NB: The two exercises are independent.

Exercise 1:

Figures 1 and 2 represent a cam-shaft (with two identical cams). The solid is homogeneous of density $\rho$ and all the relevant geometrical data are given in Figs. 1&2.

1 – Give the position of the centre of inertia of the solid (1pt)

2 – Give the components of the matrix of inertia in $O_A$ (middle of the axle) in terms of the geometrical parameters and $\rho$. (6pts)

3 – Is the cam-shaft statically balanced with respect to the axis $(O, \bar{z}_{0,1})$? Is it dynamically balanced with respect to $(O, \bar{z}_{0,1})$? (1pt)

4 – Determine the moment of inertia of the cam-shaft with respect to the axis $(O_A, \bar{z}_{0,1})$ (1pts)

Figure 1 – Cam-shaft (general view)
Exercise 2:

Figure 3 shows a simplified representation of a roundabout. The system is made of:

- solid 0: ground, \( \vec{z}_0 \) is the upward vertical
- solid 1: bar of mass \( m_1 \), centre of inertia \( O_1 \), its moment of inertia with respect to \( (O_1, \vec{z}_1) \) is \( C_1 \).
- solid 2: wheel of radius \( R \), mass \( m_2 \), centre of inertia \( O_2 \), its matrix of inertia in \( O_2 \) is 
  \[
  [I]_{O_2,s_1} = \begin{bmatrix}
  A_2 & B_2 \\
  B_2 & B_2
  \end{bmatrix}
  \]
- solid 3: basket assimilated to a point mass concentrated in \( G_3 \) of mass \( m_3 \).

All relevant geometrical data are given in Figure 1.

Links:

1/0: revolute joint of axis \( (O_1, \vec{z}_{0,1}) \), parameter \( \theta \)
2/1: cylindrical joint of axis \((O_1, \bar{x}_{1,2})\), parameters \[
\begin{aligned}
X &= O_1^* \hat{O}_2 \cdot \bar{x}_{1,2} \\
\varphi
\end{aligned}
\]

\textit{NB:} \(O_1^*\) is the orthogonal projection of \(O_1\) on the \((O_1, \bar{x}_{1,2})\) axis

3/1: revolute joint of axis \((O_3, \bar{z}_{1,3})\), parameter \(\psi\)

Questions:

1 – Mass geometry:

1 – a Calculate the matrix of inertia of solid 3 in \(O_3\) (2pts)

1 – b Using Binet’s notations \((A_1, B_1, \ldots)\), give the simplest form of the matrix of inertia of solid 1 in \(O_1\). (1,5pts)

2 – Determine the dynamic wrench of solid 3 in \(O_3\) (2+4 pts)

3 – Determine the total kinetic energy with respect to \((R_0)\) (2,5pts)

Figure 3 – Roundabout.