

Conception Non Conventionnelle : Art, Vivant, Design, Environnement

Unconventional conception: Art, living, Design, environment

GM 5 – CE

# Cellular (solids) materials

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<i>Contributors:</i> <i>Jérôme</i> <i>Florian</i>	<i>Adrien</i> <i>Martoia</i>	<i>Mateis</i> <i>LaMCoS</i>

# I. Introduction

## I.1. Context & objective

**Arts, Vivant, Design,  
Environnement**

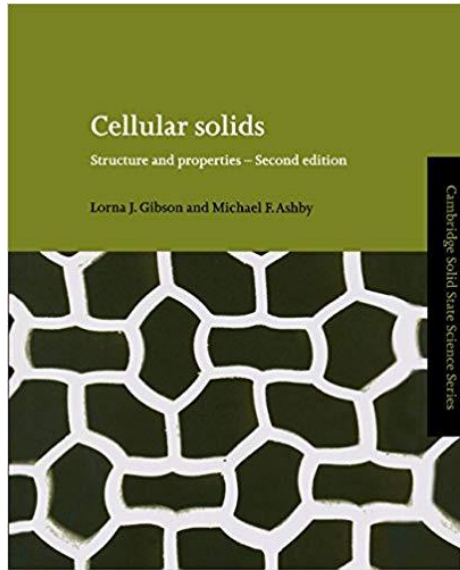
**Matière &  
perception**

Perception sensorielle

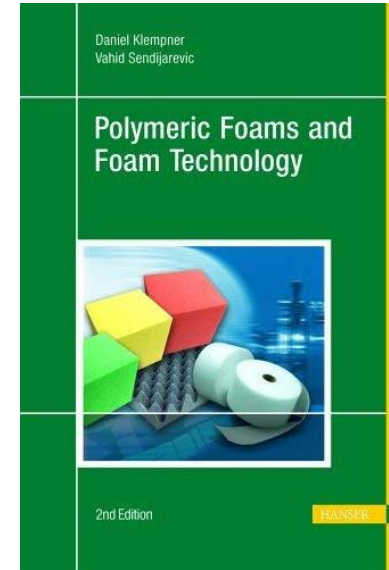
Matériaux non conventionnels :  
Bois, Verres, Tissus humains,  
**Matériaux Cellulaires**, Céramiques ...

# I. Introduction

## I.2. References



[1] Gibson, L., Ashby, M. F. (1988).  
Cellular Solids, Structure and Properties.  
Cambridge Solid State Science Series

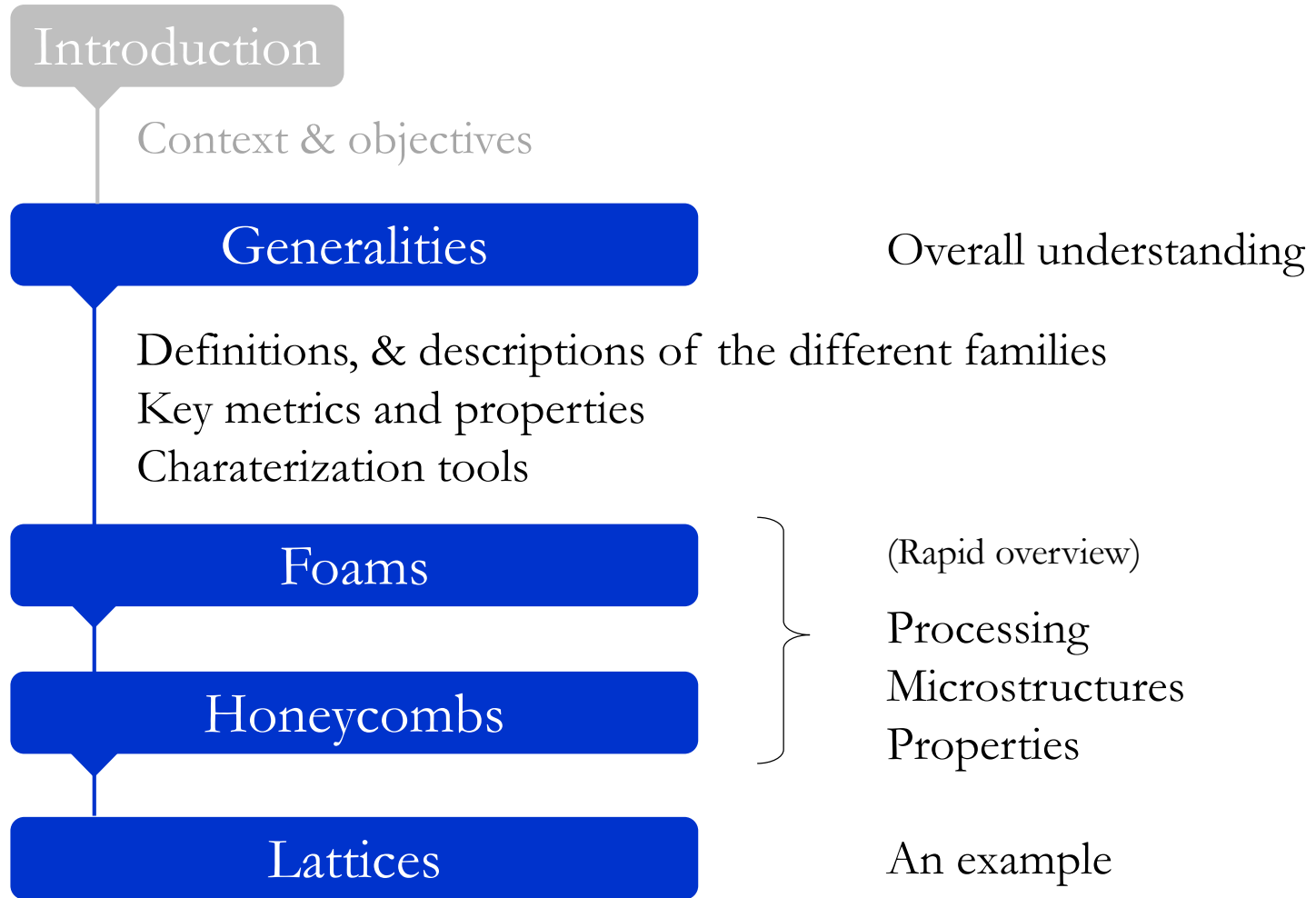


[2] Klemperer, D., & Sendjarevic, V. (2000).  
Handbook Of Polymeric Foams  
And Foam Technology.  
Hanser Publishers.

[...] *Specialized literature and internet sources that will be listed throughout the document.*

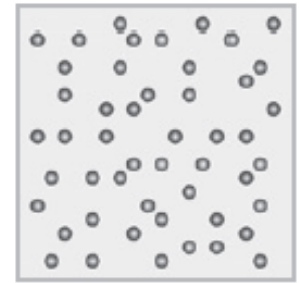
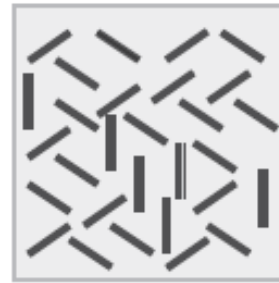
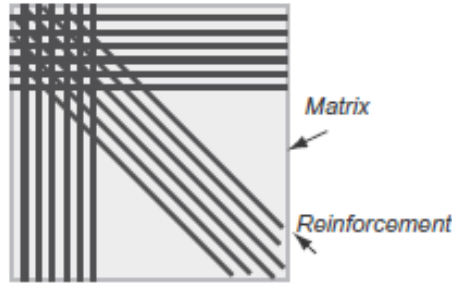
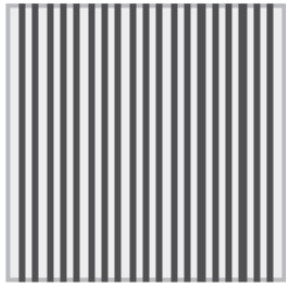
# I. Introduction

## I.3. Outline

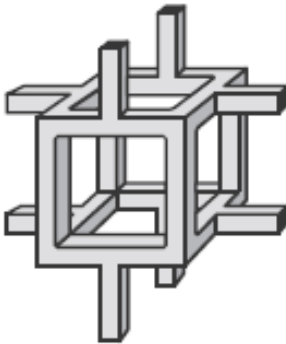


## II. Generalities

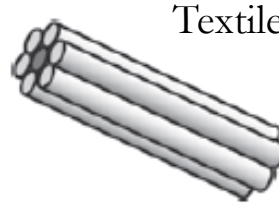
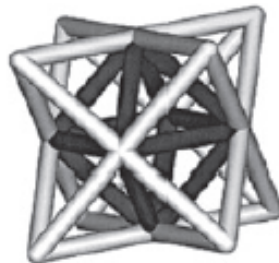
### II.1. Architected materials: family picture



Reinforced materials



Cellular materials



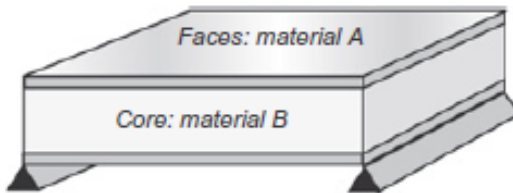
Textile



Segmented structure

### Architected materials

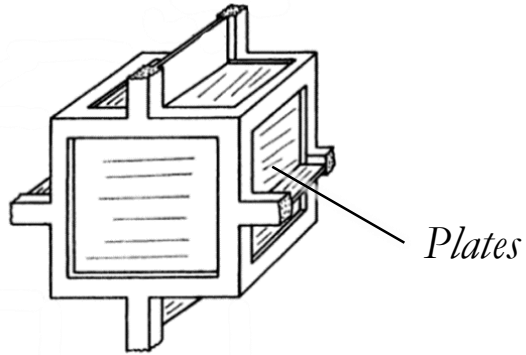
Sandwich structures



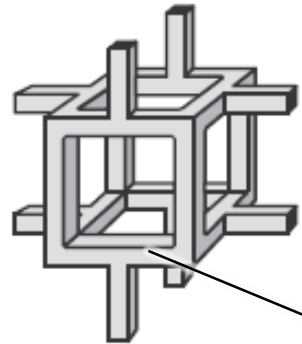
## II. Generalities

### II.2. Cellular (solids) materials

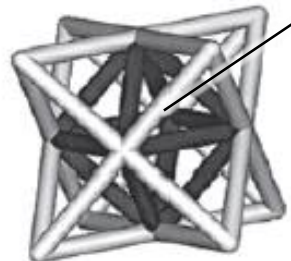
#### II.2.1. Definition



*Plates*



*Struts*



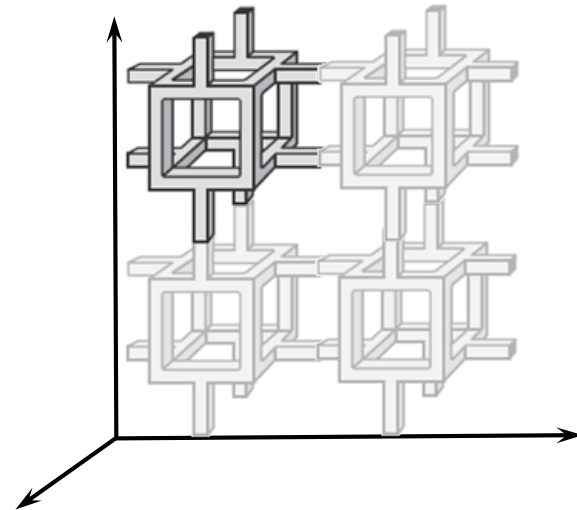
*Octet-truss  
Unit cell*

#### ***Cella*** (Latin):

a small compartment,  
enclosed **space = voids**  
(gaseous phase)

#### **Cellular solids:**

an assembly of packed cells  
within 3D space.



Interconnected network of solid  
struts or plates composing the  
edges and/or faces of the  
numerous cells.

# II. Generalities

## II.2. Cellular (solids) materials

## II.2.2. Classification

- Foams (Mousses)

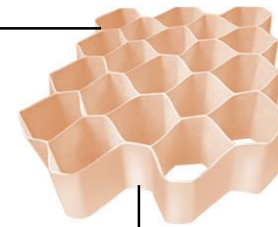
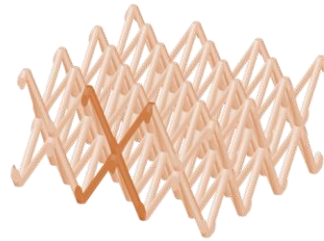


Stochastic

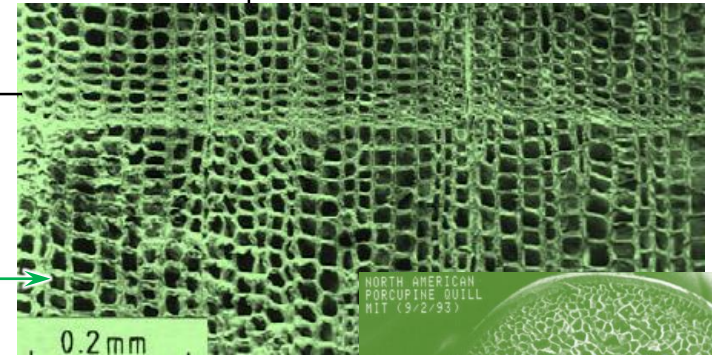
- Honeycomb structures (Structures en nid d'abeilles)

Periodic

- Lattices (Treillis)

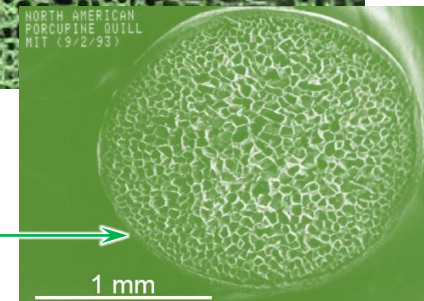


- Wood, leaves, fruits (bois, feuilles, fruits...)



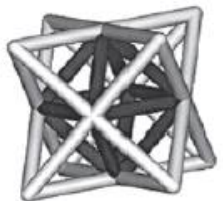
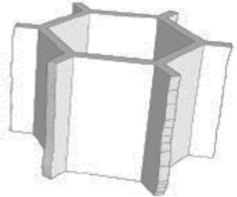
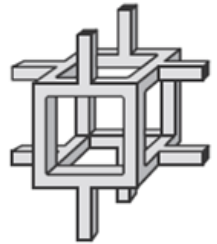
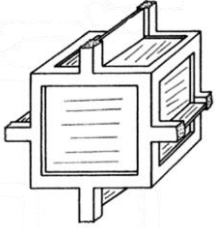
- Cellulose, bone, hair, feathers (cellules, os, poils, plumes,...)

Porcupine



## II. Generalities

### II.3. Cellular material properties



The Cellular solids can be seen as **an assembly of materials and voids** in 3D space.

⇒ Peculiar **properties** that first come to mind:

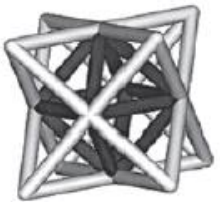
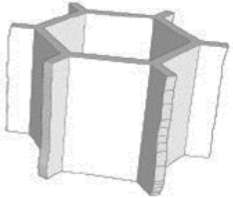
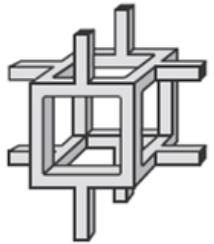
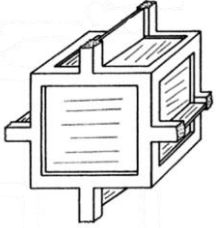
And the less obvious:



## II. Generalities

### II.3. Cellular material properties

#### II.3.1. Key ingredients



The Cellular solids can be seen as **an assembly of materials and voids** in 3D space.

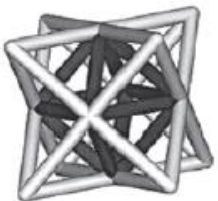
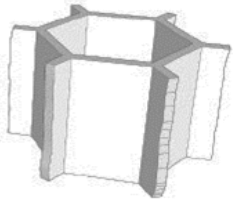
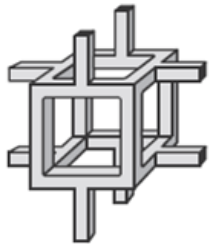
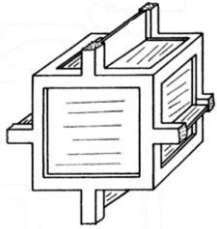
⇒ Its **properties** mainly depend on:



## II. Generalities

### II.3. Cellular material properties

#### II.3.2a. Relative density



The **volume fraction** or **relative density** characterizes the ratio between matter and “void”:

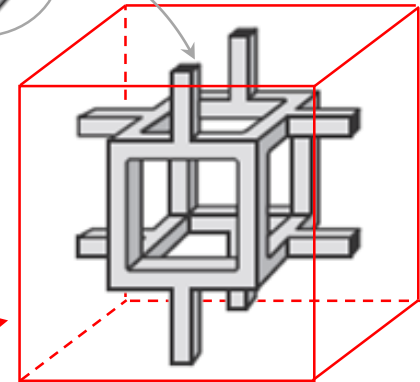
*Density of the cellular solid ( $\text{kg}/\text{m}^3$ )*

*Volume of the solid matter*

$$\bar{\rho} = \frac{\rho}{\rho_s} = \frac{V_s}{V_t}$$

*Density of the parent material ( $\text{kg}/\text{m}^3$ )*

*Total volume (occupation)*



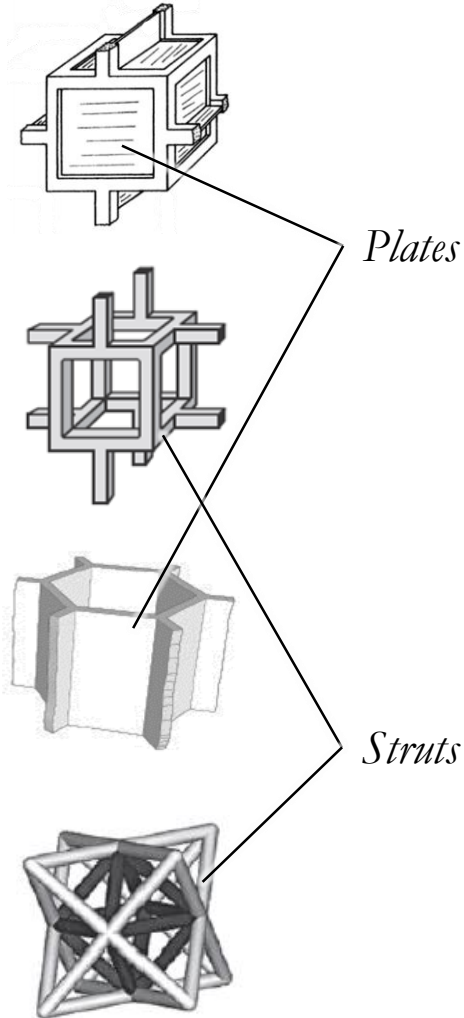
$$\bar{\rho} = 1 - p$$

*Porosity: volume fraction of pores*

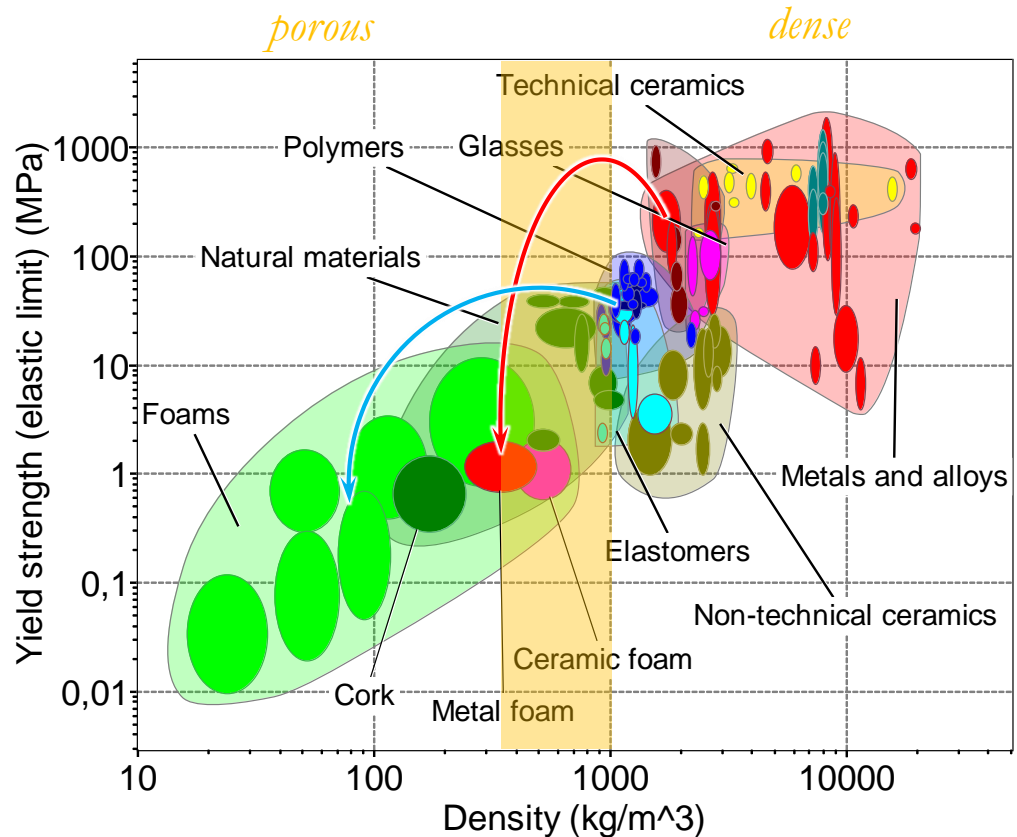
## II. Generalities

### II.3. Cellular material properties

### II.3.2b. Parent material properties



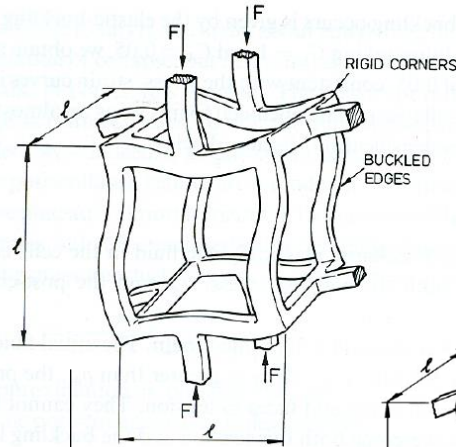
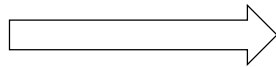
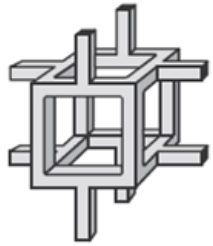
The constitutive material(s) has(ve) its own **intrinsic properties**:



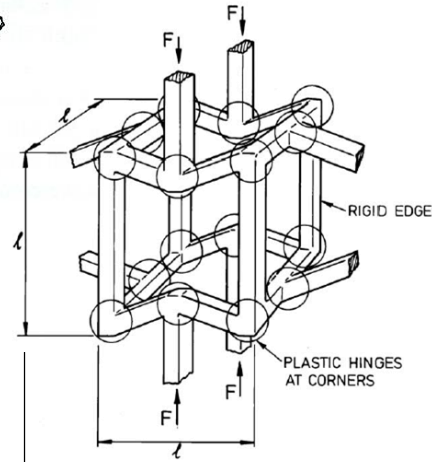
# II. Generalities

## II.3. Cellular material properties

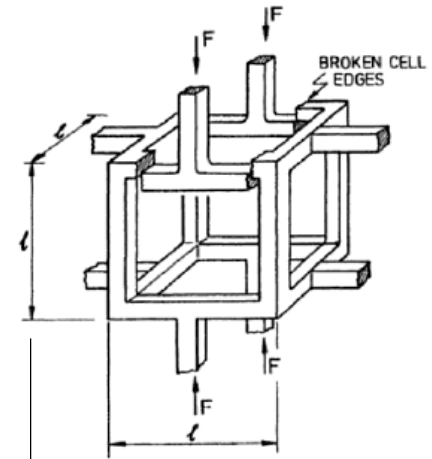
## II.3.2b. Parent material properties



*“soft”*

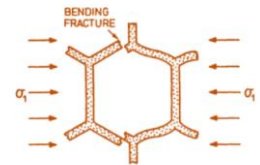
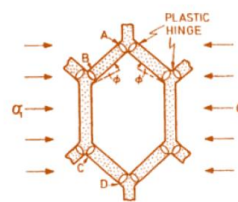
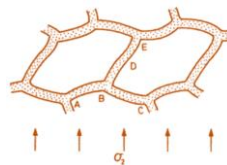
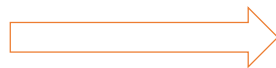


*“ductile”*



*“brittle”*

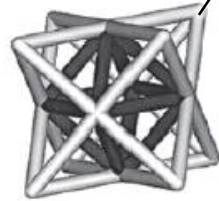
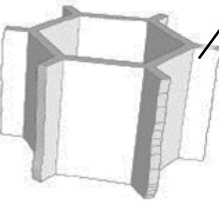
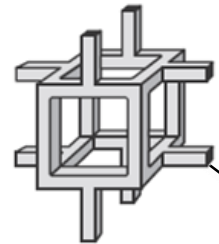
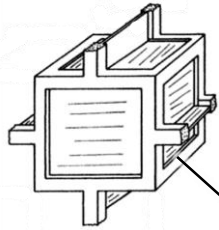
The yielding (onset of non-linear behaviour) of the cellular material does not necessarily depend on the yield strength of the parent material.



## II. Generalities

### II.3. Cellular material properties

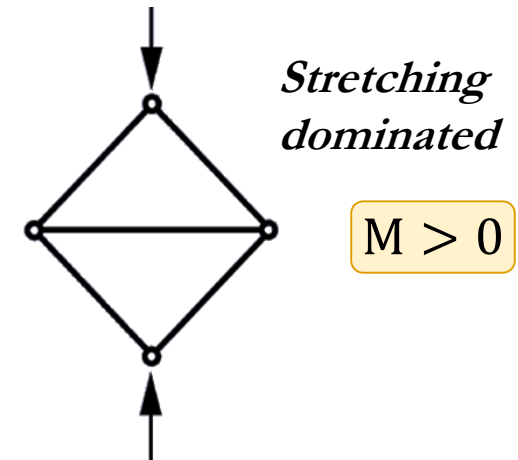
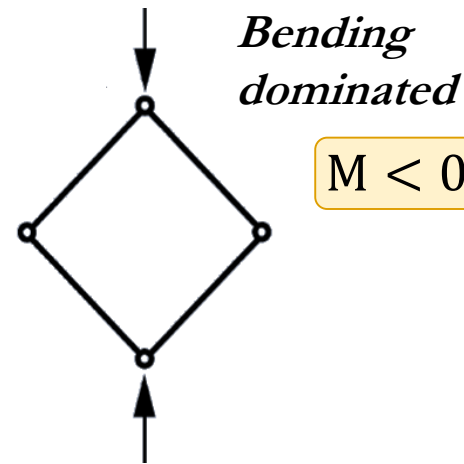
#### II.3.2c. Architecture



*Closed cells*

*Open cells*

The **architecture**, *i.e.* the way the matter is distributed within space:



The **connectivity** has a crucial effect on how the deformation takes place within the structure  
(*Maxwell criteria*)

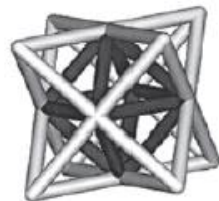
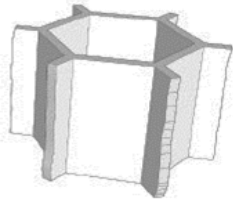
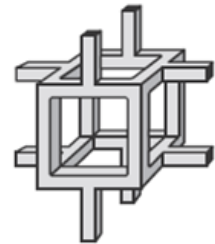
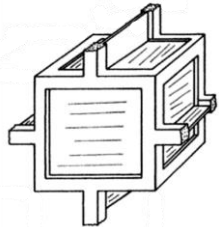
$$M = b - 3j + 6$$

$b$ : nb of struts  
 $j$ : nb of frictionless joints

# II. Generalities

## II.4. Cellular material characterization

### II.4.1. Geometry (architectures)



**Material**  
 Density,  $\rho^*$  ( $\text{kg/m}^3$ )  
 Open or closed cells  
 Edge connectivity,  $Z_e$   
 Face connectivity,  $Z_f$   
 Mean edges/face,  $\bar{n}^*$   
 Mean faces/cell,  $\bar{f}^*$   
 Cell shape\*  
 Symmetry of structure  
 Cell edge thickness,  $t_e$  ( $\mu\text{m}$ )  
 Cell face thickness,  $t_f$  ( $\mu\text{m}$ )  
 Fraction of material in cell edges,  $\phi$   
 Largest principal cell dimension,  $\bar{L}_1$  (mm)  
 Smallest principal cell dimension,  $\bar{L}_3$  (mm)  
 Intermediate principal cell dimension,  $\bar{L}_2$  (mm)  
 Shape anisotropy ratios,  $R_{12} = \bar{L}_1/\bar{L}_2$  and  
 $R_{13} = \bar{L}_1/\bar{L}_3$   
 Standard deviation of cell size (mm)  
 Other specific features  
 (periodic variations in density, cell size, etc.)

**Rigid polyurethane (Fig. 2.19)**

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Closed

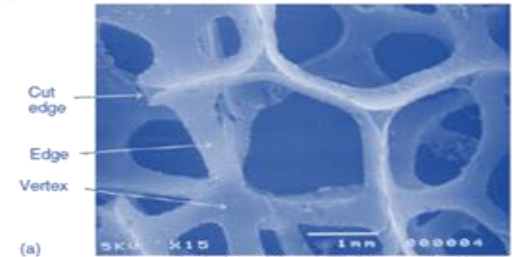
4

3

—

—

—



**Axisymmetric**

30

3

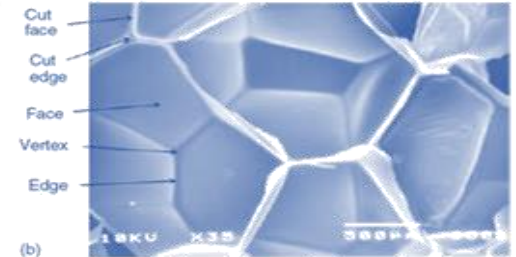
0.70

0.53

0.44

0.53

$R_{12} = 1.0$ ;  $R_{13} = 1.2$



0.075

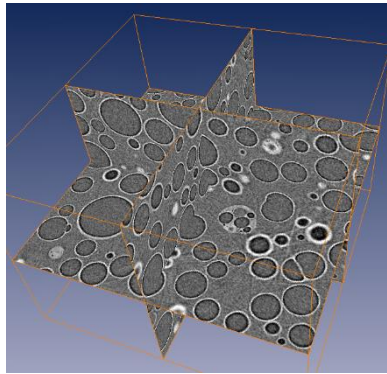
None

Tricky to characterize such microstructures based on 2D imaging techniques (SEM, Optical microscope ...)

## II. Generalities

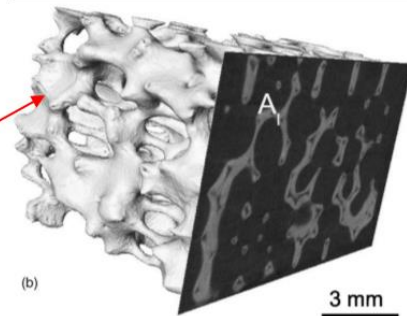
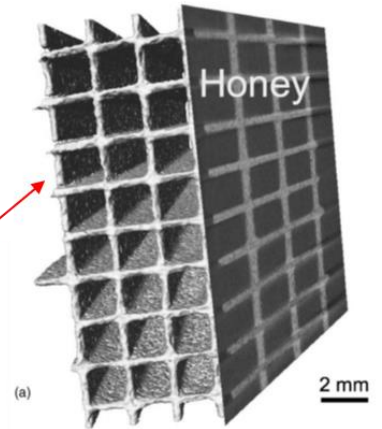
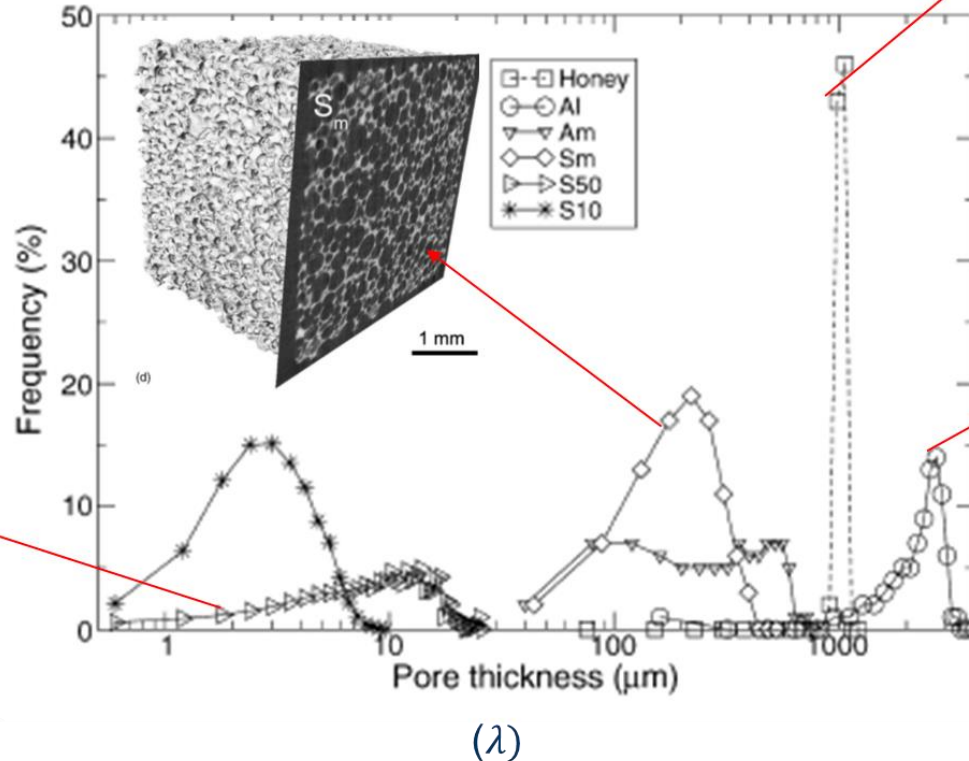
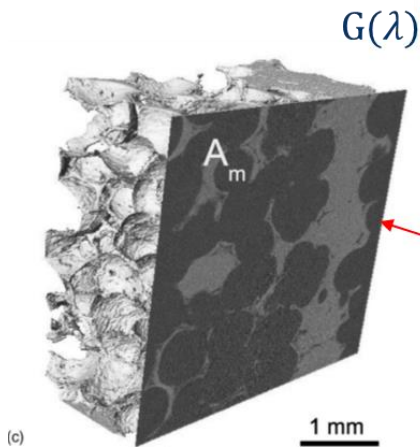
### II.4. Cellular material characterization

### II.4.1. Geometry (architectures)



### 3D imaging techniques & associated image processing

(X-ray computed tomography)



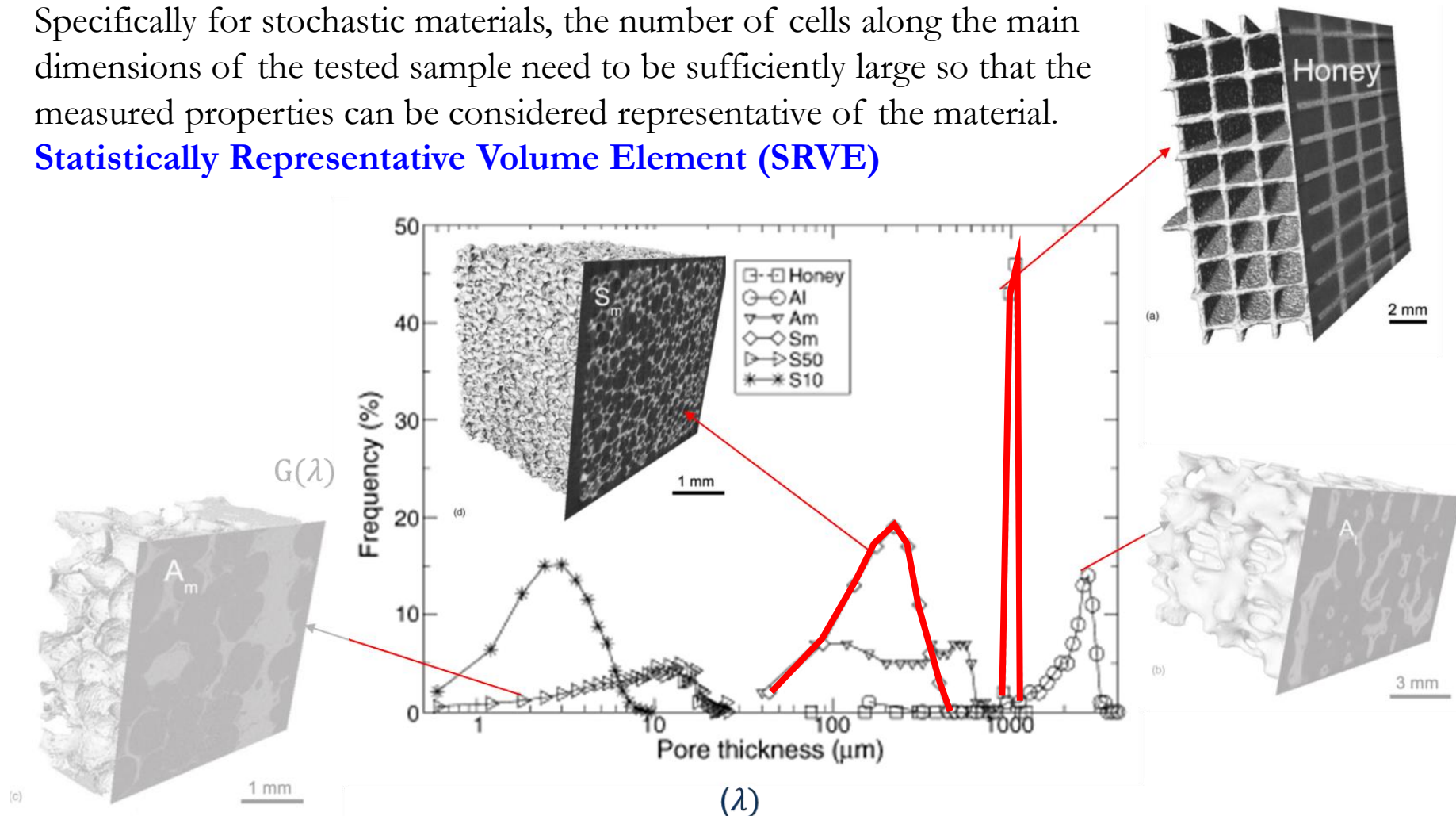
## II. Generalities

### II.4. Cellular material characterization

### II.4.2. Mechanical properties

Specifically for stochastic materials, the number of cells along the main dimensions of the tested sample need to be sufficiently large so that the measured properties can be considered representative of the material.

### Statistically Representative Volume Element (SRVE)

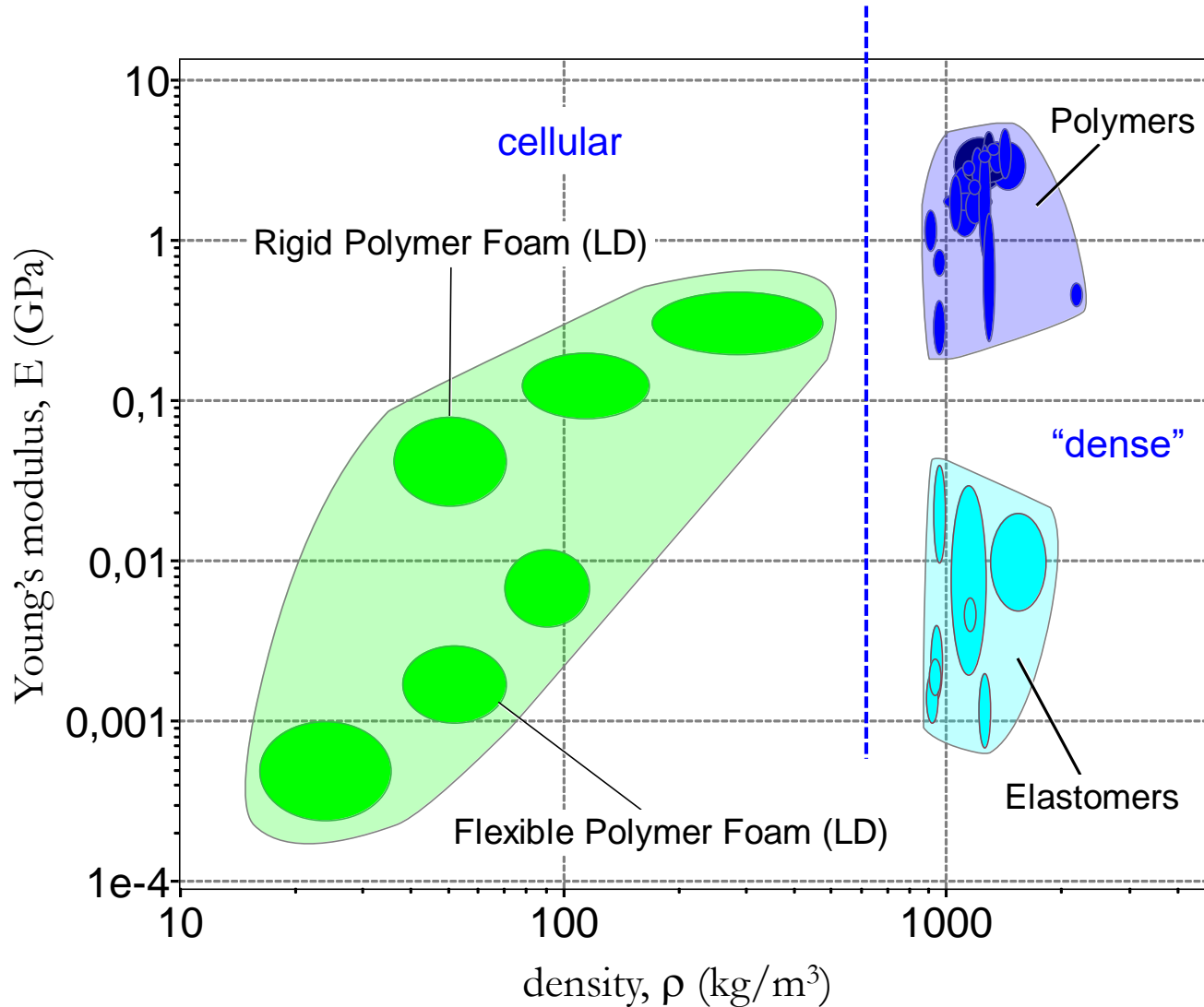




## II. Generalities

### II.5. Material & geometrical contributions

#### II.5.1. Open question



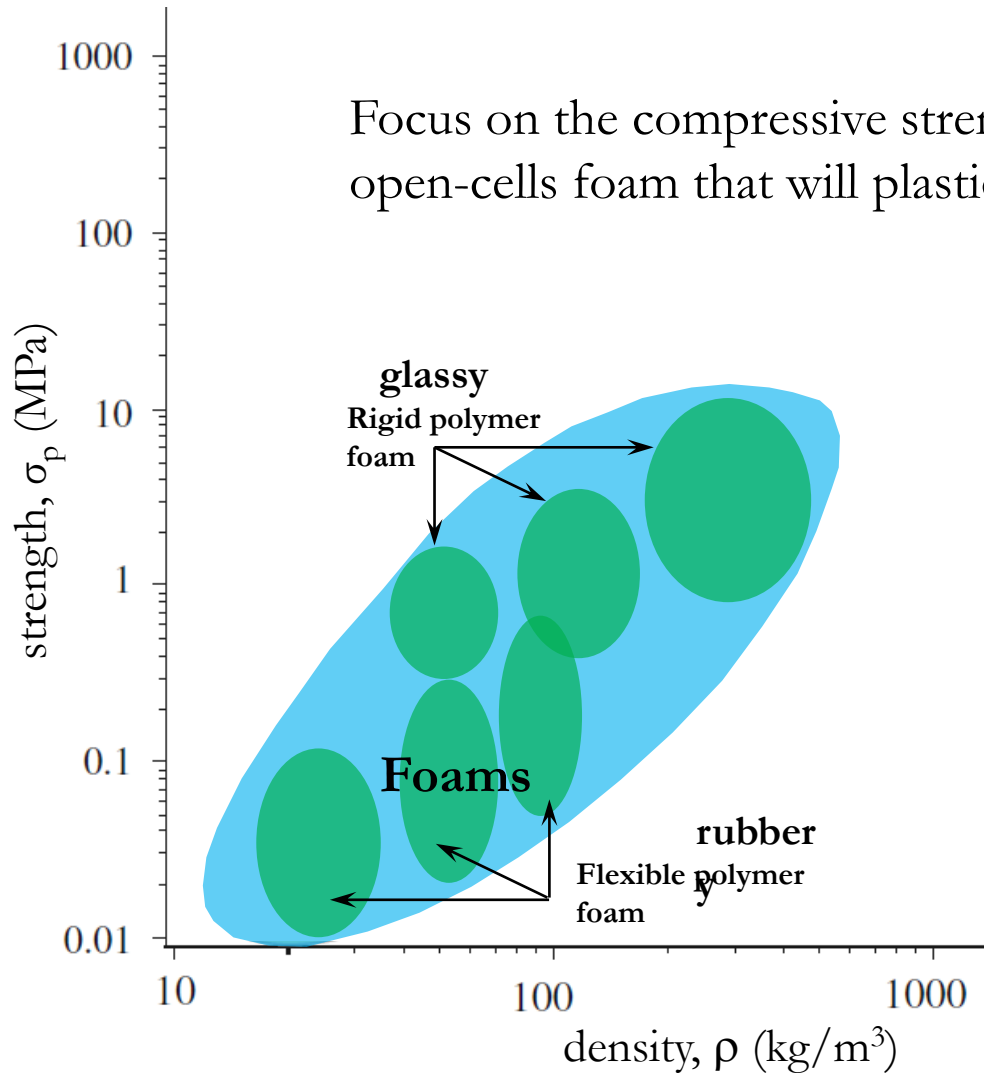
- Same architecture
- Different parent material
- Different architecture
- Same parent material
- Different architecture
- Different parent material

Dissociate the 2 contributions in order to “rank” the architectures

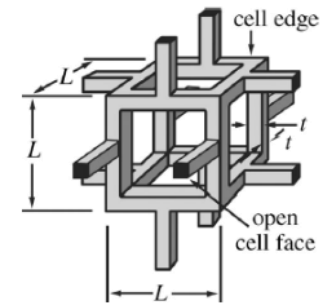
## II. Generalities

### II.5. Material & geometrical contributions

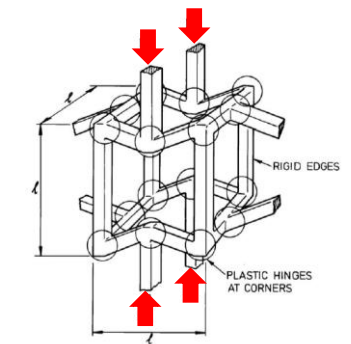
### II.5.2. Specific properties



*Ideal RVE*



*(at rest)*

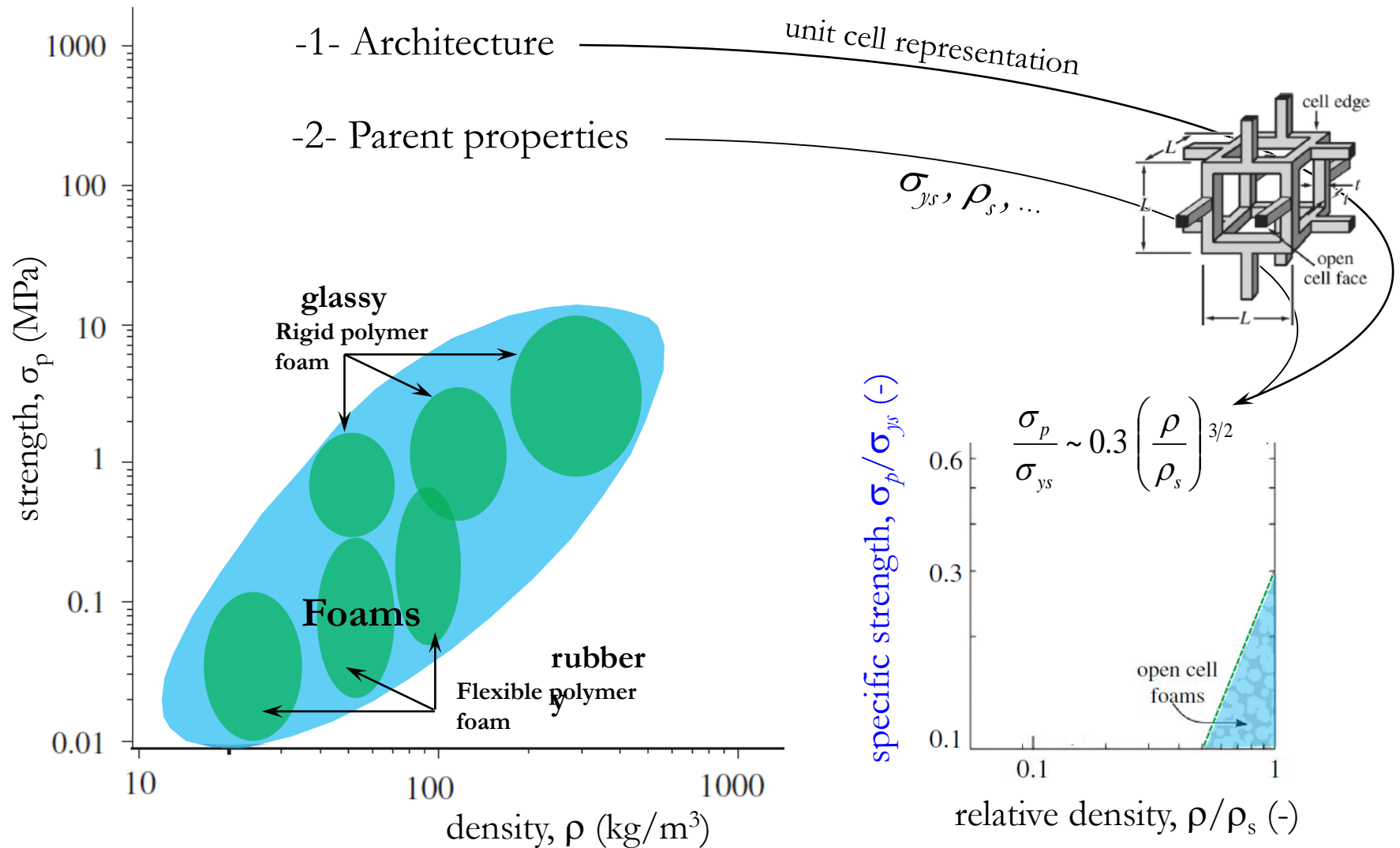


*(yielded)*

## II. Generalities

### II.5. Material & geometrical contributions

### II.5.2. Specific properties

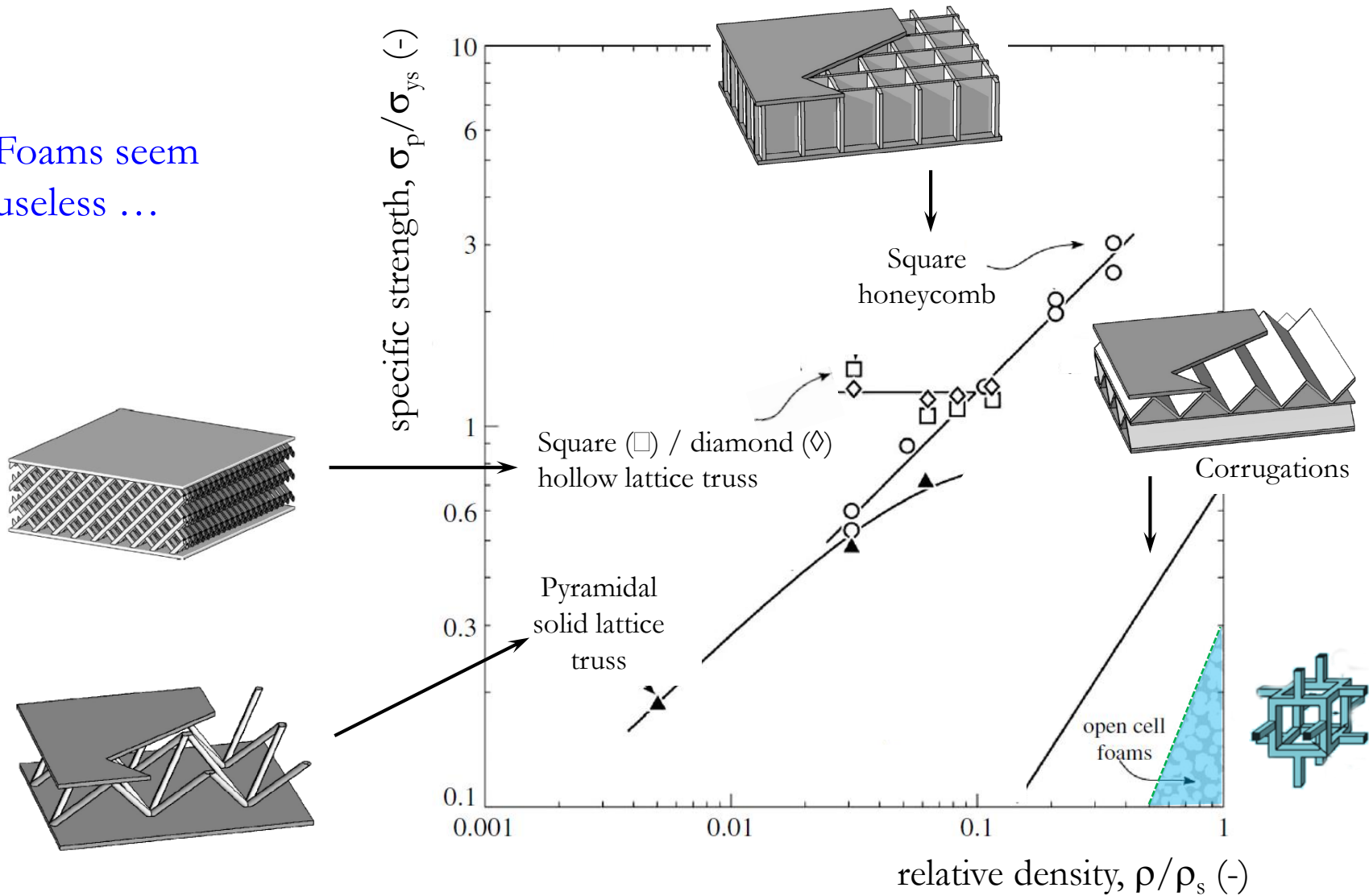


## II. Generalities

### II.5. Material & geometrical contributions

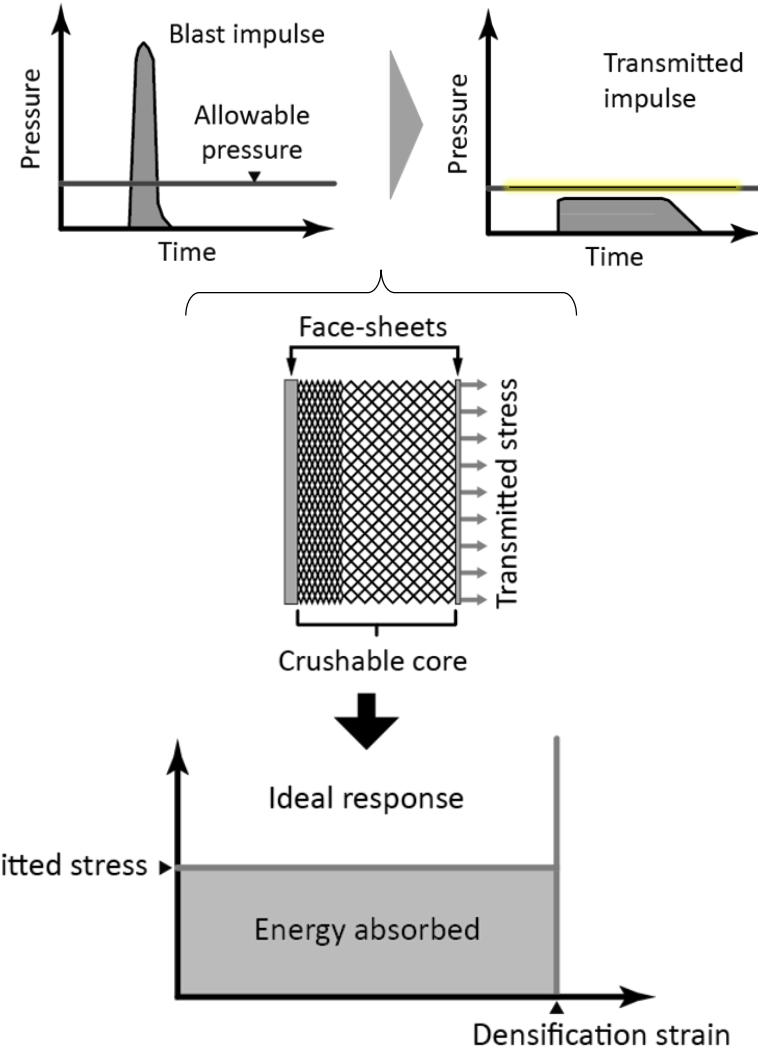
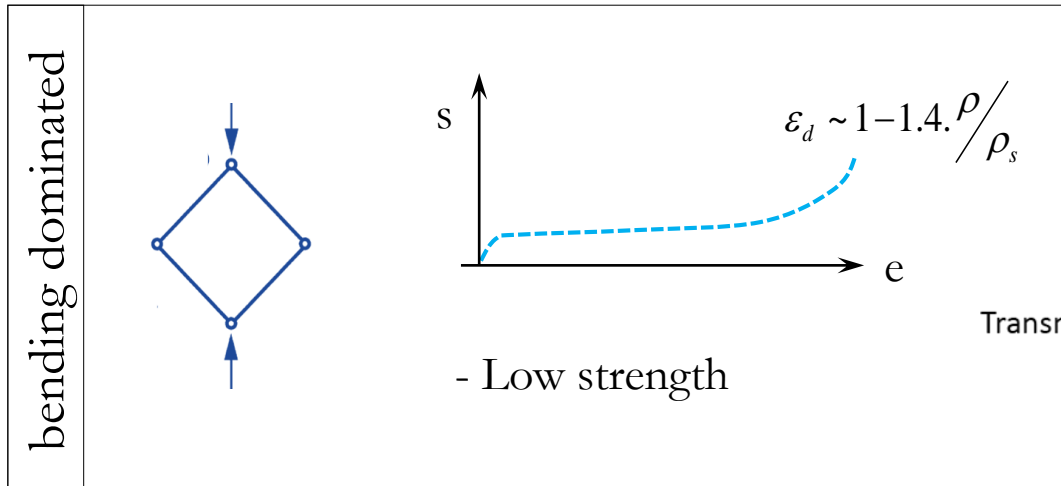
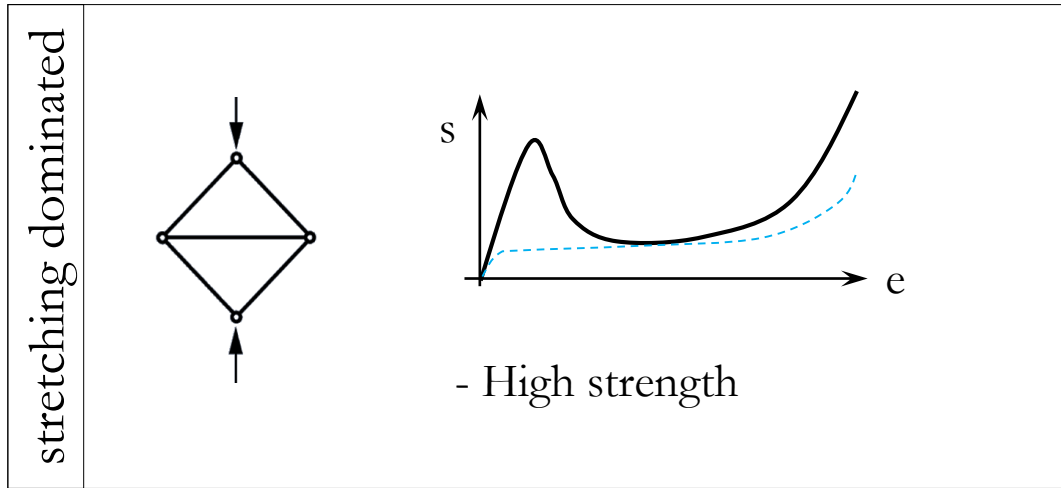
### II.5.2. Specific properties

Foams seem  
useless ...



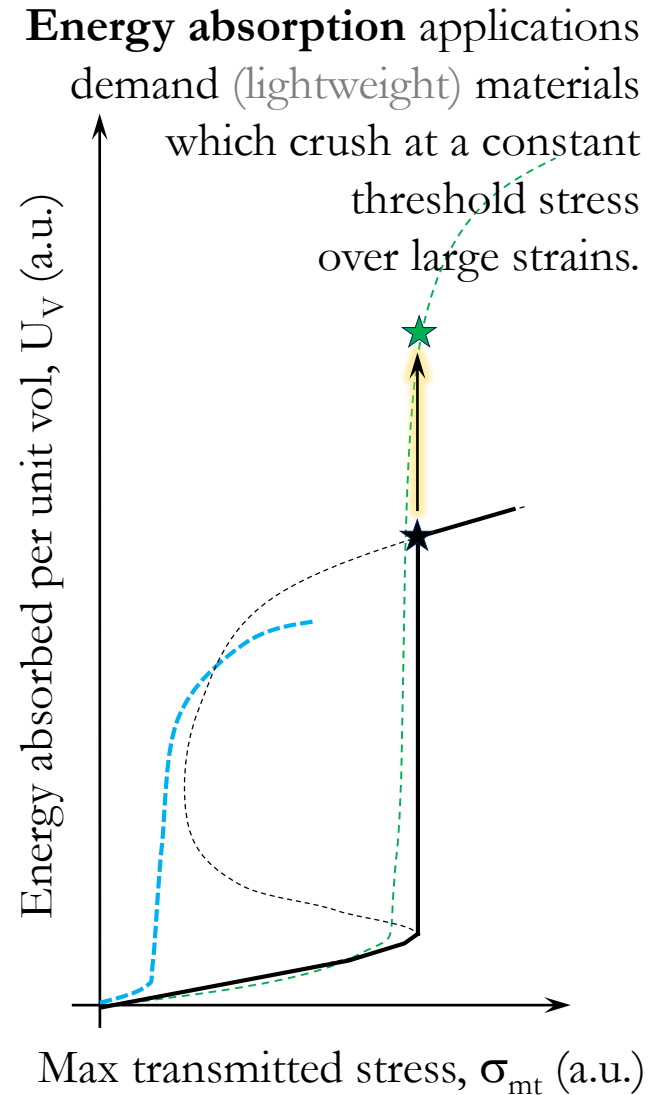
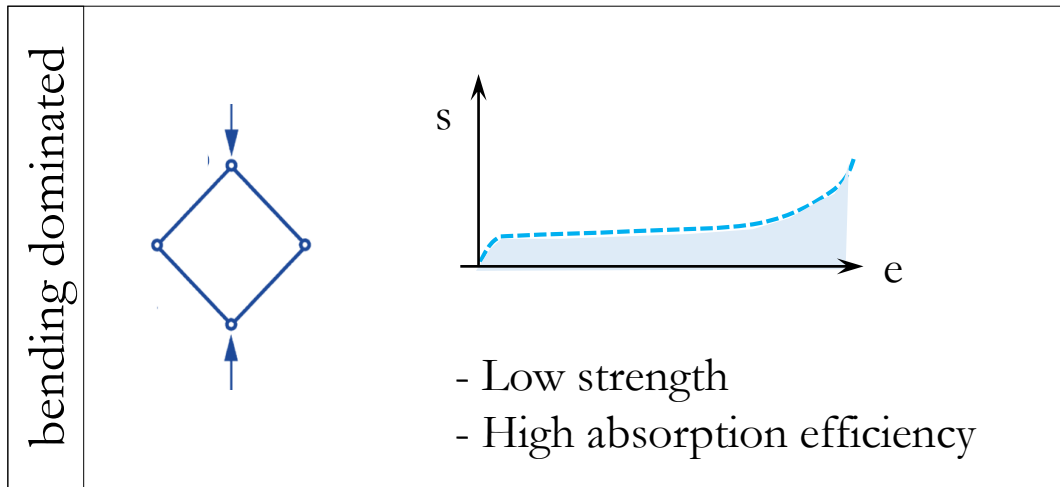
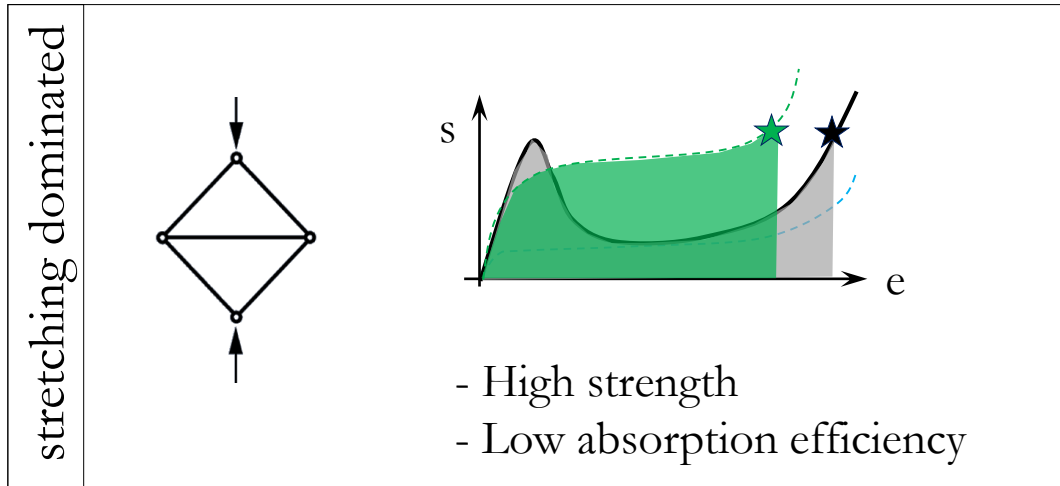
# II. Generalities

## II.6. Crushing response

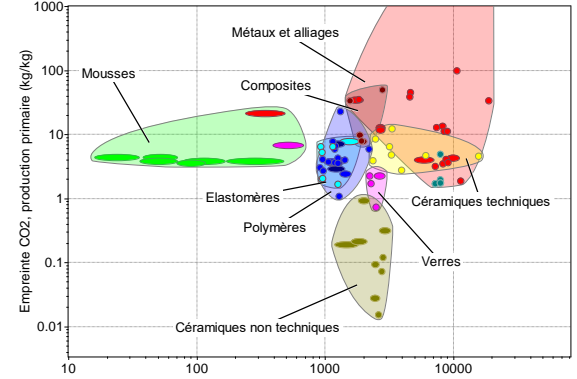
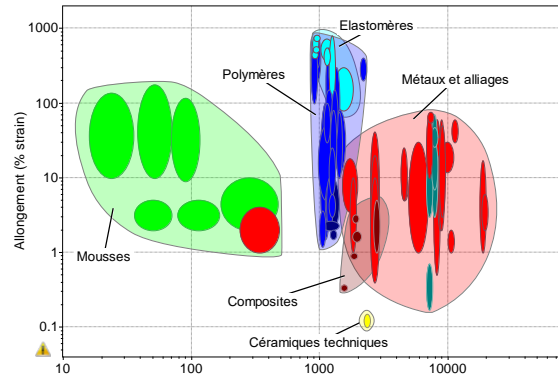
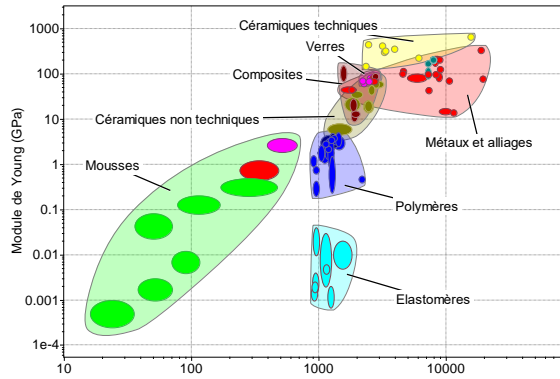


## II. Generalities

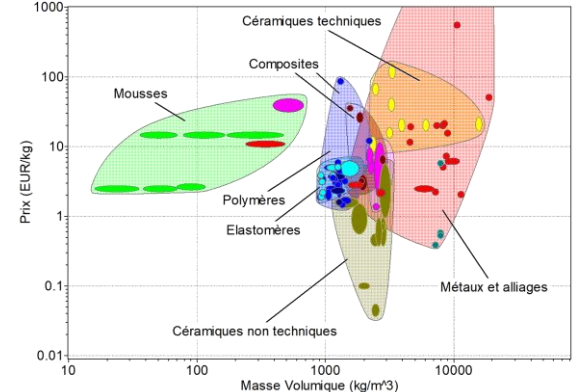
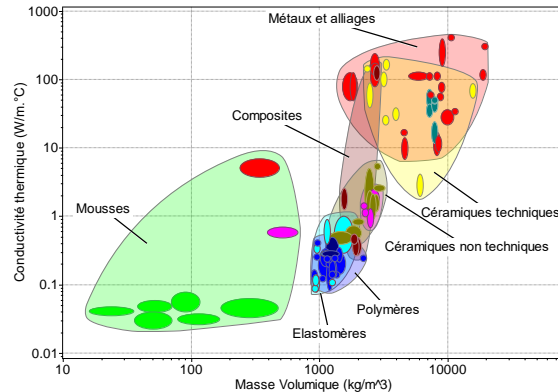
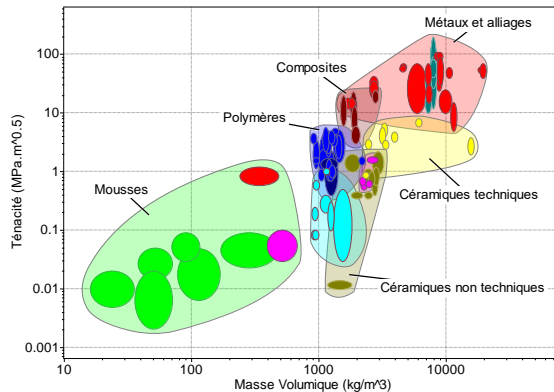
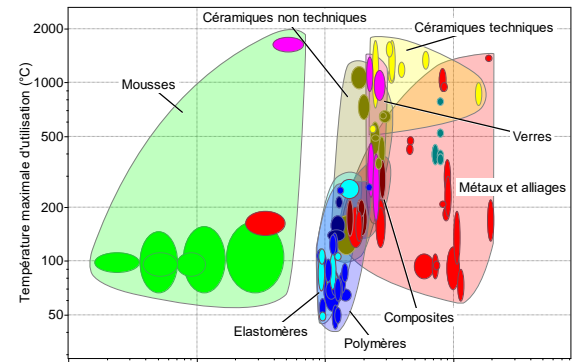
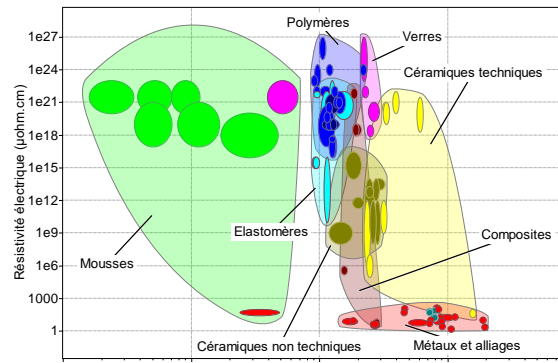
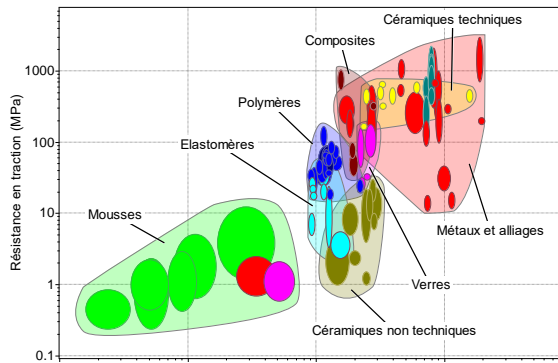
### II.6. Crushing response



## II. Generalities: Summary



Parent material properties & geometry trigger the cellular materials properties



# Cellular (solids) materials

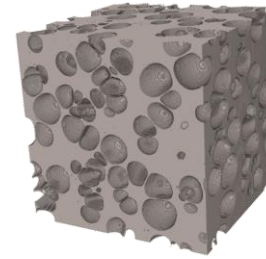
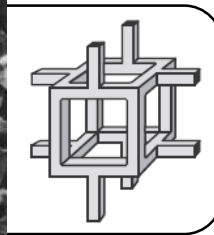
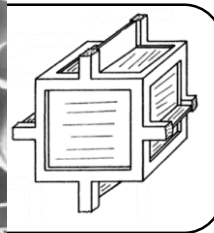
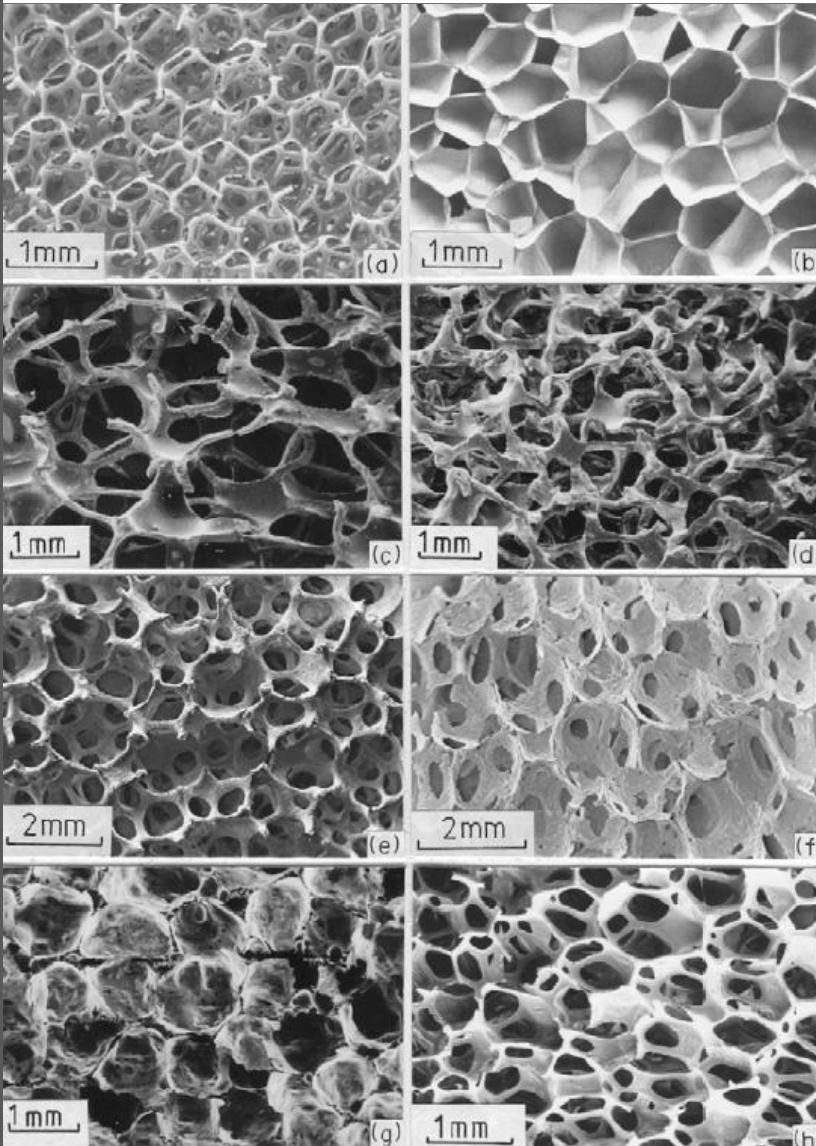
## GM5 – CE

### III - FOAMS



# III. Foams

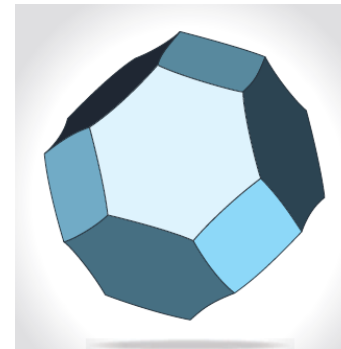
## III.1. Examples



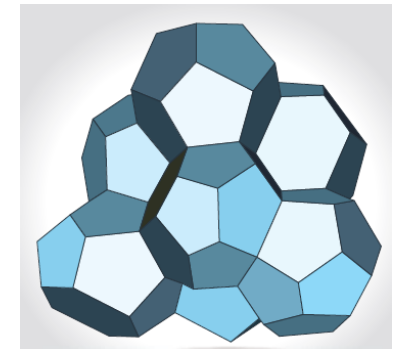
← *Elastomeric foam of a table tennis racket pad*

Various microstructures justifying the use/definition of different ideal unit cells (RVE)

Kelvin cell  
Tetrakaidecahedron  
(14 faces)

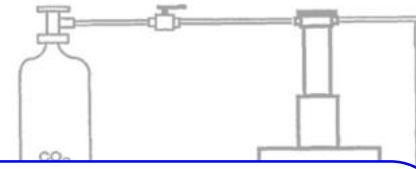


Weaire & Phelan cells  
(isovolume and maximized specific surface area)



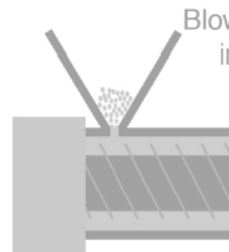
# III. Foams

## III.2. Processing



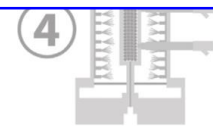
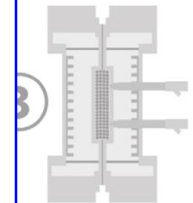
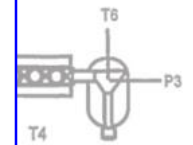
Gas has to be incorporated within the monomers or the molten polymers  $\Rightarrow$  **blowing agent**

- **Physical agents** (e.g.  $\text{CO}_2$ ,  $\text{N}_2$ ) introduced under high pressure into molten polymers (or polymer solutions) which then expand to form bubbles by progressive pressure reduction.
- **Physical agents** vaporizing with heat.
- **Chemical agents** which decompose or recombine under the influence of heat to release gases.



Foam extrusion system

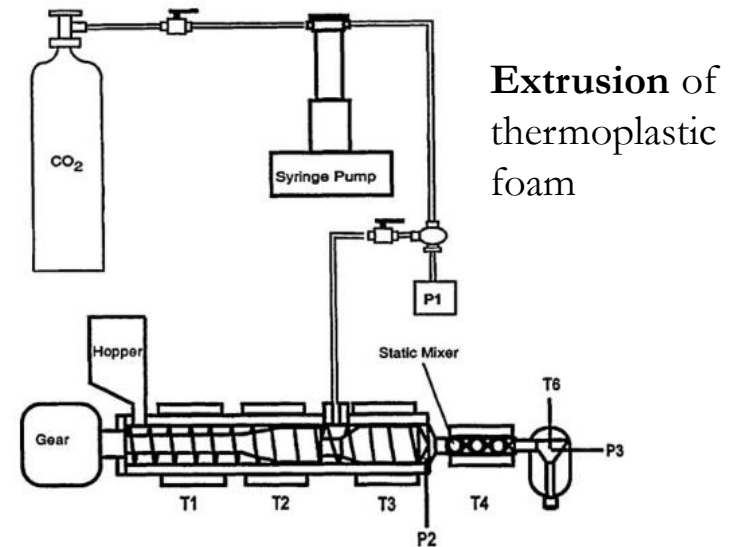
Rotating knives



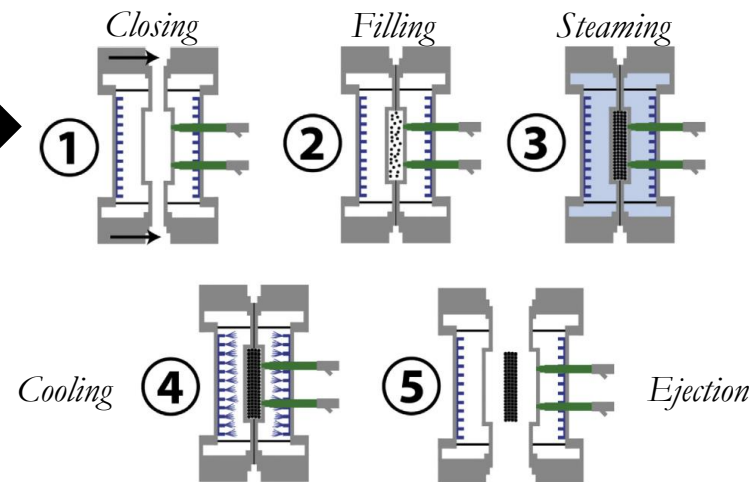
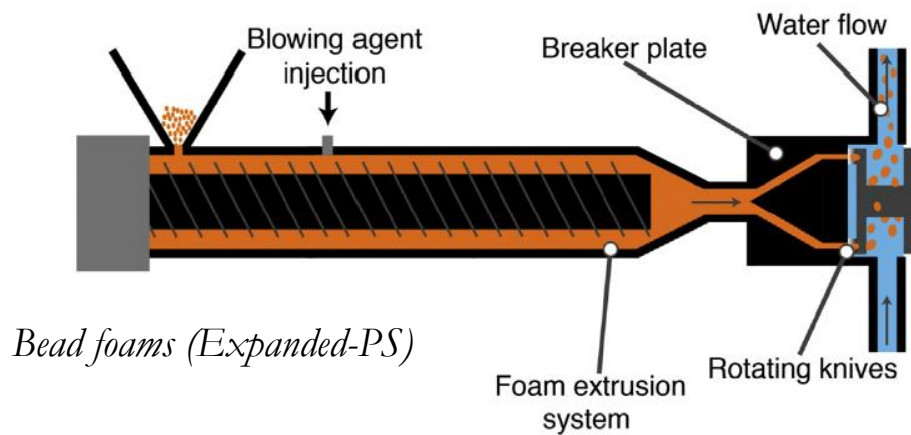
# III. Foams

## III.2. Processing

Part A & B are **mixed** + blowing agent +



### Steam chest molding

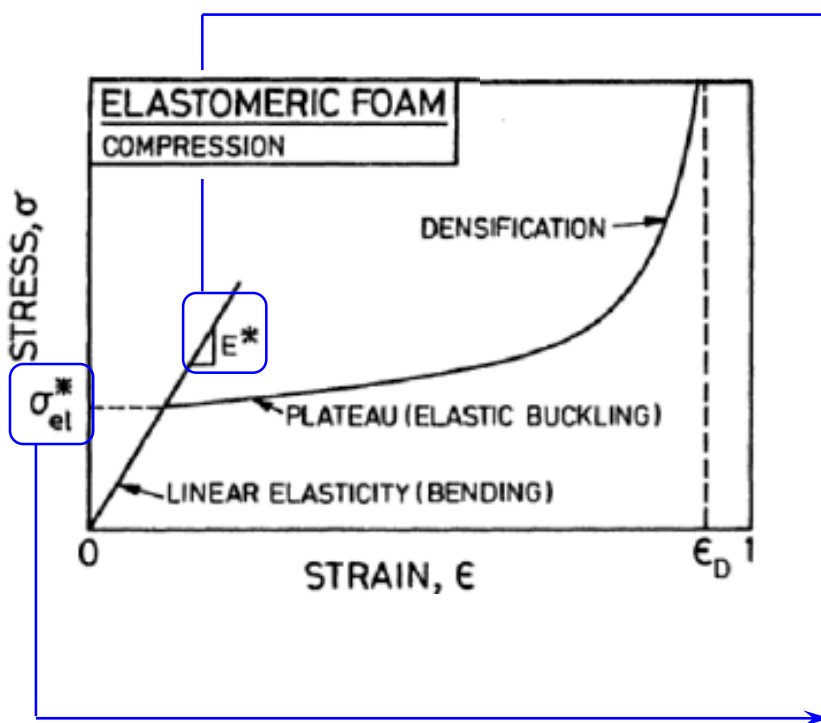


# III. Foams

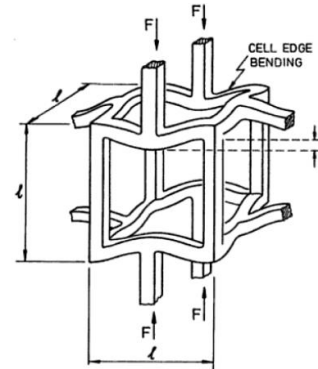
## III.3. Mechanical properties: compression

### Macroscopic behavior

3 regimes are evidenced:  
Linear, Plateau, densification



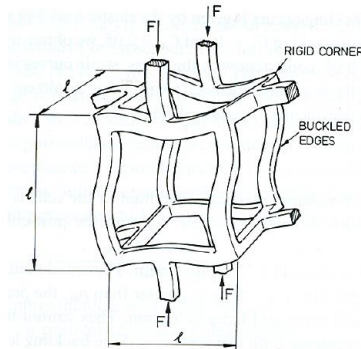
### Micromechanical predictions



Bending of the edges

$$\frac{E}{E_s} = C_1 \left[ \frac{\rho}{\rho_s} \right]^2$$

$C_1 \approx 1$  (exp. data)



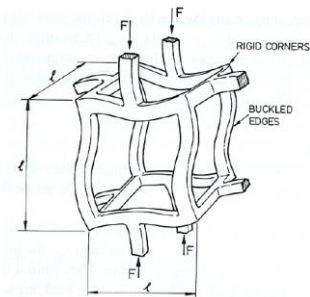
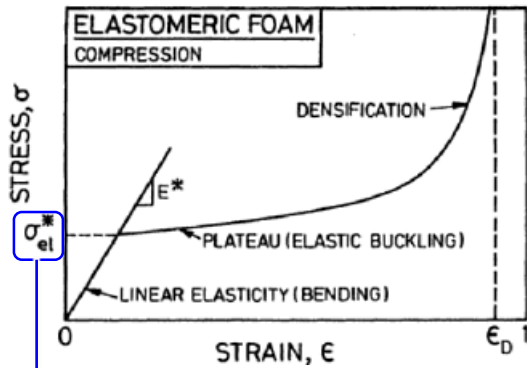
Elastic buckling of the edges

$$\frac{\sigma_y}{E_s} = C_4 \left[ \frac{\rho}{\rho_s} \right]^2$$

$C_4 \approx 0.05$  (exp. data)

# III. Foams

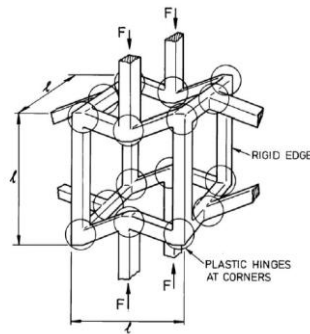
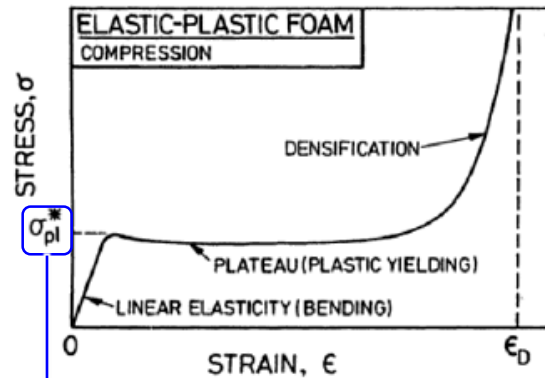
## III.3. Mechanical properties: compression



Elastic buckling of the edges

$$\sigma_y = C_4 \left[ \frac{\rho}{\rho_s} \right]^2 E_s$$

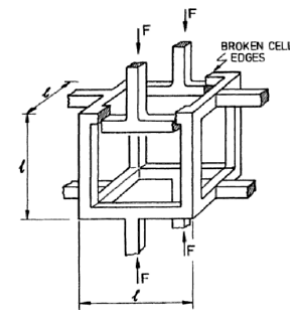
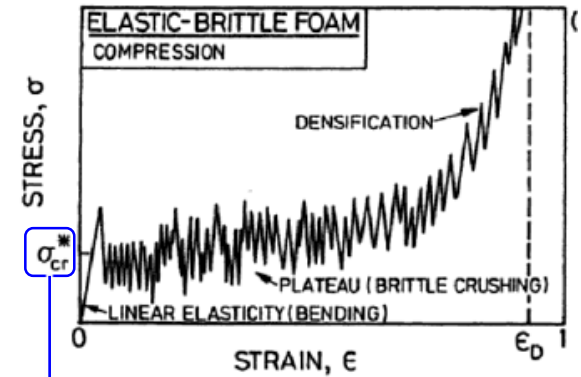
$C_4 \approx 0.05$  (exp. data)



Plastic hinging of the edges

$$\sigma_y = C_5 \left[ \frac{\rho}{\rho_s} \right]^{3/2} \sigma_{ys}$$

$C_5 \approx 0.3$  (exp. data)



Brittle failure of the edges

$$\sigma_y = C_6 \left[ \frac{\rho}{\rho_s} \right]^{3/2} \sigma_{rs}$$

$C_6 \approx 0.2$  (exp. data)

# Cellular (solids) materials

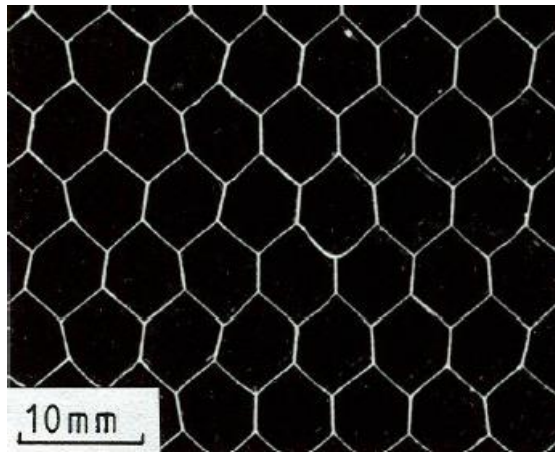
## GM5 – CE

### IV - HONEYCOMBS

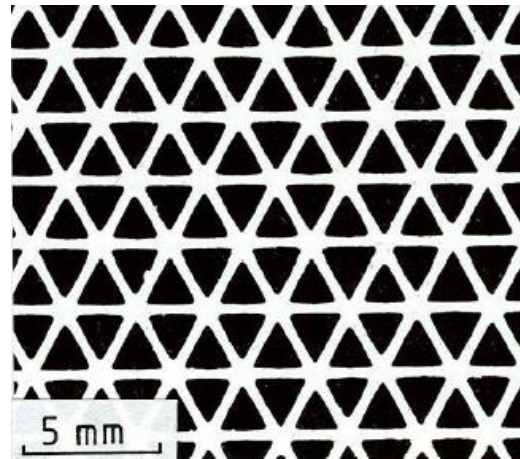
# IV. Honeycombs

## IV.1. Examples

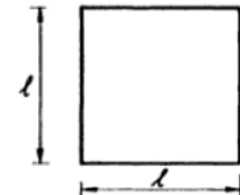
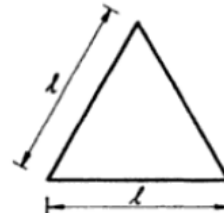
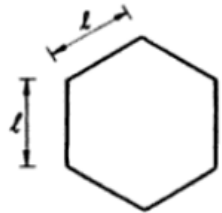
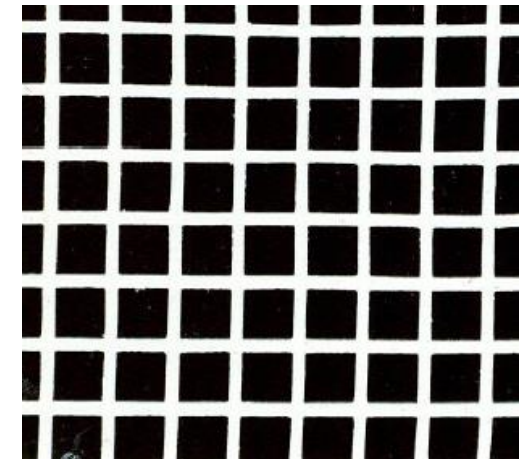
*Hexagonal / Aluminium*



*Triangular / Aluminium*



*Cubic / Ceramic*

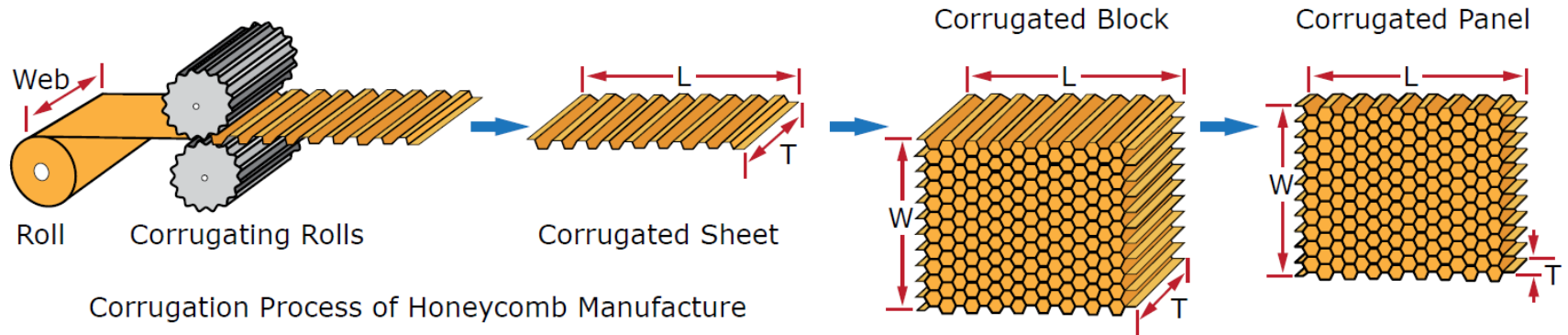


Relative density:  $\phi = \frac{\rho}{\rho_s} = C_1 \frac{t}{l}$

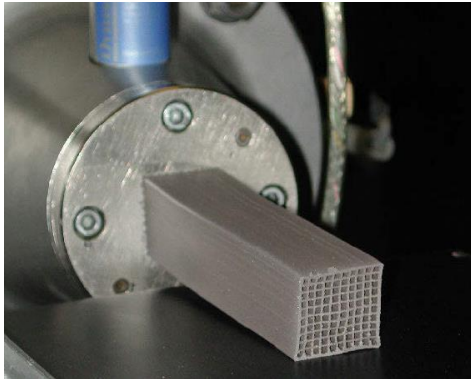
$t$  — Walls thickness  
 $l$  — Walls length  
 $C_1$  — Constant depending on the cells geometry

# IV. Honeycombs

## IV.1. Processing



### Extrusion



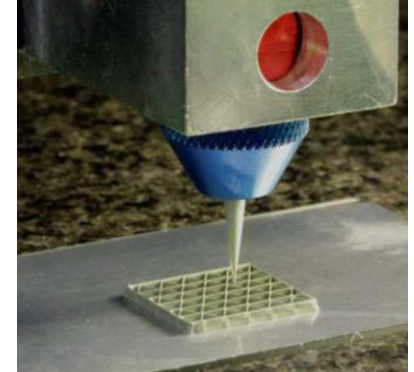
<http://ocw.mit.edu/help/faq-fair-use/>

### Injection molding

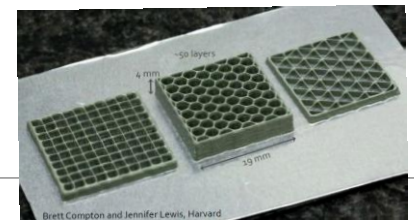


(BMW i3)

### Casting, ...



### 3D printing

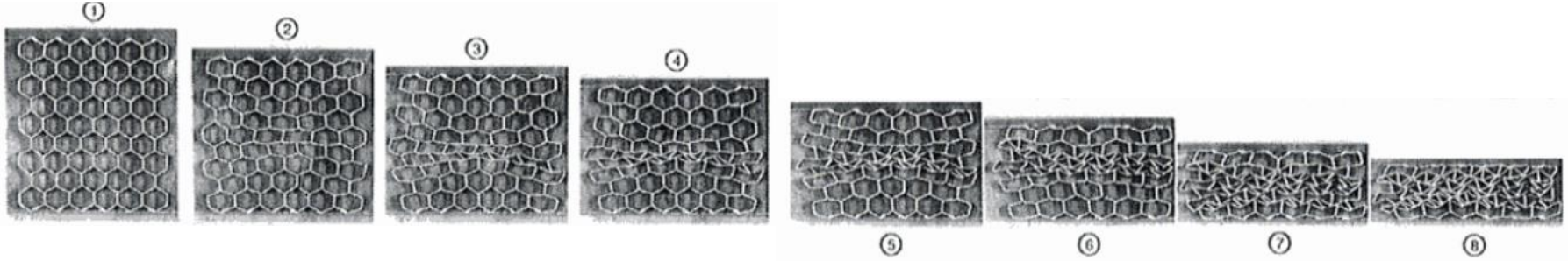


Brett Compton and Jennifer Lewis, Harvard

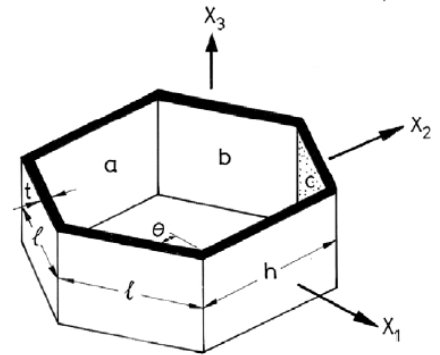
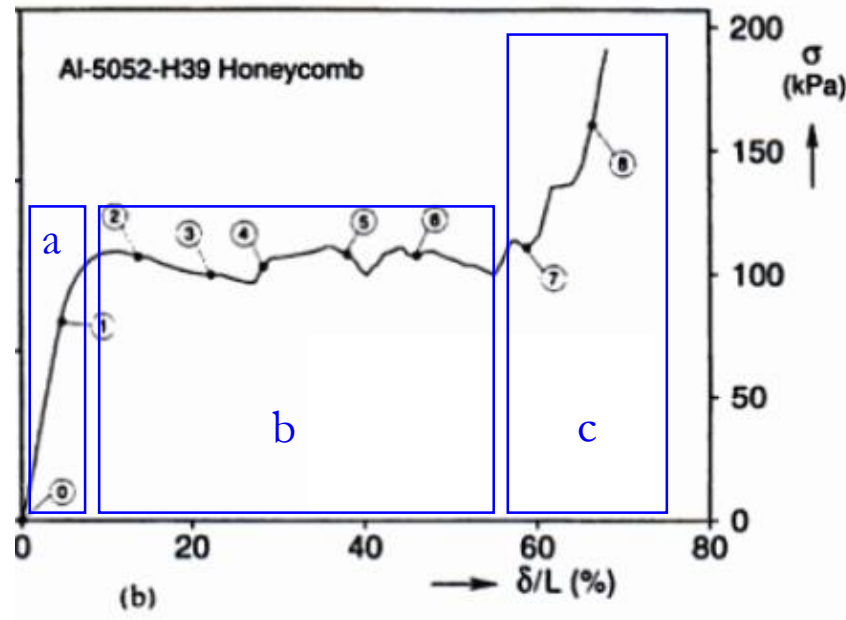


# IV. Honeycombs

## IV.1. Mechanical properties: compression



- a. Linear regime
- b. Plateau regime
- c. Densification



$$\frac{E_3}{E_2} \approx \left[ \frac{l}{t} \right]^2 \gg 1$$

Pronounced anisotropy

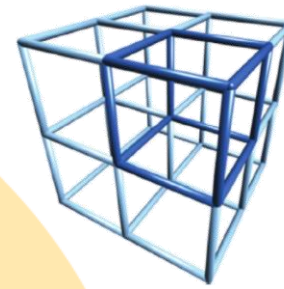
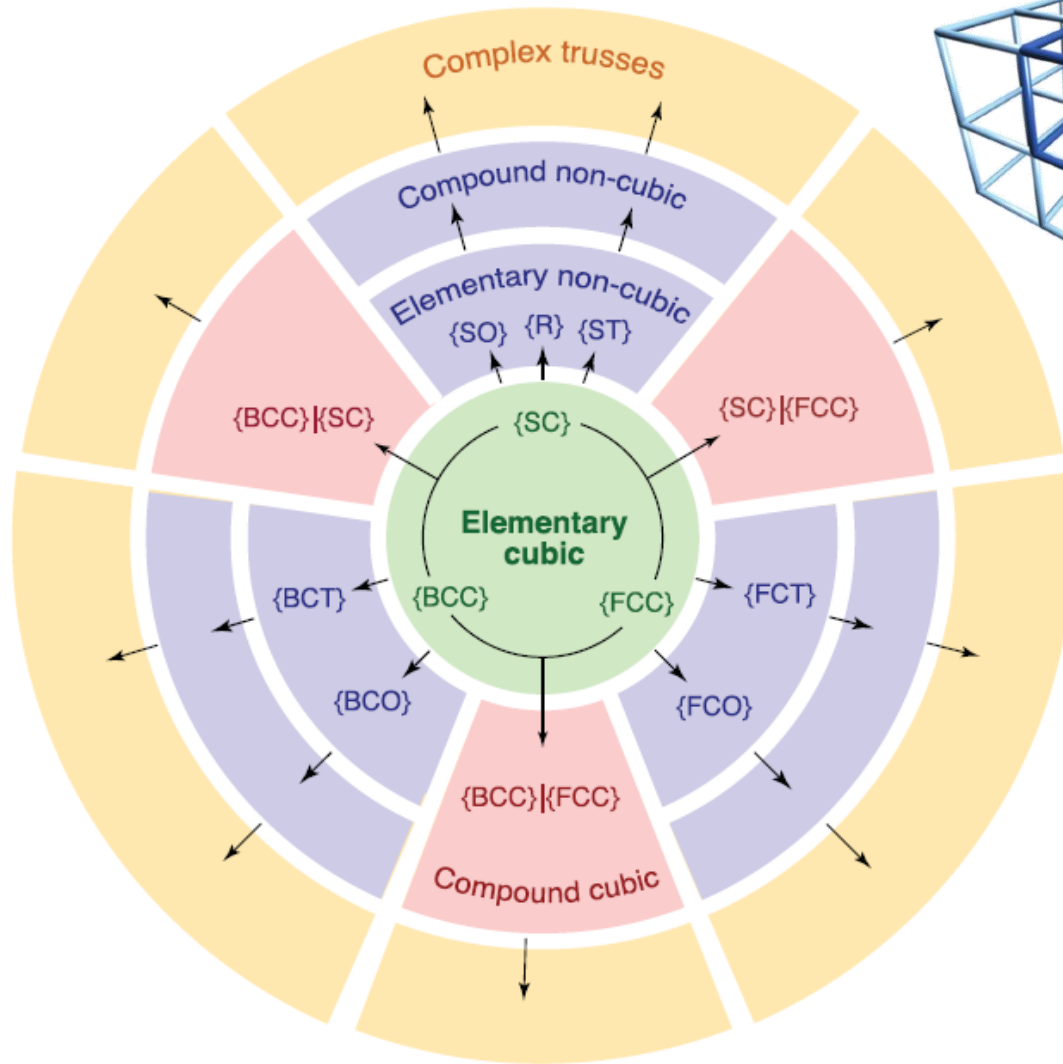
# Cellular (solids) materials

## GM5 – CE

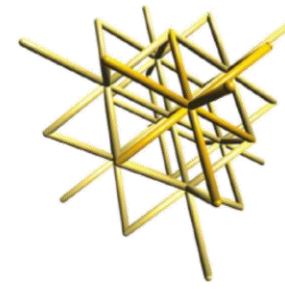
# V - LATTICES

# V. Lattices

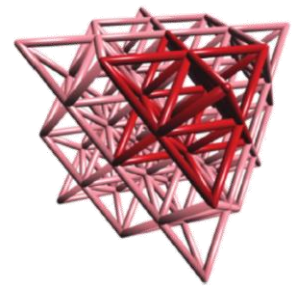
## V.1. Taxonomy



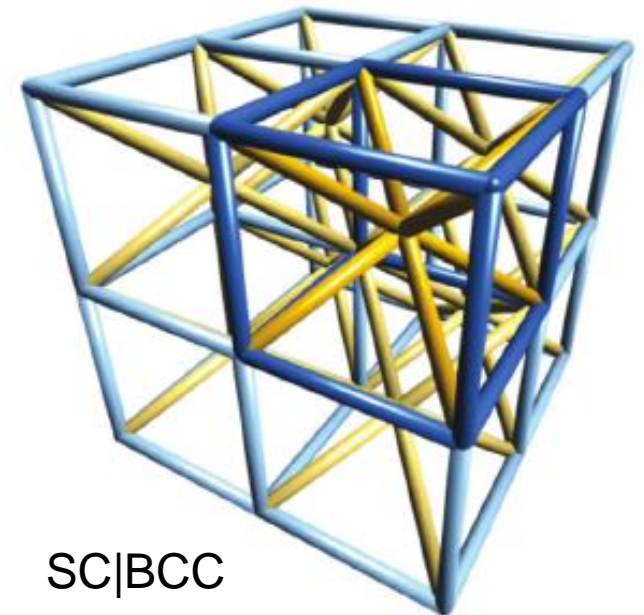
SC



BCC



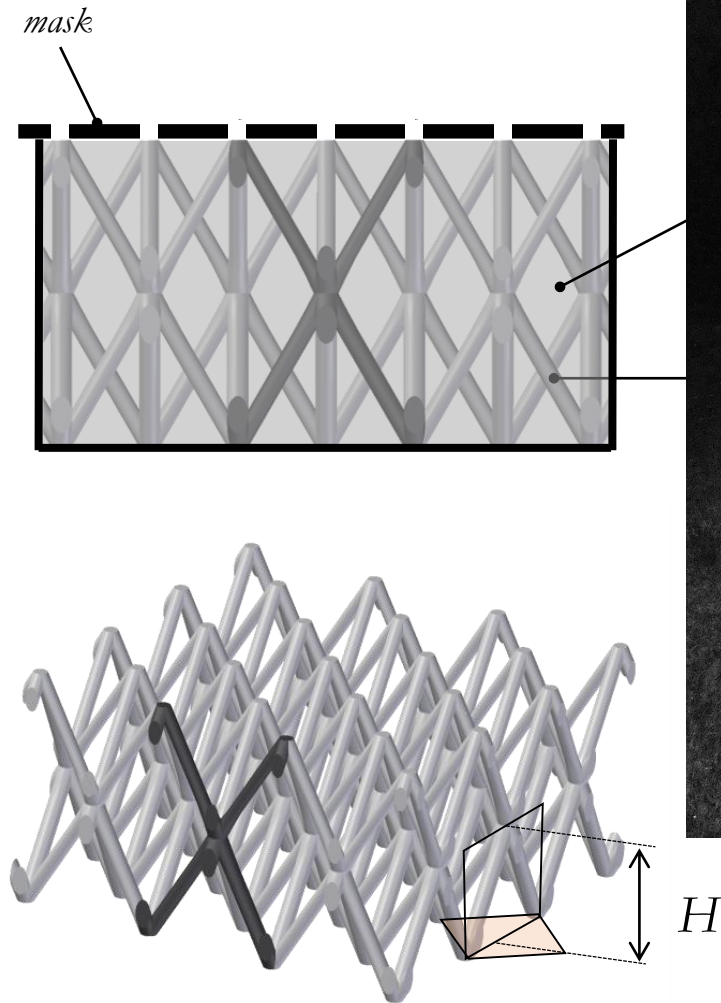
FCC



SC|BCC

# V. Lattices

## V.2. Materials by design: an example



Mask (apertures, dispersion)

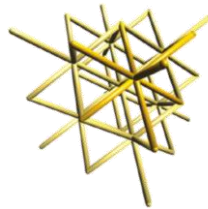
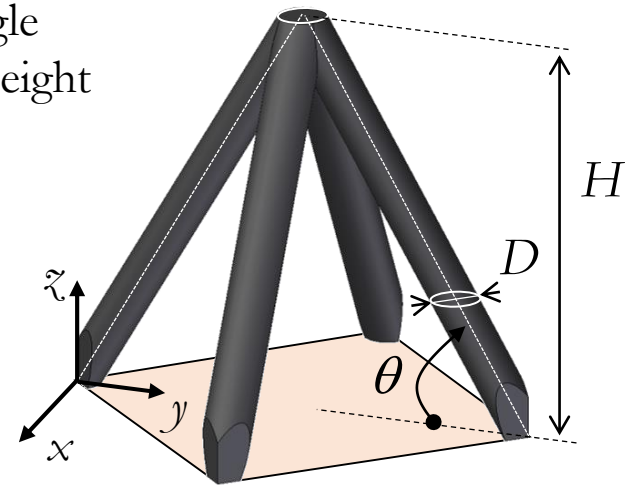
$N_l$ : Number of layers

$N_s$ : Number of merging struts

$\theta$ : Strut angle

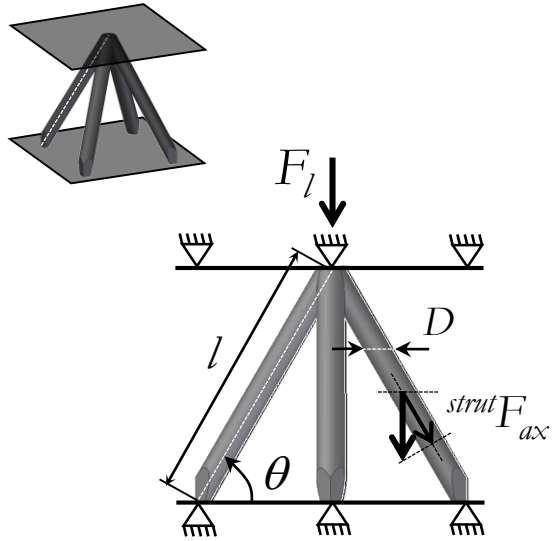
$H$ : Lattice height

R.V.E.



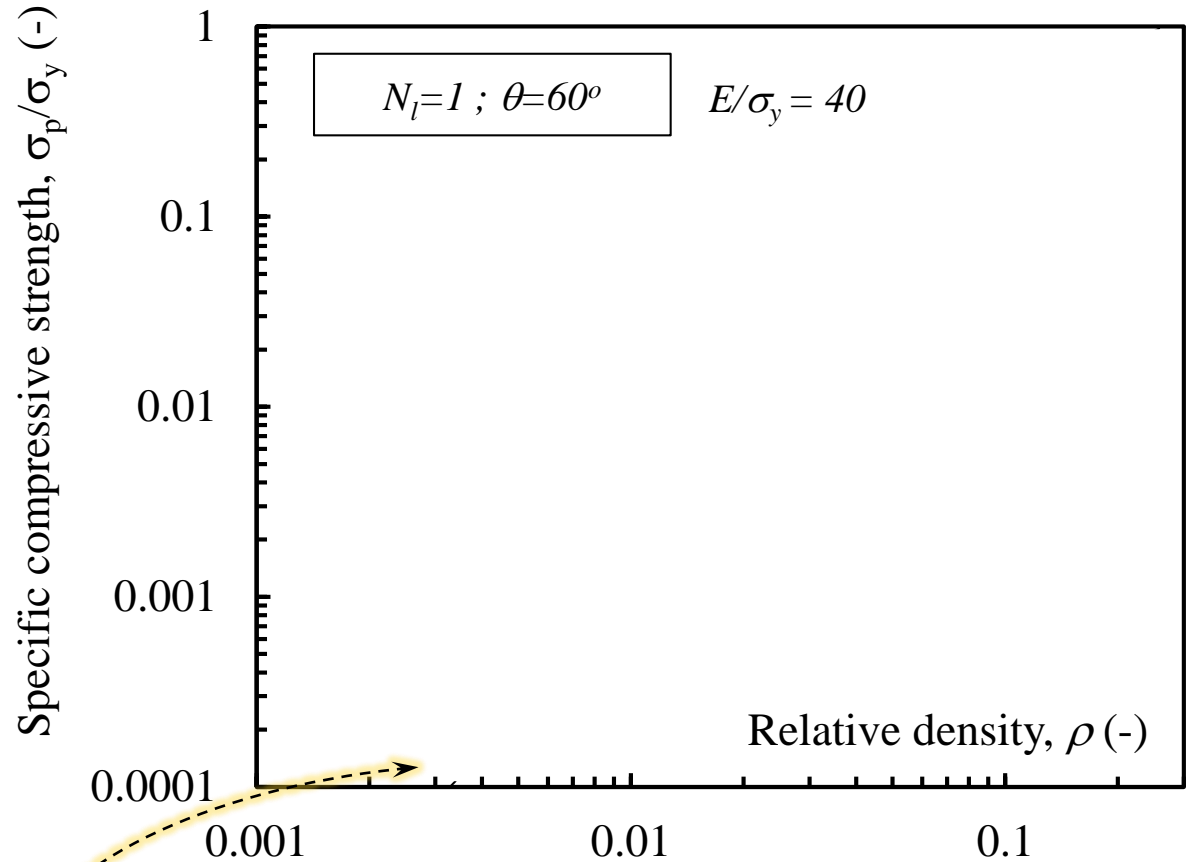
# V. Lattices

## V.2. Materials by design: an example

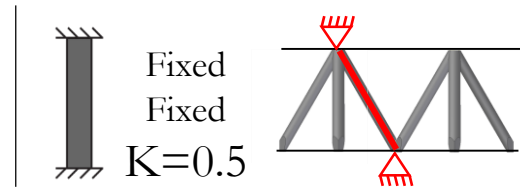


**Buckling:**

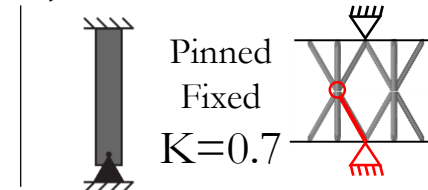
$$strut F_{ax} = \frac{\pi^3 E D^4}{(8Kl)^2 \cos^3 \theta}$$



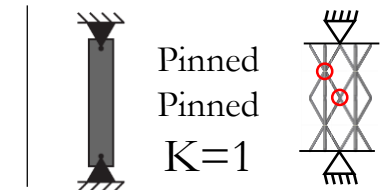
$N_l = 1$



$N_l = 2$

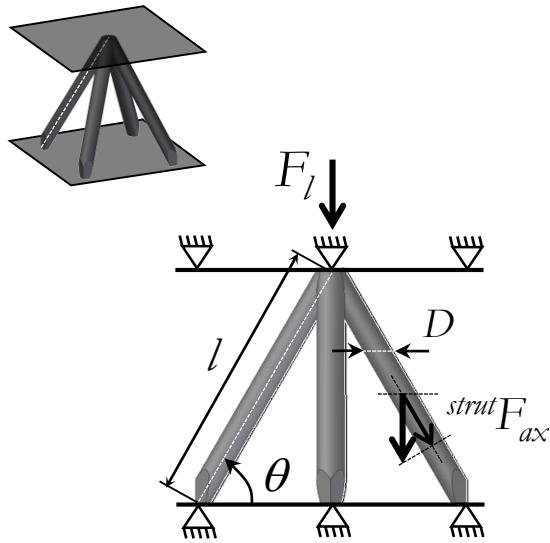


$N_l > 2$



# V. Lattices

## V.2. Materials by design: an example

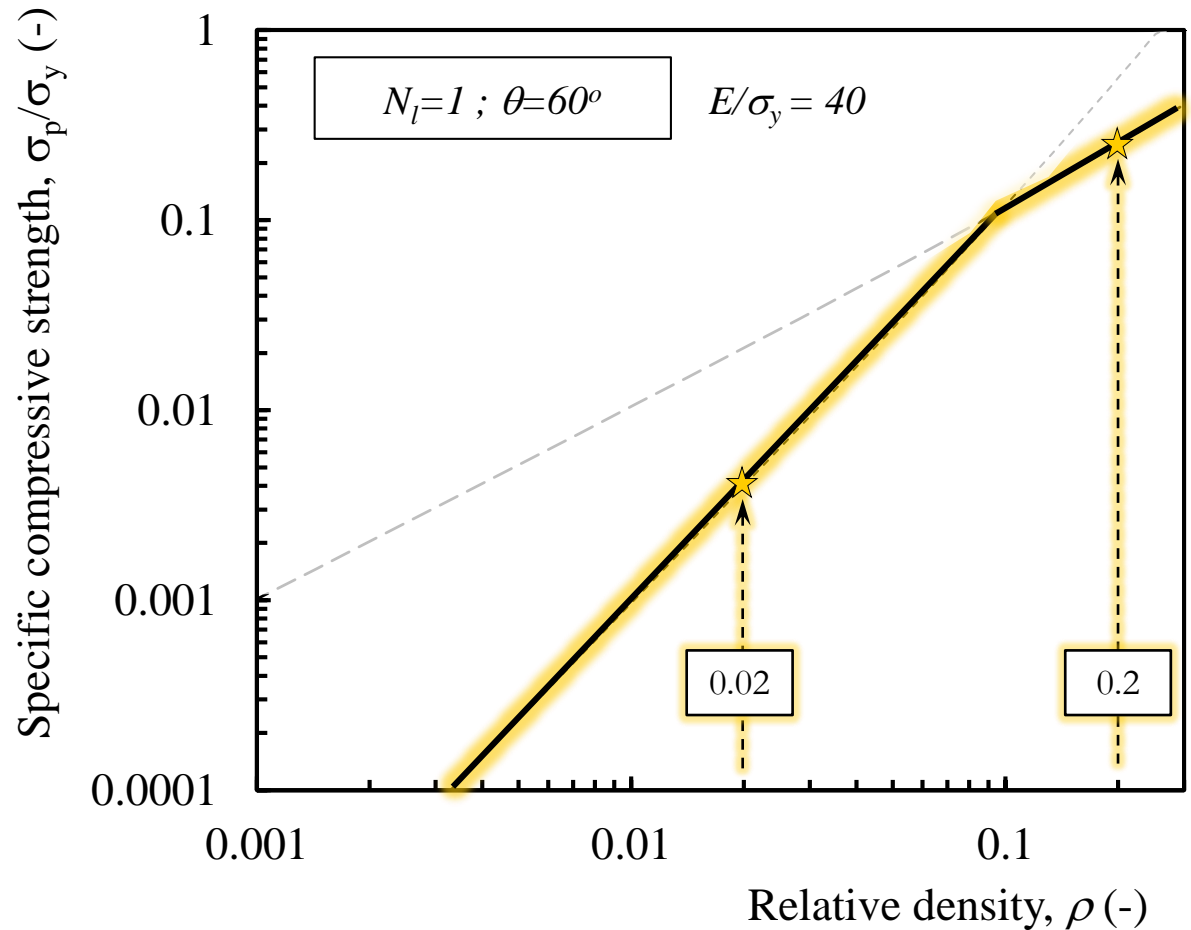


Buckling:

$$strut F_{ax} = \frac{\pi^3 E D^4}{(8Kl)^2 \cos^3 \theta}$$

Yielding:

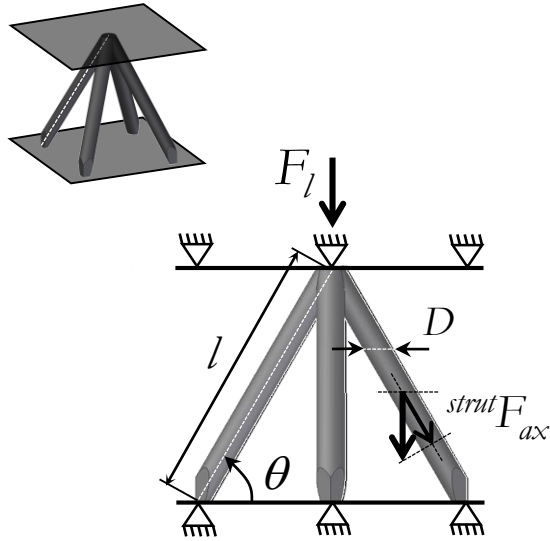
$$strut F_{ax} = \sigma_y \left( \frac{\pi}{4} D^2 \cos \theta \right)$$



The failure mode is triggered by the **geometry** of the lattice

# V. Lattices

## V.2. Materials by design: an example

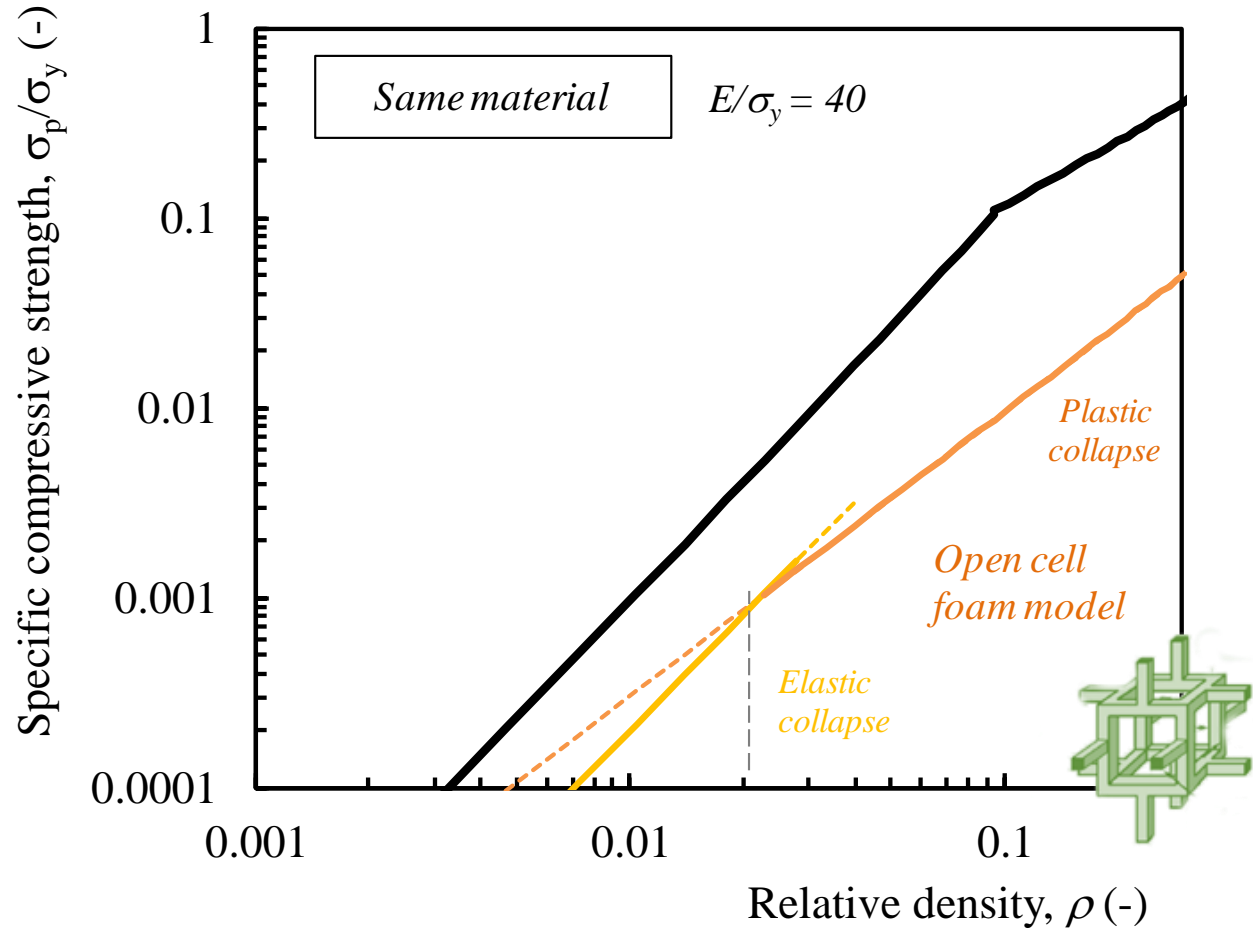


Buckling:

$$strut F_{ax} = \frac{\pi^3 E D^4}{(8Kl)^2 \cos^3 \theta}$$

Yielding:

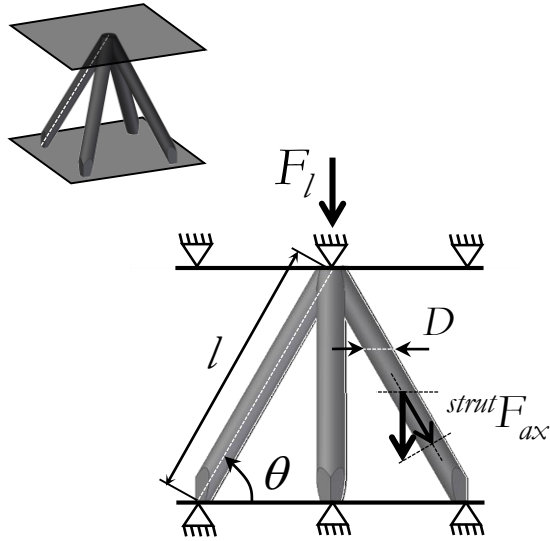
$$strut F_{ax} = \sigma_y \left( \frac{\pi}{4} D^2 \cos \theta \right)$$



The failure mode is triggered by the **geometry** of the lattice

# V. Lattices

## V.2. Materials by design: an example

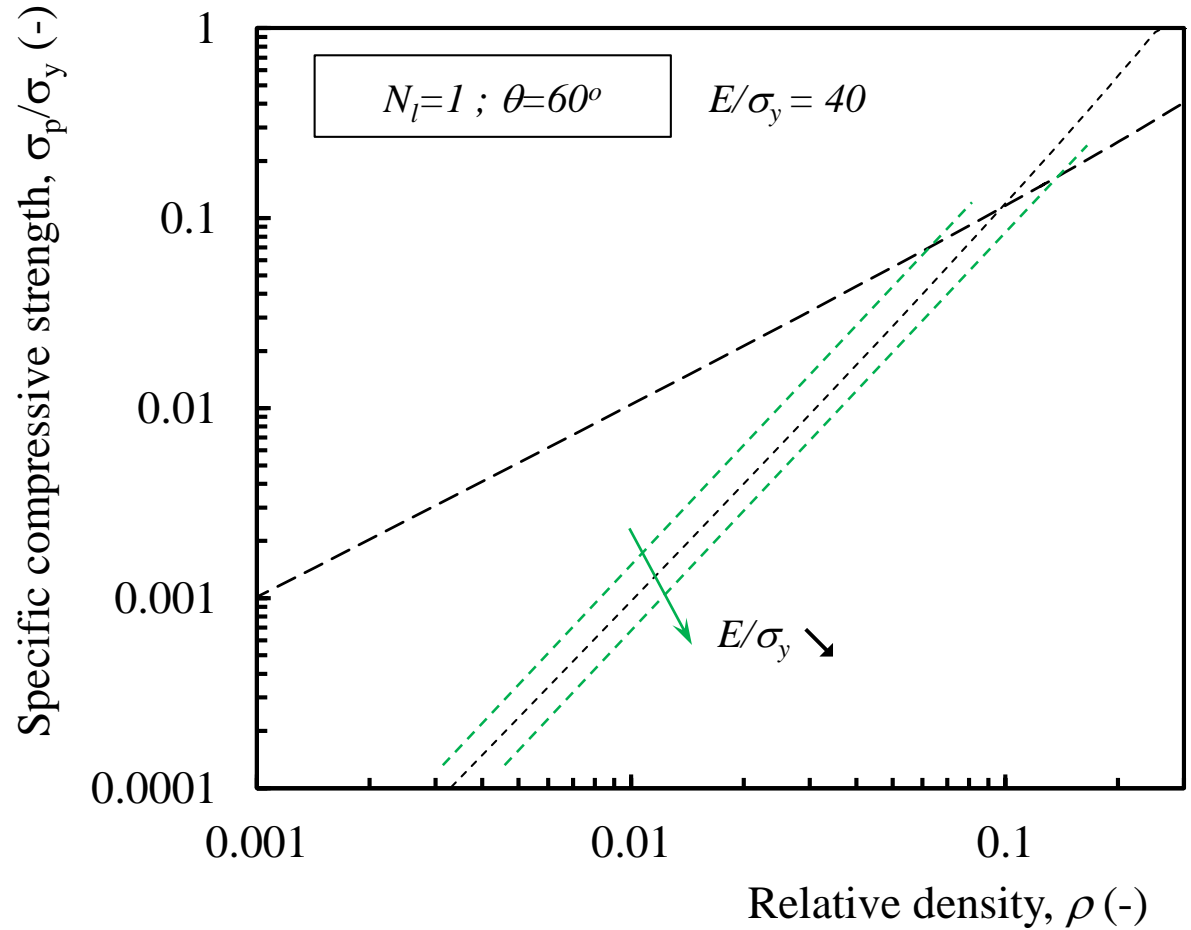


Buckling:

$$strut F_{ax} = \frac{\pi^3 E D^4}{(8Kl)^2 \cos^3 \theta}$$

Yielding:

$$strut F_{ax} = \sigma_y \left( \frac{\pi}{4} D^2 \cos \theta \right)$$

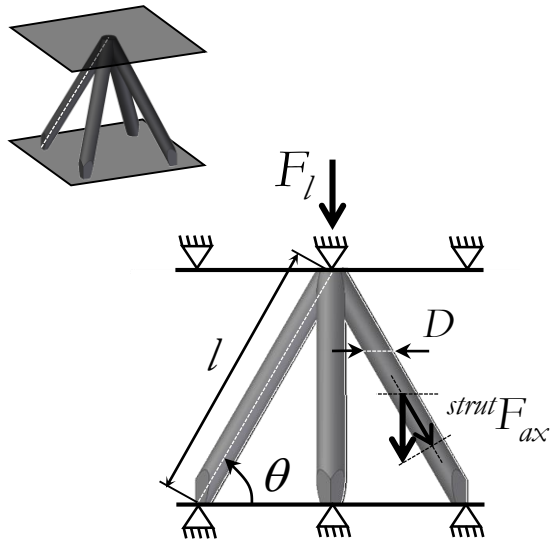


The failure mode is triggered by the parent **material properties** (and its sensitivities)



# V. Lattices

## V.2. Materials by design: an example

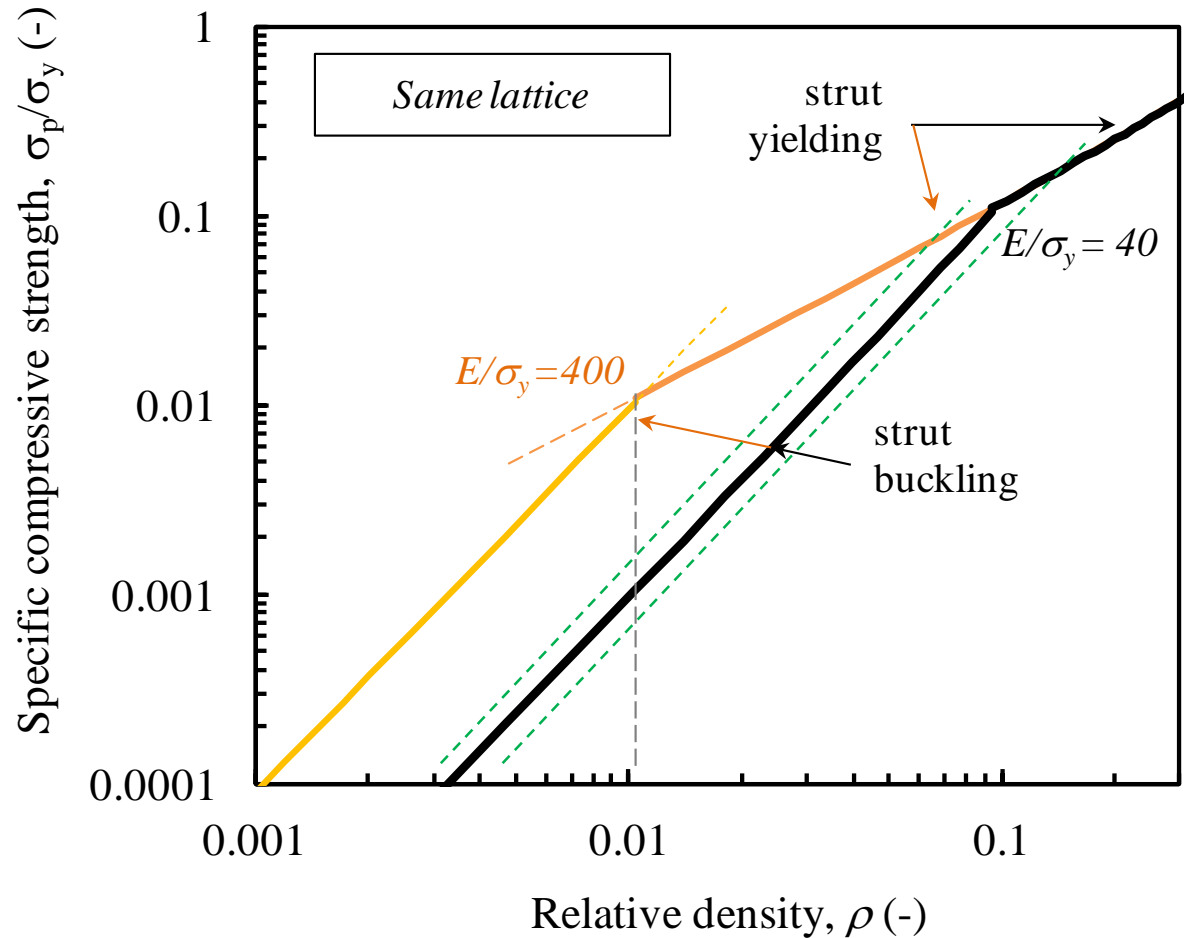


Buckling:

$$strut F_{ax} = \frac{\pi^3 E D^4}{(8Kl)^2 \cos^3 \theta}$$

Yielding:

$$strut F_{ax} = \sigma_y \left( \frac{\pi}{4} D^2 \cos \theta \right)$$

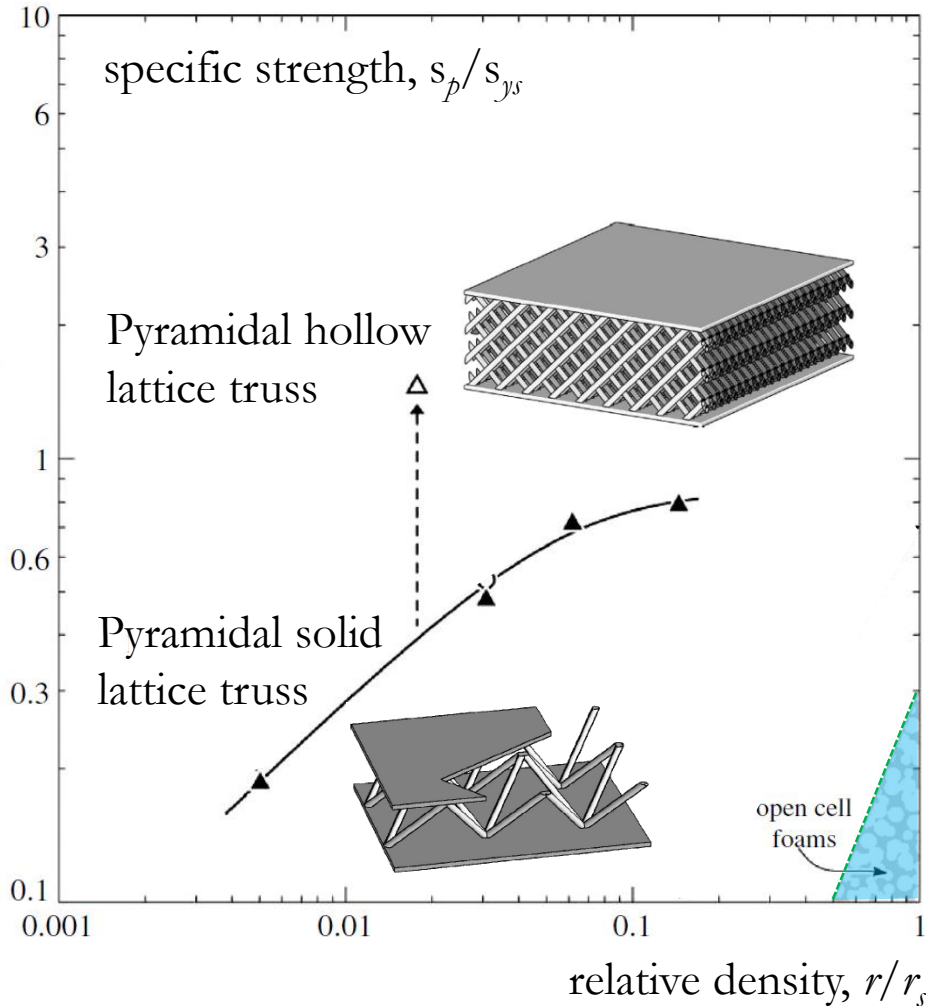


The failure mode is triggered by the parent **material properties** (and its sensitivities)

glassy polymer  
*vs*  
 conventional metal 41

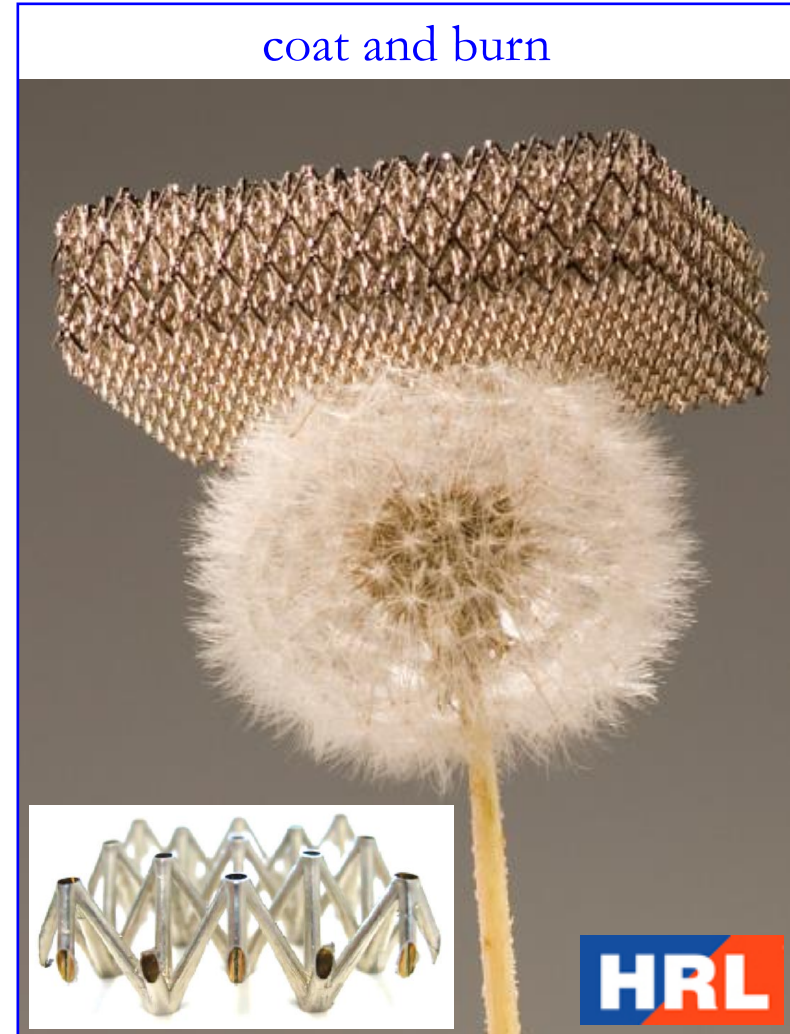
# V. Lattices

## V.2. Materials by design: an example



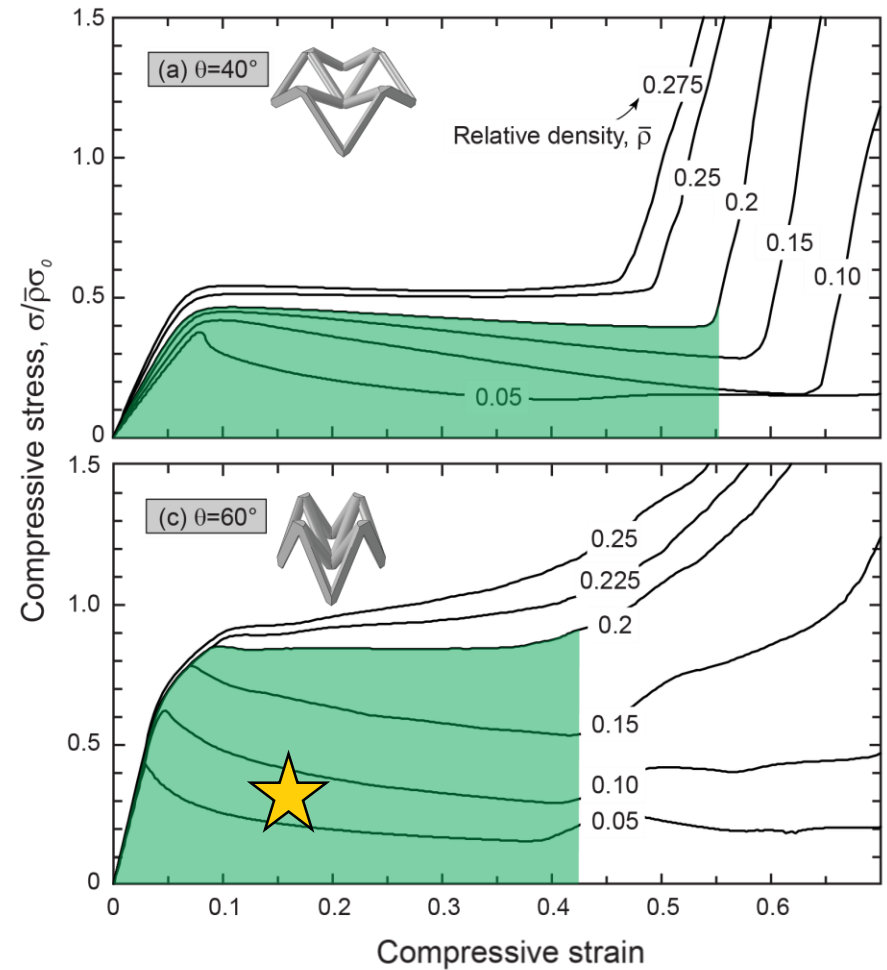
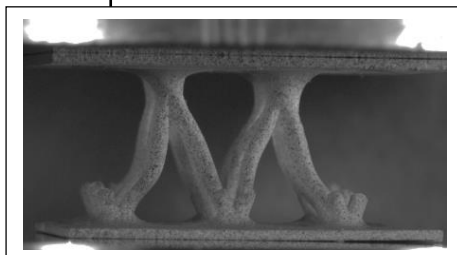
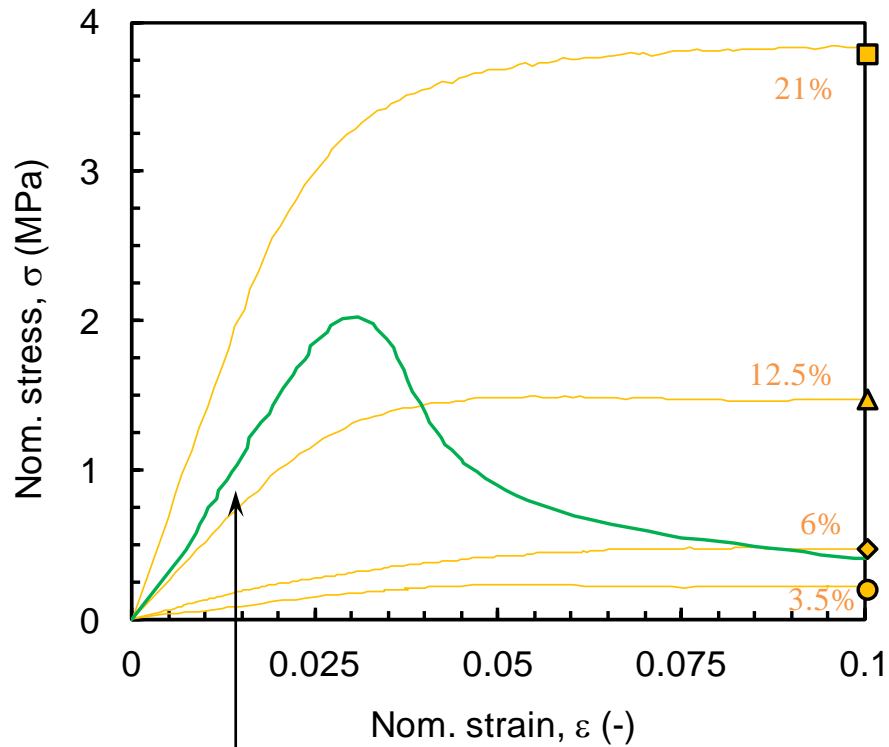
Absolute & Specific strength enhancement

coat and burn



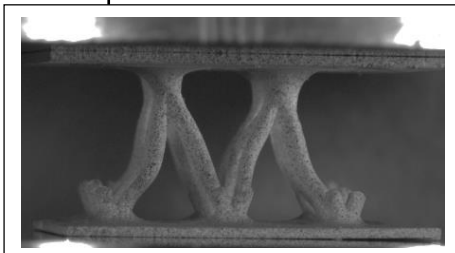
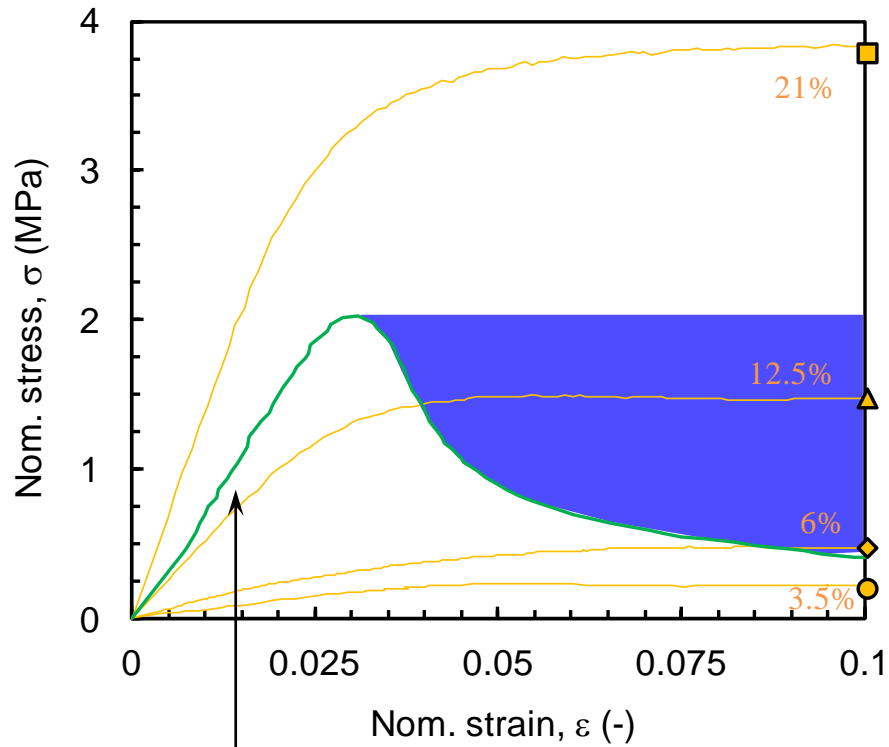
# V. Lattices

## V.2. Materials by design: an example

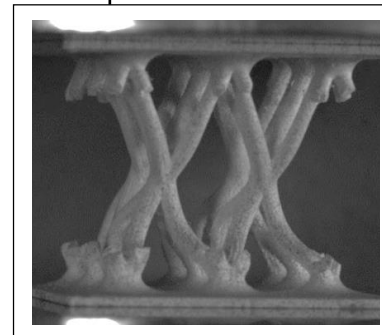
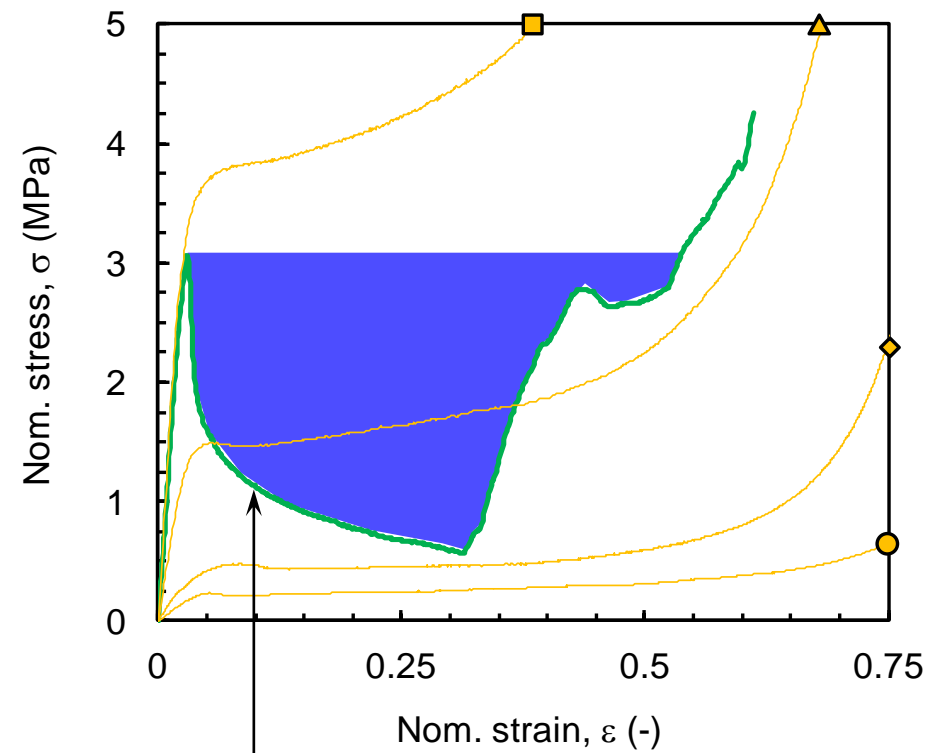


# V. Lattices

## V.2. Materials by design: an example



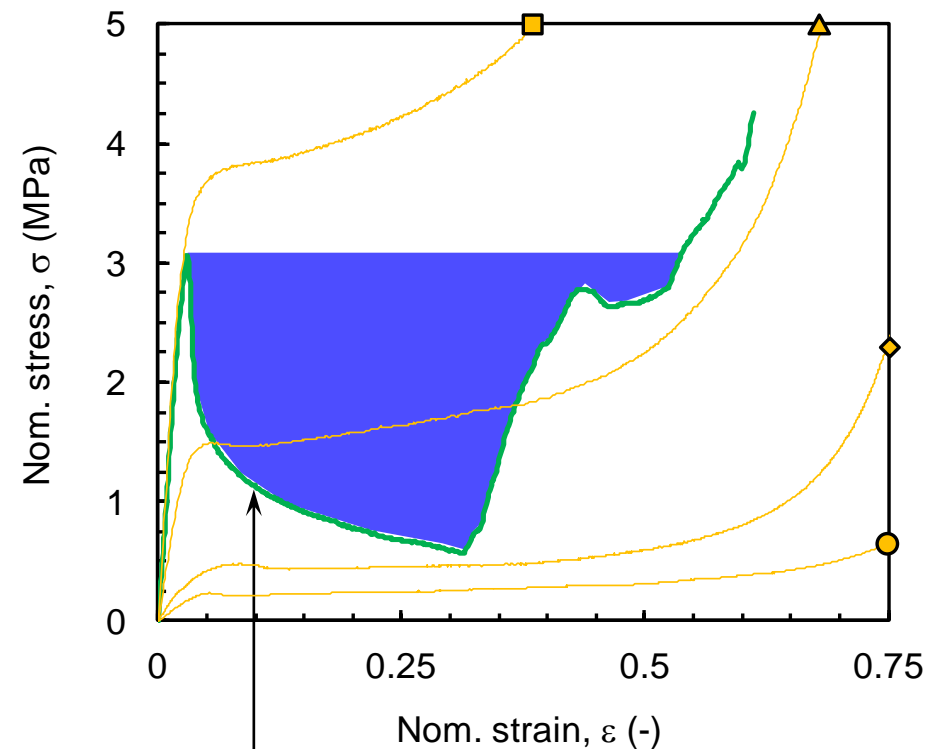
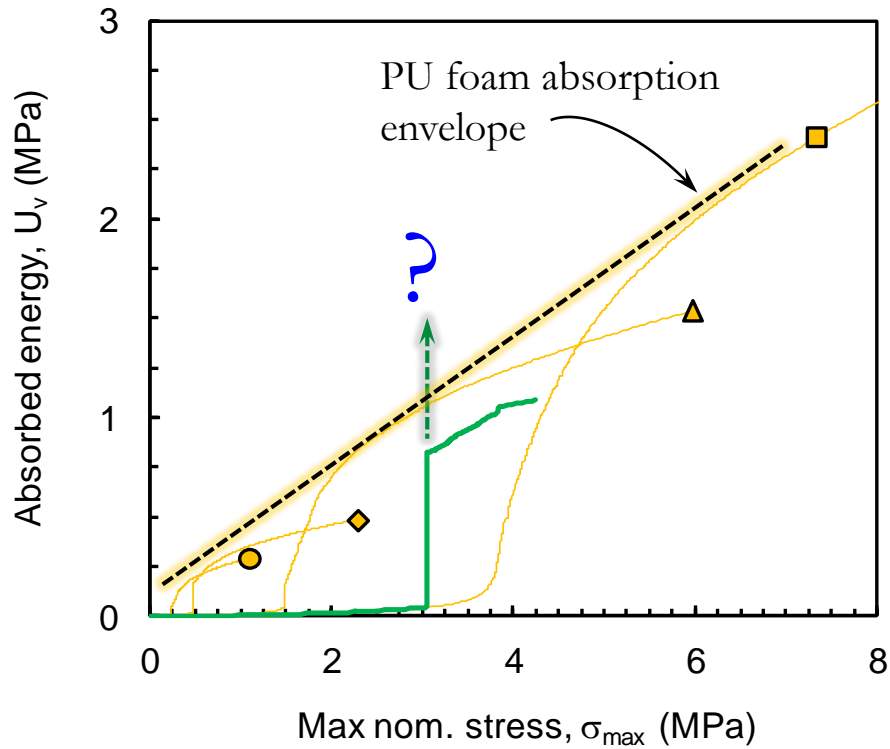
$N_l = 1$   
 $\theta = 60^\circ$   
 $\rho = 8\%$



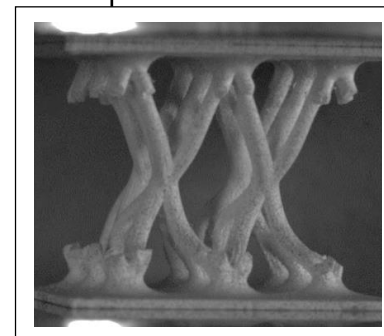
$N_l = 2$   
 $\theta = 60^\circ$   
 $\rho = 12\%$

# V. Lattices

## V.2. Materials by design: an example



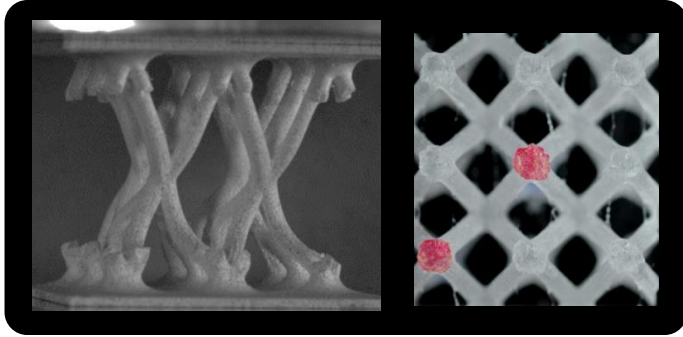
Exploring the possible adjustments to enhance the energy absorption capability of pyramidal lattice systems



$N_l = 2$   
 $\theta = 60^\circ$   
 $\rho = 12\%$

# V. Lattices

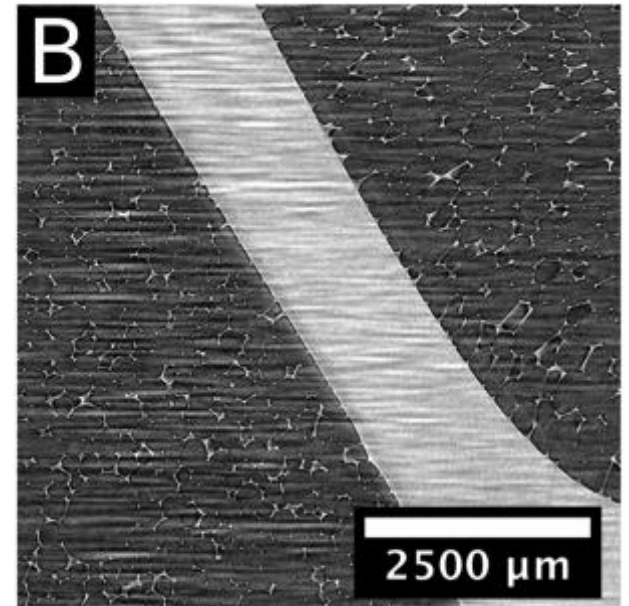
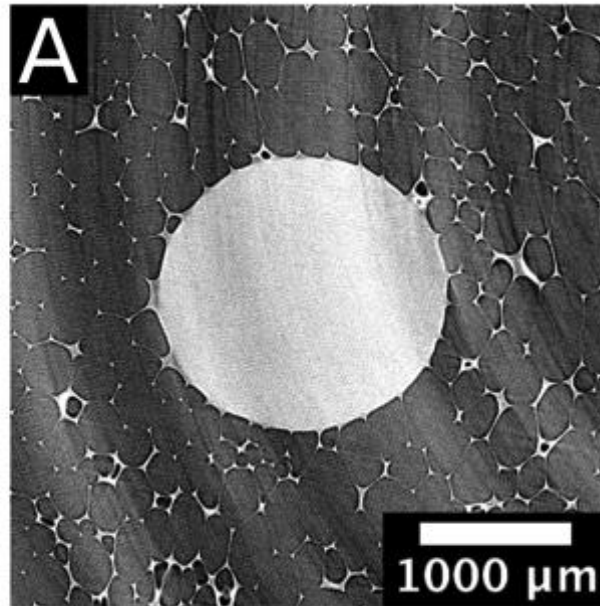
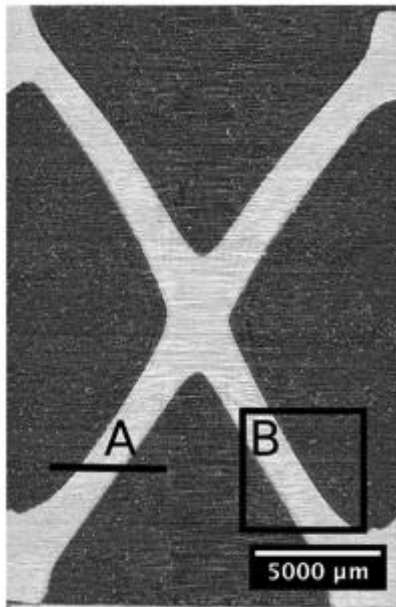
## V.2. Materials by design: an example



Macro-lattice



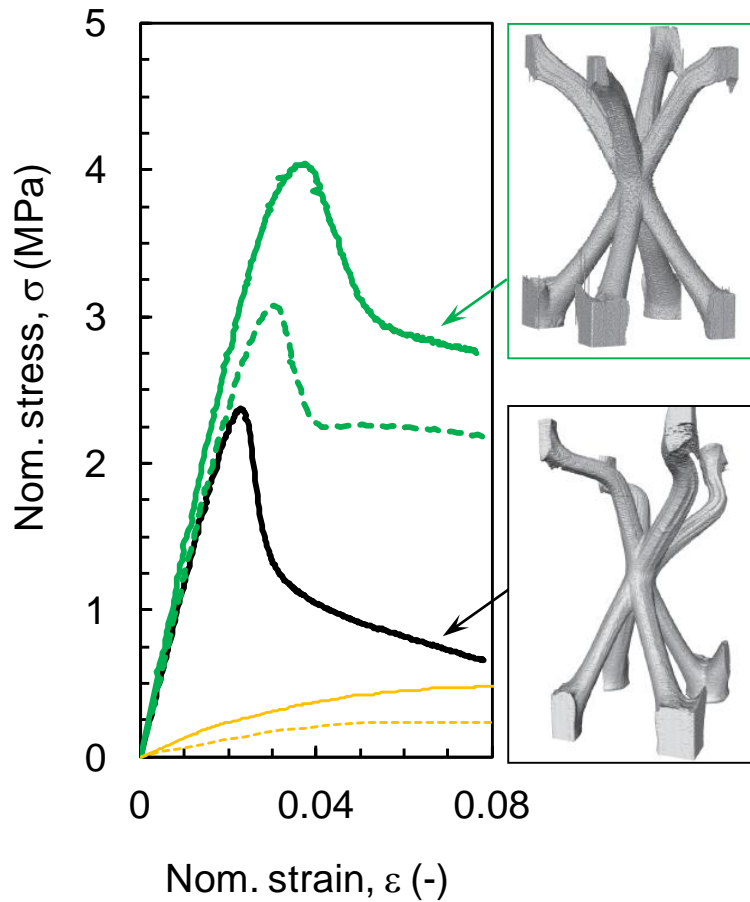
Foamed  
macro-lattice



# V. Lattices

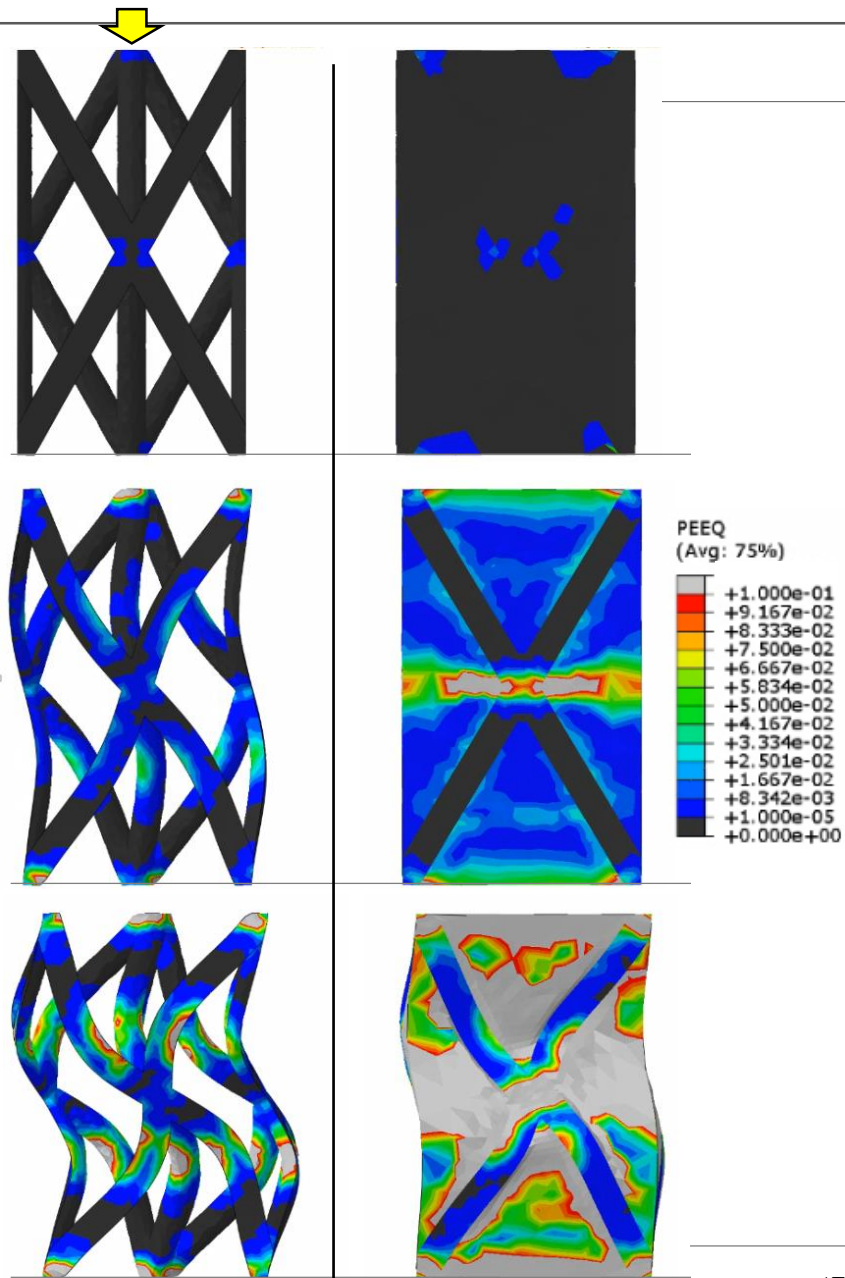
## V.2. Materials by design: an example

Experiments ( $\mu$ -CT characterization)



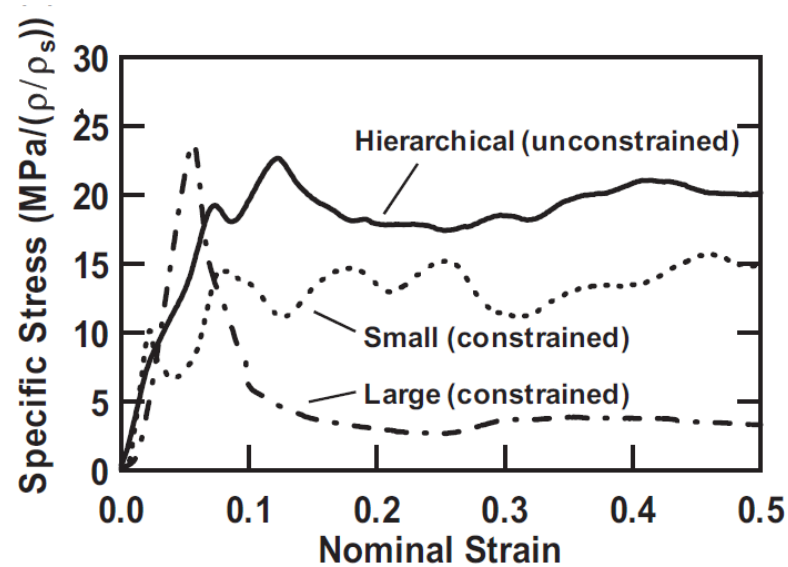
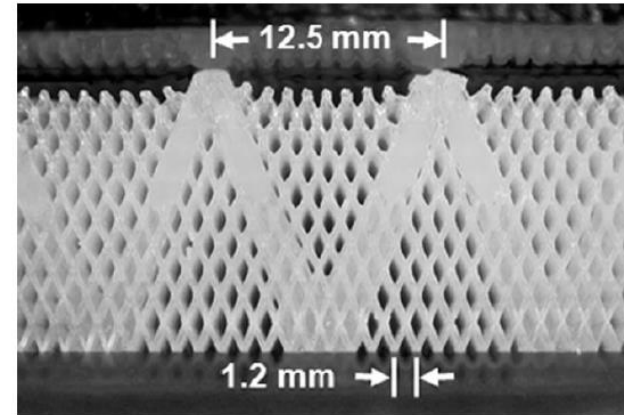
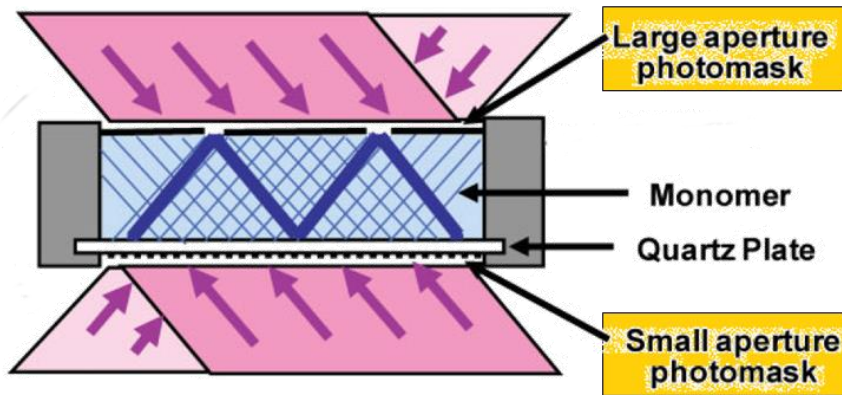
F.E. simulations

(Equiv. Plastic strain (PEEQ) profiles at given times along the comp test)



# V. Lattices

## V.2. Materials by design: an example



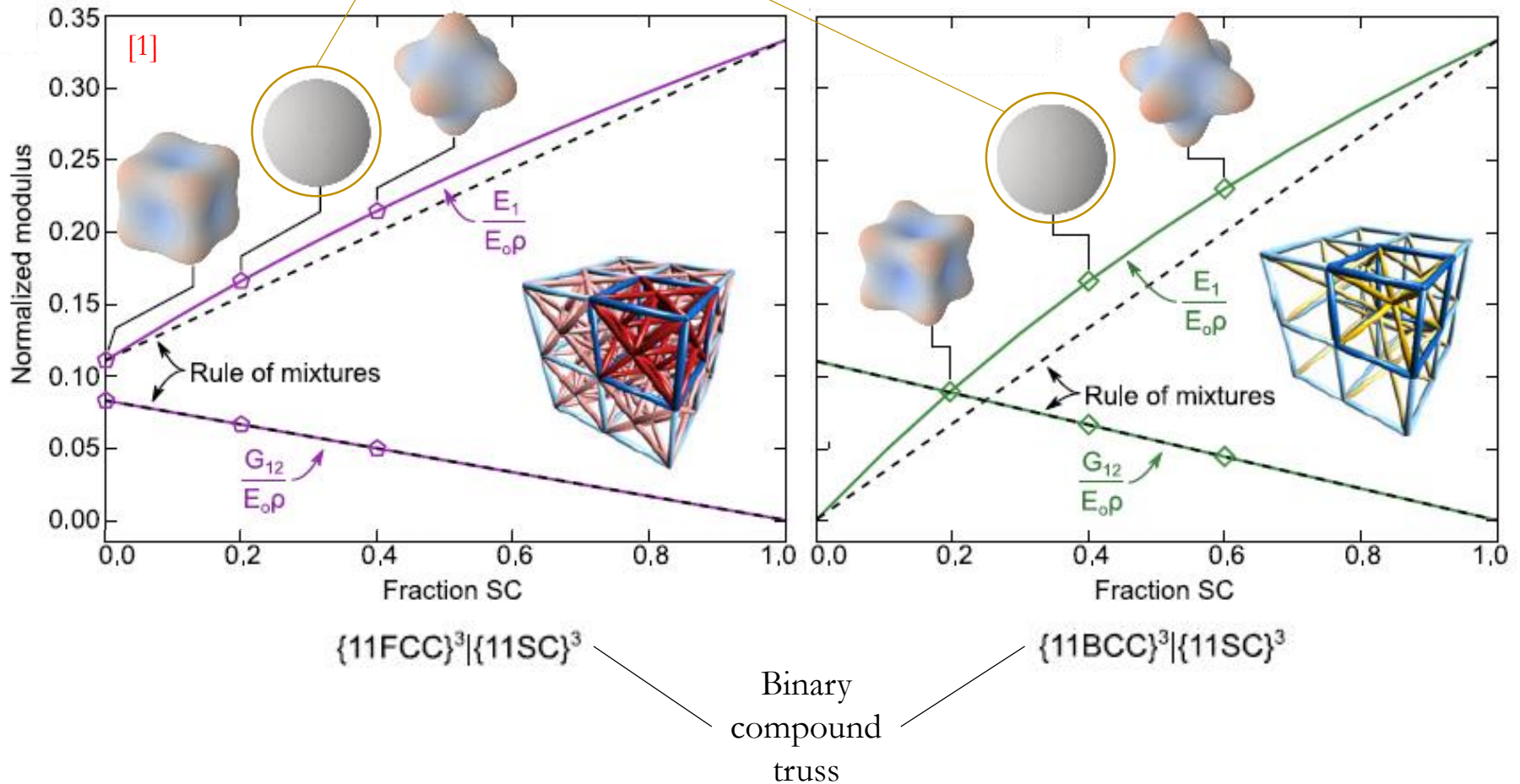


# V. Lattices

## V.3. Materials by design: Enhancement / Optimization

Polar plots of axial stiffness  
(isotropic lattice materials)

$$[2] \frac{1}{E_{ijk}} = \frac{1}{E_1} - 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2) \left( \frac{1}{E_1} - \frac{1}{2G_{12}} - \frac{\nu_{12}}{E_1} \right)$$

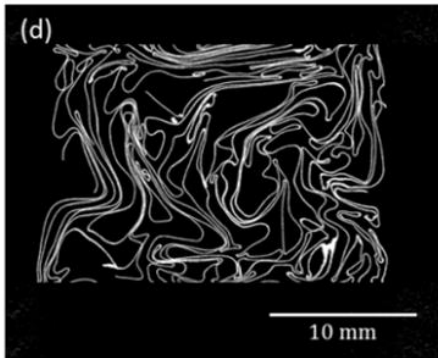


[1] 2018-Latture\_J-Mat-Res\_249-262.pdf

[2] 1985-Nye: Physical Properties of Crystals: (Oxford University Press).

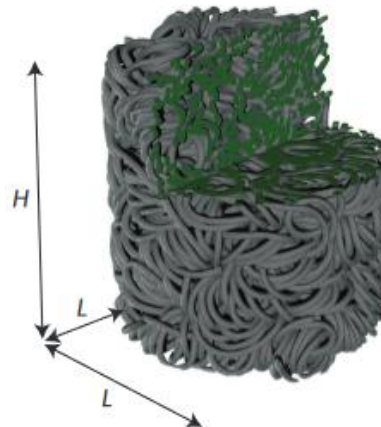
# Conclusion: Limited limitations

Crumpled material



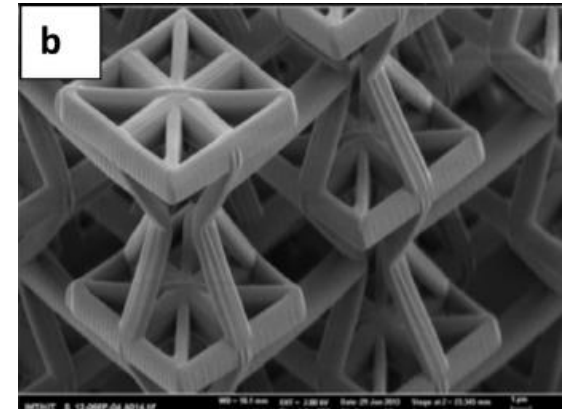
*Martoia et al., Mater&Design, 2017*

Entangled monofilament



*Rodney et al., Nature Materials, 2016*

Auxetic materials



*Saxena et al., Adv. Eng. Mater., 2016*