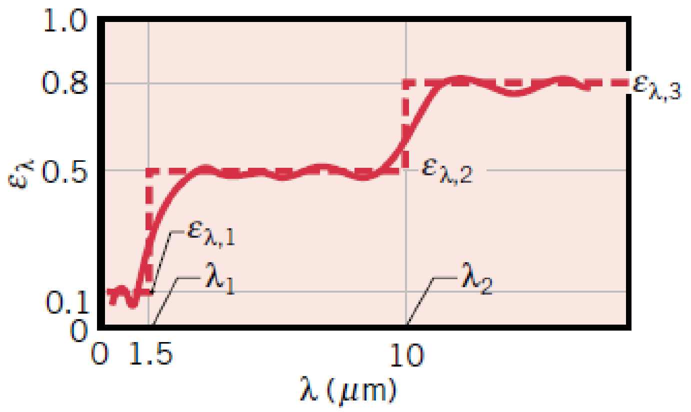
Emissivité hémisphérique totale

Example 12.9

A diffuse, fire brick wall of temperature $T_s = 500$ K has the spectral emissivity shown and is exposed to a bed of coals at 2000 K.



Determine the total, hemispherical emissivity and emissive power of the fire brick wall. What is the total absorptivity of the wall to irradiation resulting from emission by the coals?

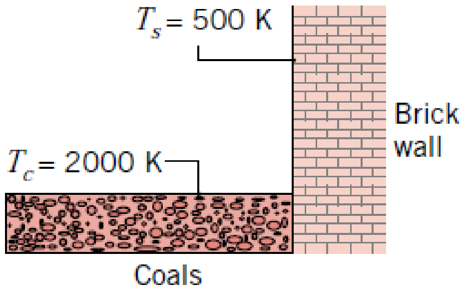
Solution

Known: Brick wall of surface temperature $T_s = 500$ K and prescribed $\varepsilon_\lambda(\lambda)$ is exposed to coals at $T_c = 2000$ K.

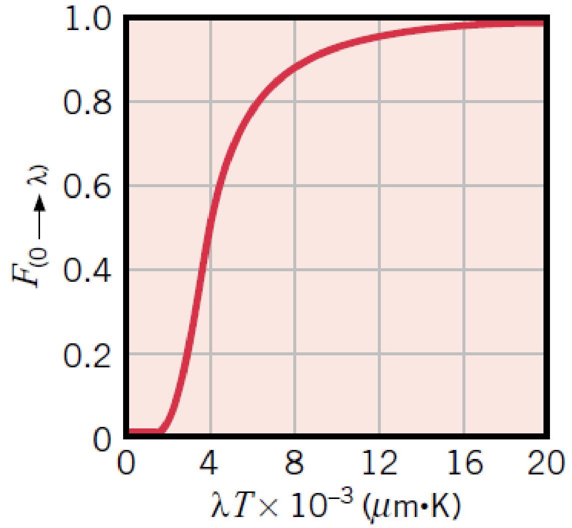
Find:

1. Total, hemispherical emissivity of the fire brick wall.
2. Total emissive power of the brick wall.
3. Absorptivity of the wall to irradiation from the coals.

Schematic:



Fraction d’émittance totale :



$$F_{\lambda_1-\lambda_2} = \frac{\int_{\lambda_1}^{\lambda_2} M^0_{\lambda,T}}{M^0}$$

flux émis par un corps à la température T

dans un domaine de longueur d'onde λ_1, λ_2

$$M^0 = \int_0^{\infty} M^0_{\lambda,T}\, d\lambda$$

Exemple : pour le Soleil à 5800 K , fraction d’émittance entre 0 et 3 µm :

λ T = 5800*3=17400 >> $F_{0\text{-}3}$= 98%

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$	λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$	λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$
200	0.000000				
400	0.000000	4,400	0.548796	9,500	0.903085
600	0.000000	4,600	0.579280	10,000	0.914199
800	0.000016	4,800	0.607559	10,500	0.923710
1,000	0.000321	5,000	0.633747	11,000	0.931890
1,200	0.002134	5,200	0.658970	11,500	0.939959
1,400	0.007790	5,400	0.680360	12,000	0.945098
1,600	0.019718	5,600	0.701046	13,000	0.955139
1,800	0.039341	5,800	0.720158	14,000	0.962898
2,000	0.066728	6,000	0.737818	15,000	0.969981
2,200	0.100888	6,200	0.754140	16,000	0.973814
2,400	0.140256	6,400	0.769234	18,000	0.980860
2,600	0.183120	6,600	0.783199	20,000	0.985602
2,800	0.227897	6,800	0.796129	25,000	0.992215
2,898	0.250108	7,000	0.808109	30,000	0.995340
3,000	0.273232	7,200	0.819217	40,000	0.997967
3,200	0.318102	7,400	0.829527	50,000	0.998953
3,400	0.361735	7,600	0.839102	75,000	0.999713
3,600	0.403607	7,800	0.848005	100,000	0.999905
3,800	0.443382	8,000	0.856288		
4,000	0.480877	8,500	0.874608		
4,200	0.516014	9,000	0.890029		

(A)

1°) $$\varepsilon = \frac{\int_0^{\infty} \varepsilon_\lambda \mathring{M}_{\lambda,T}\, d\lambda}{\int_0^{\infty} \mathring{M}_{\lambda,T}\, d\lambda} \quad ; \quad \mathring{M}_{\lambda T} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

ε : Emissivité hémisphérique totale

$$\varepsilon = \left(\varepsilon_1 \int_0^{1,5} \mathring{M}_{\lambda,T}\, d\lambda + \varepsilon_2 \int_{1,5}^{10} \mathring{M}_{\lambda,T}\, d\lambda + \varepsilon_3 \int_{10}^{\infty} \mathring{M}_{\lambda,T}\right) / \sigma_0 T^4$$

(avec $\sigma_0 T^4 = \int \mathring{M}_{\lambda,T}\, dT$

$$\varepsilon = \varepsilon_1 F(0,1.5) + \varepsilon_2 F(1,5,10) + \varepsilon_3 F(10,\infty)$$

$\lambda_1 T = 1,5 \times 500 = 750\ \mu.K \Rightarrow$

$F(0,1.5) = 0$

$\lambda_2 T = 10 \cdot T = 5000\ \mu.K \Rightarrow$

$F(0,10) = 0,633747 \Rightarrow F(1,5,10) \simeq 0,634$

et $F(10,\infty) = 1 - F(1.5,10) = 0,366$

$\Rightarrow \varepsilon = 0,1 \times 0 + 0,5 \times 0,634 + 0,8 \times 0,366$

$$\boxed{\varepsilon = 0,61}$$

2°) $\Rightarrow \quad \varepsilon \sigma_0 T^4 = 0{,}61 \times 5{,}67\, 10^{-8} \times (500)^4 = 2161 \ W/m^2$

3°) $$\alpha = \frac{\int_0^\infty \alpha_\lambda \, M^\circ_{\lambda, T_c} \, d\lambda}{\sigma_0 T_c^4}$$

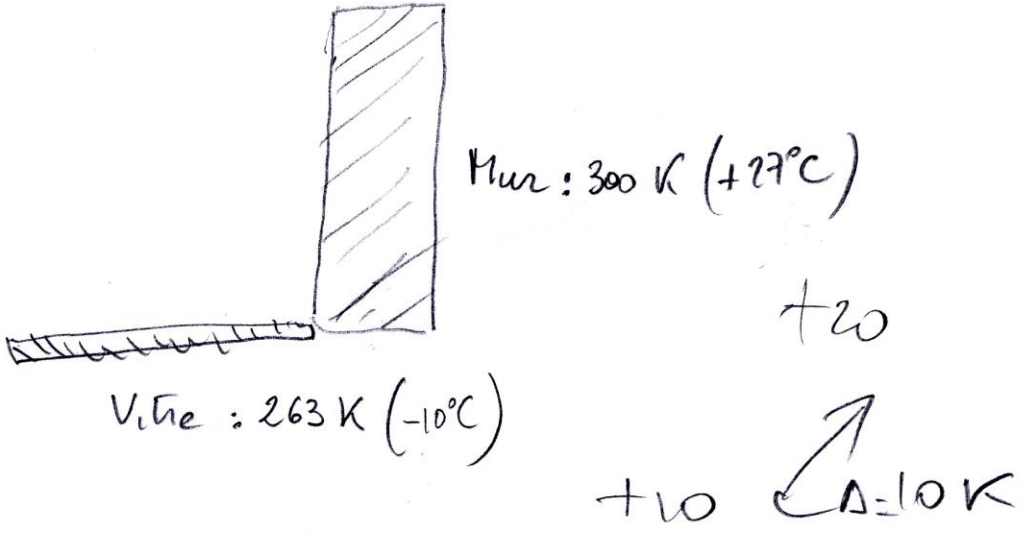
$\lambda_1 T_c = 1{,}5 \times 2000 = 3000 \, \mu K \Rightarrow F'_{(0,1.5)} = 0{,}273$

$\lambda_2 T_c = 10 \times 2000 = 20000 \, \mu K \Rightarrow F'_{(0,10)} = 0{,}986$

$\alpha = 0.1 \times 0{,}273 + 0.5\,(0{,}986 - 0{,}273) + 0{,}8\,(1 - 0.986)$

$\boxed{\alpha = 0{,}395}$

(B)



1°)

$$\varepsilon = \varepsilon_1 F(0,1.5) + \varepsilon_2 F(1.5,10) + \varepsilon_3 F(10,\infty)$$

$$\lambda_1 T = 1{,}5 \times 300 = 450\,\mu K \Rightarrow F(0,1.5) = 0$$

$$\lambda_2 T = 10 \times 300 = 3000\,\mu.K \Rightarrow F(0,10) = 0{,}273$$

$$\varepsilon = 0.1 \times 0 + 0{,}5 \times 0.273 + 0{,}8\,(1 - 0{,}273) = 0{,}7181$$

2°) $\alpha = 0.1\, F'(0, 1.5) + 0{,}5 \cdot F'(1.5, 10) + 0{,}8\, F'(10, \infty)$

$\lambda_1 T_V = 1{,}5 \times 263 = 394{,}5\ \mu K \Rightarrow F'(0, 1.5) = 0$

$\lambda_2 T_V = 10 \times 263 = 2630\ \mu K \Rightarrow F'(0, 10) = 0{,}183$

$\alpha = 0.1 \times 0 + 0.5 \times 0{,}183 + 0{,}8\,(1 - 0{,}183) = 0{,}7451$

$\boxed{\varepsilon = 0{,}72}$ et $\boxed{\alpha = 0{,}75}$