max} are the box constraints, lower and upper limits.

The Compliance minimization is equivalent to Stiffness maximization problem. The objective function of this problem is:

$$C = \frac{1}{2}\mathbf{F_g}^T\mathbf{u_g} \quad (4)$$

where C is the structure Compliance, $\mathbf{F_g}$ is the constant external loads and $\mathbf{u_g}$ is the structural displacement. We consider that the structure is modeled by Finite Element Method, and the static equilibrium equation can be written by:

$$\mathbf{K_g}\mathbf{u_g} = \mathbf{F_g} \quad (5)$$

Introducing the design variable A_e, as area cross section for the element e, the strain energy can be re-written, for the cases where the stiffness matrix is symmetric, by:

$$C = \frac{1}{2}\mathbf{F_g}^T\mathbf{u_g}(A_e) = \frac{1}{2}\mathbf{u_g}(A_e)^T\mathbf{K_g}(A_e)\mathbf{u_g}(A_e) \quad (6)$$

where A_e is the vector of bars cross section area, e indicate a specific element, nel is total number of elements. To simplify the notation we use $\mathbf{K_g}(A_e) = \mathbf{K_g}$ and $\mathbf{u_g}(A_e) = \mathbf{u_g}$. But, since $\mathbf{K_g}\mathbf{u_g} = \mathbf{F_g}$ if $\mathbf{F_g}$ is constant, than, the first derivative of the equilibrium equation with respect to the design variables A_e is:

$$\frac{d\mathbf{K_g}}{dA_e}\mathbf{u_g} + \mathbf{K_g}\frac{d\mathbf{u_g}}{dA_e} = 0 \tag{7}$$

than:

$$\mathbf{K_g}\frac{d\mathbf{u_g}}{dA_e} = -\frac{d\mathbf{K_g}}{dA_e}\mathbf{u_g} \tag{8}$$

and,

$$\frac{d\mathbf{u_g}}{dA_e} = -\mathbf{K_g}^{-1}\frac{d\mathbf{K_g}}{dA_e}\mathbf{u_g} \tag{9}$$

And calculating the derivative of C we have:

$$\frac{dC}{dA_e} = \frac{1}{2}(2 * \mathbf{u_g}^T\mathbf{K_g}\frac{d\mathbf{u_g}}{dA_e} + \mathbf{u_g}^T\frac{d\mathbf{K_g}}{dA_e}\mathbf{u_g}) \tag{10}$$

Replacing the equation (9) in (10) we have:

$$\frac{dC}{dA_e} = \frac{1}{2}(-2 * \mathbf{u_g}^T \mathbf{K_g} \mathbf{K_g}^{-1} \frac{d\mathbf{K_g}}{dA_e} \mathbf{u_g} + \mathbf{u_g}^T \frac{d\mathbf{K_g}}{dA_e} \mathbf{u_g}) \tag{11}$$

and we arrive to:

$$\frac{dC}{dA_e} = -\frac{1}{2} \mathbf{u_g}^T \frac{d\mathbf{K_g}}{dA_e} \mathbf{u_g} \tag{12}$$

The stiffness Matrix variation with respect to the element surface A_e can be done by:

$$\frac{d\mathbf{K_g}}{dA_e} = \frac{1}{A_e} \mathbf{K_e} \tag{13}$$

where $\mathbf{K_e}$ is the stiffness matrix of an element e.

In an displayed form,

$$\frac{d\mathbf{K_g}}{dA_e} = \frac{E}{L}\begin{bmatrix} \lambda^2 & & & \text{Sim.} \\ \lambda\mu & \mu^2 & & \\ -\lambda^2 & -\lambda\mu & \lambda^2 & \\ -\lambda\mu & -\mu^2 & \lambda\mu & \mu^2 \end{bmatrix} \tag{14}$$

we can use the quadratic form, as a positive number that represent the variations of the objective function C with respect to A_e:

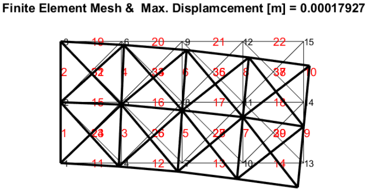
$$\frac{dC}{dA_e} = \frac{1}{2}\mathbf{u_e}^T \frac{1}{A_e}\mathbf{K_e}\mathbf{u_e} \tag{15}$$

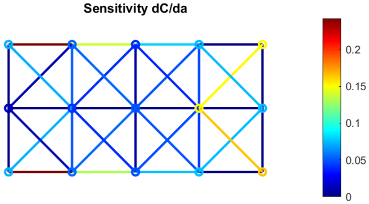
where $\mathbf{u_e}$ is the displacement vector of one element e. In this contest, to minimize the compliance, we need decreased the A_e of the elements with the lowest values of $\dfrac{dC}{dA_e}$ and increase the area cross section of the elements with the highest value of the sensitivity function.

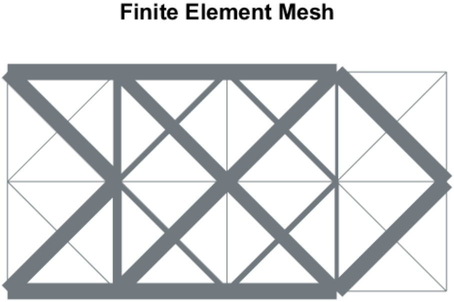
The Evolutionary Algorithm can be stated as:

1. Discretize the design domain using a finite element method using bars elements. Impose boundary conditions and loads;
2. Solve the static structural analysis, finding the nodal displacements;
3. Calculate the sensitivity number $\frac{dC}{dA_e}$, using the equation 15,
4. Reduce the cross section area A_e by a constant value $A_e - \delta A_e$ for a fixed number of elements with smaller $\frac{dC}{dA_e}$ values,
5. Increase the cross section area A_e by a constant value $A_e + \delta A_e$ for a fixed number of elements with higher $\frac{dC}{dA_e}$ values,
6. Repeat pass 2 and 5 until reach the final volume or an limit value for the compliance.

Example - cantilever beam







Two-material truss topology optimization for minimum compliance with weight and/or Embodied Carbon constraints.

To implement a two materials models, we need to use a pseudo density variable define as x_e one value per element. We consider that, if $x_e = 0$ means that the element is of timber and $x_e = 1$ means that the element is of steel.

$$
\begin{array}{ll}
Minimize & C = \mathbf{F_g}^T \mathbf{u_g} \\
Subjectto & \mathbf{K_g}(A_e, x_e)\ \mathbf{u_g} = \mathbf{F_g} \\
 & g \leq g_{lim} \\
 & A_{min} \leq A_e \leq A_{max} \\
 & 0 \leq x_e \leq 1
\end{array}
\tag{16}
$$

The equation (16) can be solve with two different constraints, $g = g_W$ the weight constraint and $g = g_{EC}$ the global warning potential. The g_{EC} is calculated by:

$$g_{EC} = \sum_{e=1}^{nel} = A_e l_e (\rho_e ECC_e) \tag{17}$$

and the limit values are W_{lim} and ECC_{lim}. To simulate the material variations we use a material interpolation model: the SIMP model.

$$\begin{aligned} &E_e = (x_e)^p \Delta E + E_{timber} \\ &\rho_e = (x_e)^p \Delta \rho + \rho_{timber} \\ &(\rho_e ECC_e) = (x_e)^p \Delta(\rho ECC) + (\rho ECC)_{timber} \end{aligned} \tag{18}$$

where:

$$\begin{aligned} &\Delta E = E_{steel} - E_{timber} \\ &\Delta \rho = \rho_{steel} - \rho_{timber} \\ &\Delta(\rho ECC) = (\rho ECC)_{steel} - (\rho ECC)_{timber} \end{aligned} \tag{19}$$

Project definitions:

- Choose one initial truss design, considering two material (steel and Wood) and perform a finite element analysis to find displacements, compliance, weight, and ECC footprint.
- Inspired in the BESO Algorithm, using the gradient information, try to find a solution that minimize the compliance, subject to EEC limit, the equilibrium equation, and box constraints for the area of elements.
- Please, show the intermediate values obtained for objective function and constraints as a function of A_e and x_e along the try and error procedure.
- Use the ECO-TOP Matlab script, to perform and to implement one strategy to find a good engineering solution that combine the compliance and the Carbon footprint minimization.
- Write a report with assumed hypotheses and principal obtained results.