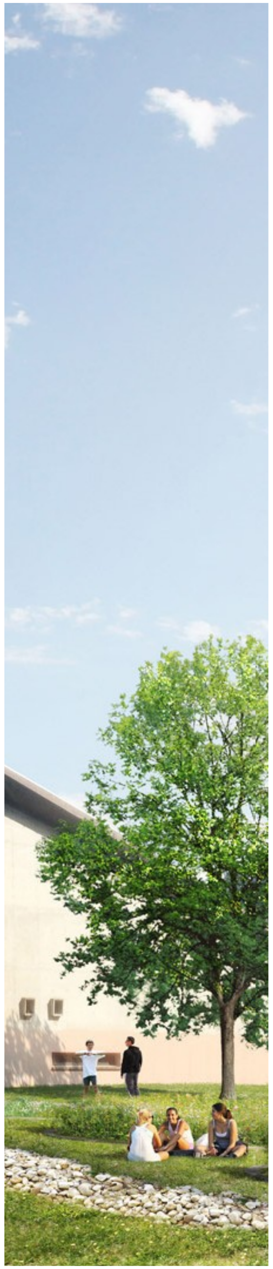
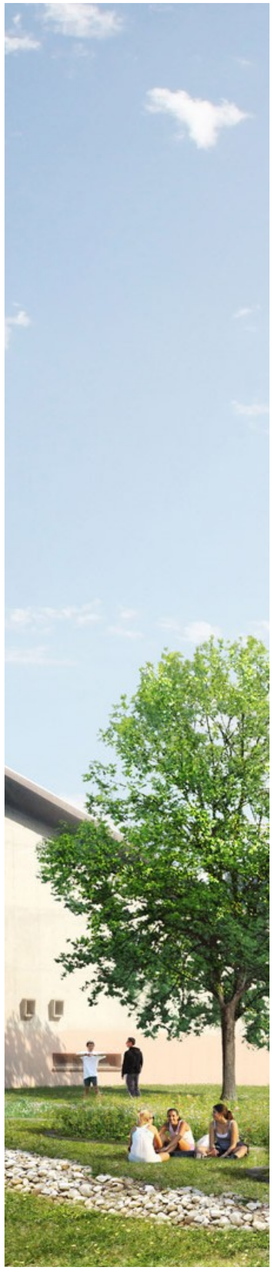


Equation de la chaleur : Cartésien, cylindrique, sphérique



Équation de la chaleur

$$\iiint_V div(\lambda \overrightarrow{grad}(T))\, dv + \iiint_V p\, dv = \iiint_V \rho\, c \frac{\partial T}{\partial t} dv$$

vraie pour tout V et donc :

$$div(\lambda \overrightarrow{grad}(T)) + p = \rho\, c \frac{\partial T}{\partial t}$$

$$\vec{q} = \lambda\, \overrightarrow{grad\ T}$$

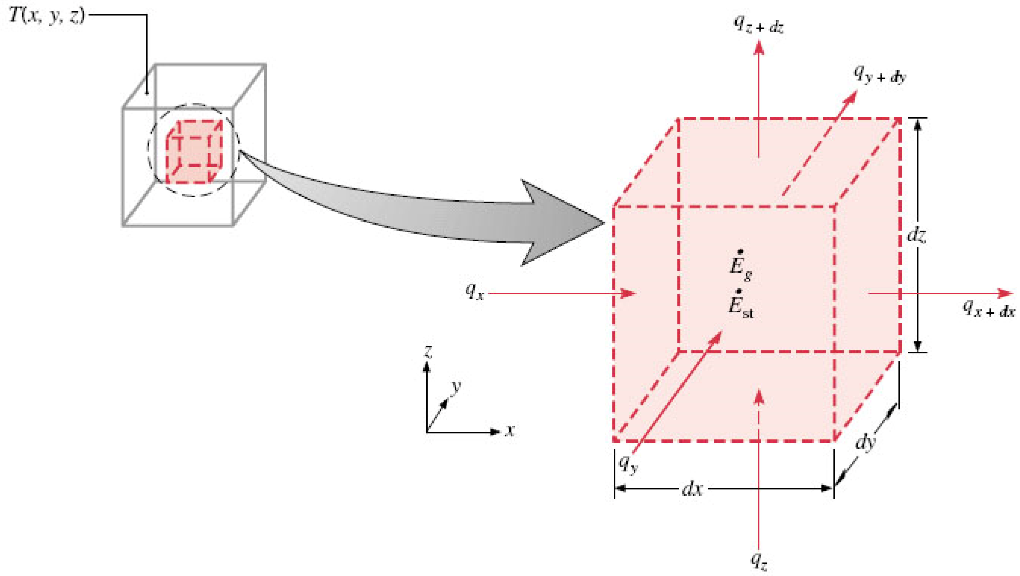
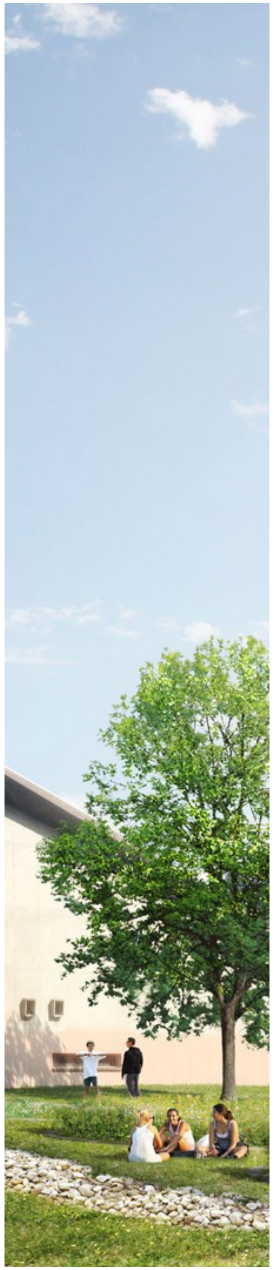


Figure 2.11 Differential control volume, $dx\ dy\ dz$, for conduction analysis in Cartesian coordinates.

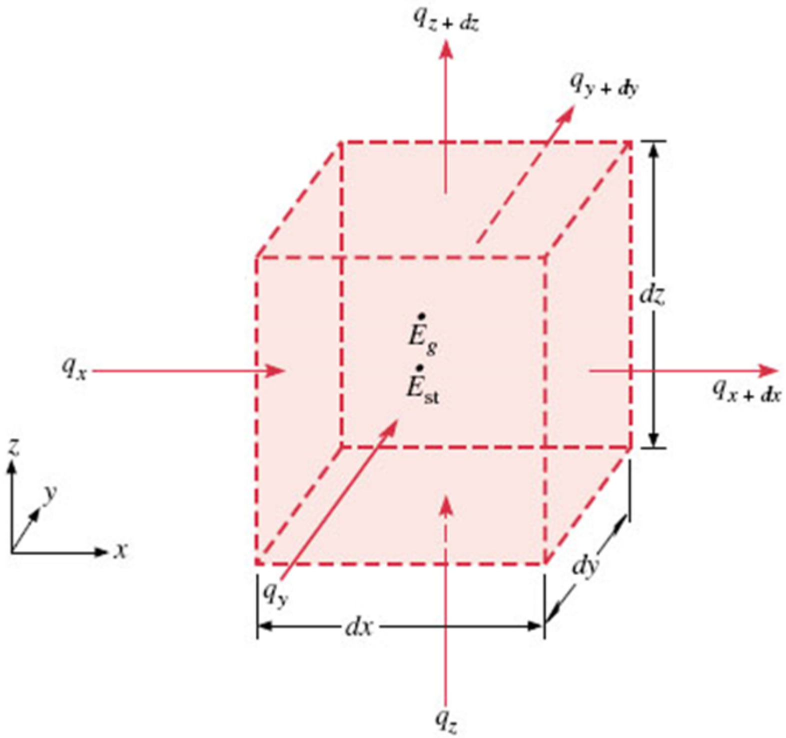


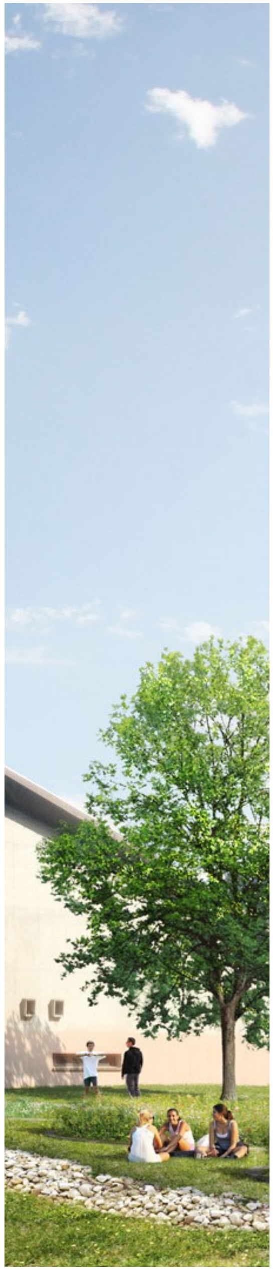
$$div(\lambda \,\overrightarrow{grad}(T)) + p = \rho\, c \frac{\partial T}{\partial t}$$

$$div(\lambda \,\overrightarrow{grad}(T))\, dv + p\, dv = \rho\, c \frac{\partial T}{\partial t} dv$$

$$\vec{q} = \lambda\, \overrightarrow{grad\, T}$$

$$div(\vec{q}\,)\, dv + p\, dv = \rho\, c \frac{\partial T}{\partial t} dv$$





$\vec{\phi} = (q_x\ ,\ q_y\ ,\ q_z)^t \qquad q_x = -\left(\lambda \frac{\partial T}{\partial x}\right)_x dy\, dz$

Bilan sur le volume de contrôle :

$\rho\ c\ dx\ dy\ dz\ \frac{\partial T}{\partial t} = +p\ dx\ dy\ dz$

$[\left(\lambda \frac{\partial T}{\partial x}\right)_{x+dx} - \left(\lambda \frac{\partial T}{\partial x}\right)_x]\ dy\ dz$

$+\left[\left(\lambda \frac{\partial T}{\partial y}\right)_{y+dy} - \left(\lambda \frac{\partial T}{\partial y}\right)_y\right] dx\ dz$

$+\left[\left(\lambda \frac{\partial T}{\partial z}\right)_{z+dz} - \left(\lambda \frac{\partial T}{\partial z}\right)_z\right] dx\ dy$

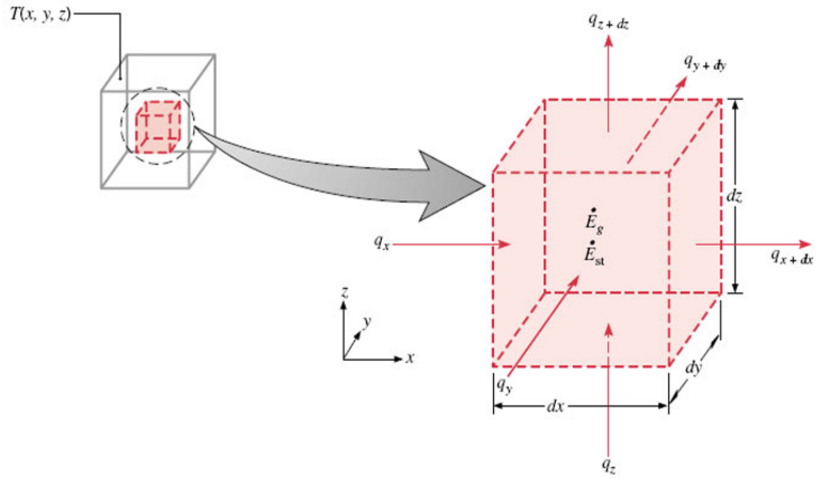
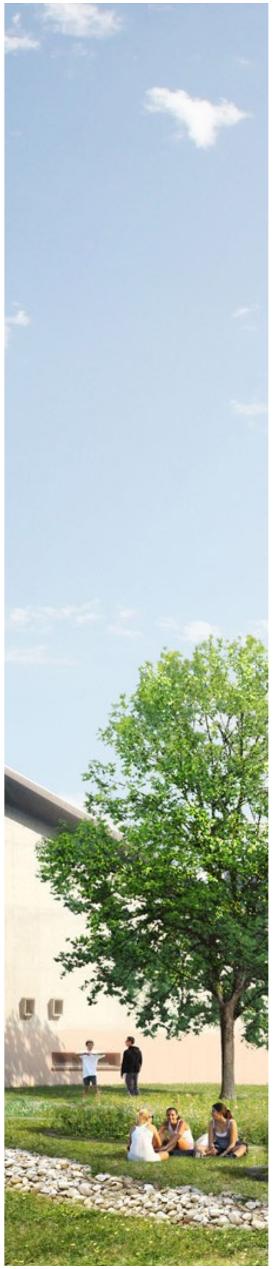


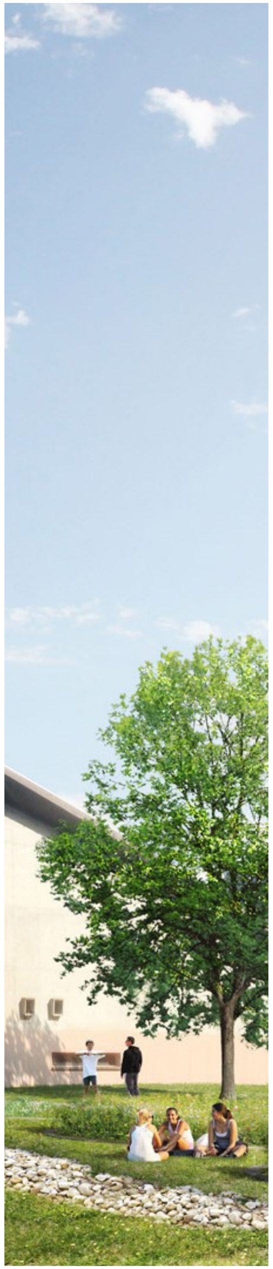
FIGURE 2.11 Differential control volume, $dx\ dy\ dz$, for conduction analysis in Cartesian coordinates.

$$\rho\ c\ \frac{\partial T}{\partial t} = \frac{[\left(\lambda \frac{\partial T}{\partial x}\right)_{x+dx} - \left(\lambda \frac{\partial T}{\partial x}\right)_x]}{dx} + \frac{[\left(\lambda \frac{\partial T}{\partial y}\right)_{y+dy} - \left(\lambda \frac{\partial T}{\partial y}\right)_y]}{dy} + \frac{[\left(\lambda \frac{\partial T}{\partial z}\right)_{z+dz} - \left(\lambda \frac{\partial T}{\partial z}\right)_z]}{dz} + p$$

$$\rho\ c\ \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + p \quad \gg \qquad div(\lambda\ \overrightarrow{grad}(T)) + p = \rho\ c\ \frac{\partial T}{\partial t}$$



Equation de la chaleur : Cylindrique



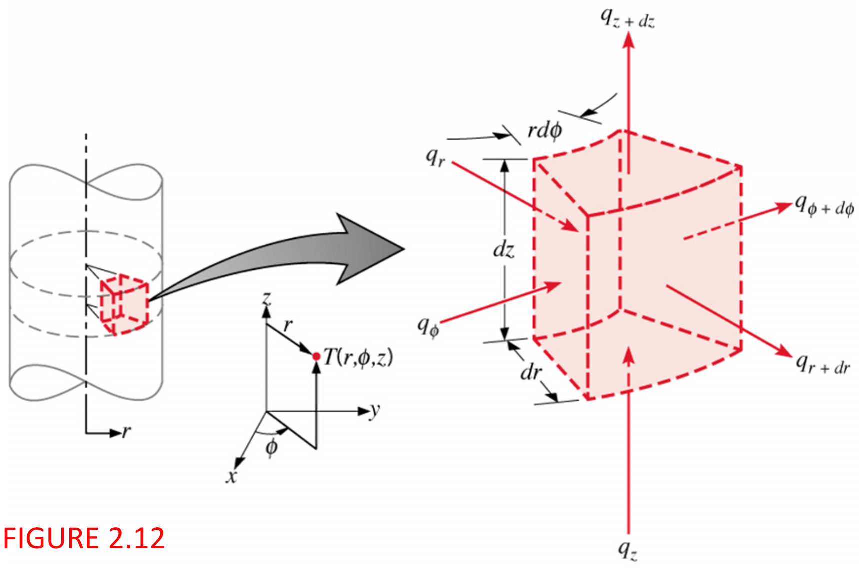


FIGURE 2.12

$$\overrightarrow{grad(T)} = \frac{\partial T}{\partial r}\vec{i}_r + \frac{\partial T}{r\partial \phi}\vec{j}_\phi + \frac{\partial T}{\partial z}\vec{z}_z$$

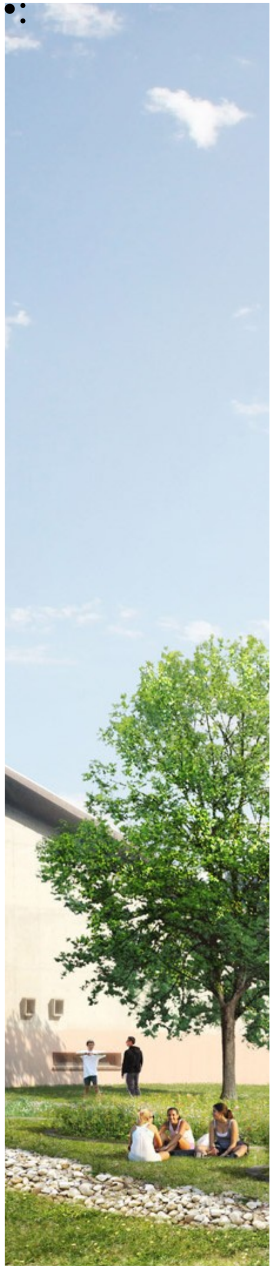
Bilan sur le volume de contrôle :

$$\rho\; c\; dx\; dy\; dz\;\frac{\partial T}{\partial t} = +p\; dx\; dy\; dz$$
$$+(-q_{r+dr} + q_r)$$
$$+(-q_{\phi+d\phi} + q_\phi)$$
$$+(-q_{z+dz} + q_z)$$

$$\rho c_p\, r\, d\phi\, dr\, dz \frac{\partial T}{\partial t} = (\lambda \frac{\partial T}{\partial r})_{r+dr}(r+dr)\, d\phi\; dz - (\lambda \frac{\partial T}{\partial r})_r\, r\, d\phi\; dz + ... + p\, r\, d\phi\, dr\, dz$$

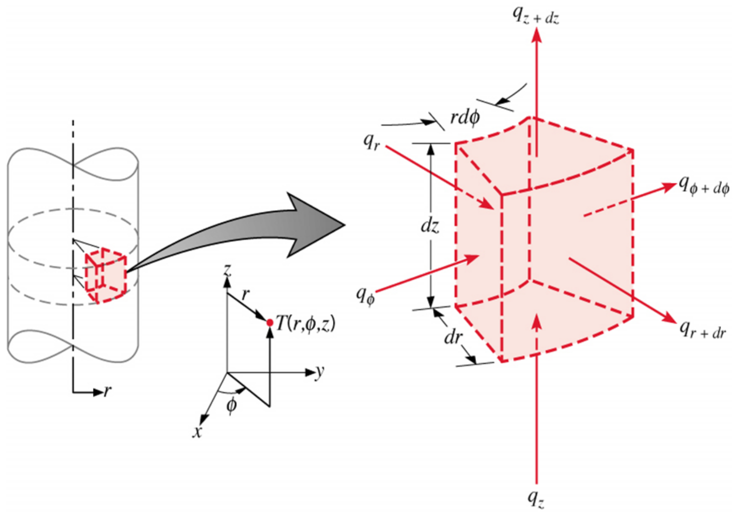
$$\rho c_p \frac{\partial T}{\partial t} = \frac{(\lambda r \frac{\partial T}{\partial r})_{r+dr} - (\lambda r \frac{\partial T}{\partial r})_r}{rdr} + ... = \frac{1}{r}\frac{\partial}{\partial r}(\lambda r \frac{\partial T}{\partial r}) + ... + p$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\lambda r \frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \phi}(\lambda \frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z}(\lambda \frac{\partial T}{\partial z}) + p$$



Cylindrique :

$$dV = r\, d\phi\, dr\, dz$$



$$q_r = -k \left(\frac{\partial T}{\partial r}\right)_r r\, d\phi\, dz$$

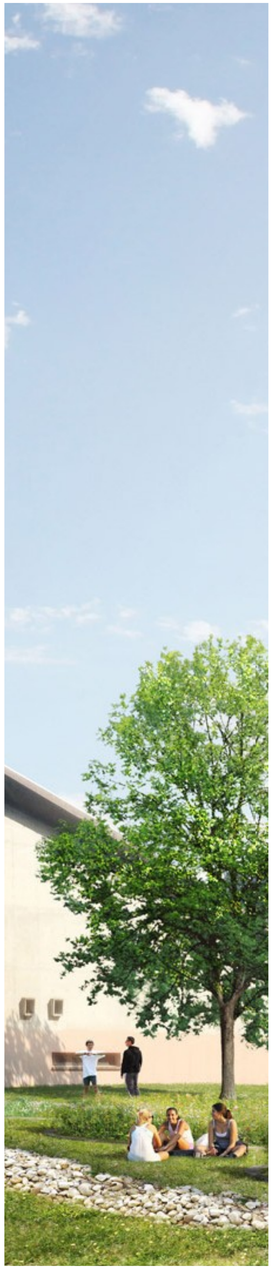
$$q_{r+dr} = -k \left(\frac{\partial T}{\partial r}\right)_{r+dr} (r+dr)\, d\phi\, dz$$

$$\rho\, c_p\, r\, d\phi\, dr\, dz \frac{\partial T}{\partial t} = \left(k \frac{\partial T}{\partial r}\right)_{r+dr} (r+dr)\, d\phi\, dz$$

$$- \left(k \frac{\partial T}{\partial r}\right)_r r\, d\phi\, dz$$

$$+ \ldots\ldots\ldots\ldots$$

$$+ \quad p\, r\, d\phi\, dz\, dr$$

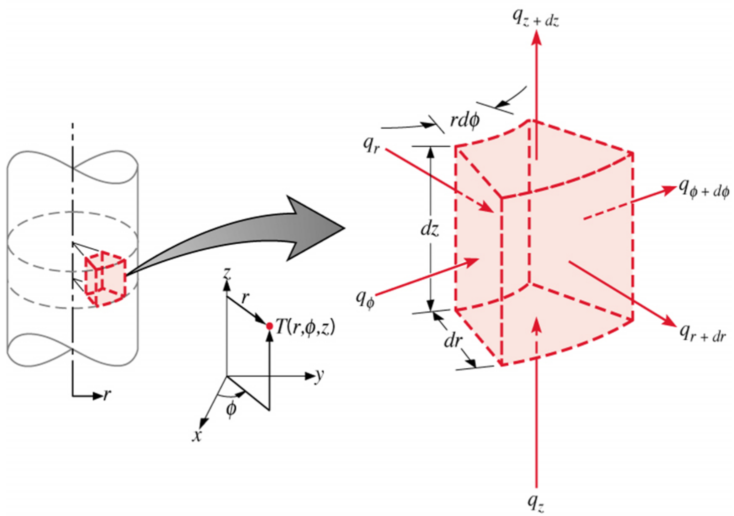


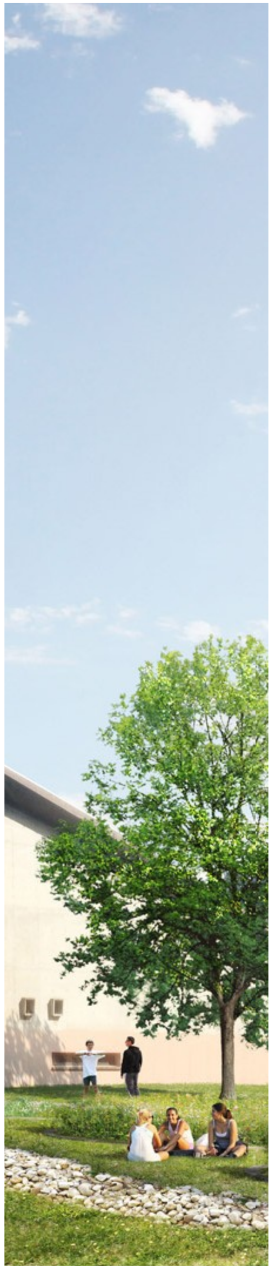
En divisant par V :

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\left(\lambda r \frac{\partial T}{\partial r}\right)_{r+dr} - \left(\lambda r \frac{\partial T}{\partial r}\right)_r}{r\,dr}$$

$$+ \cdots\cdots$$

$$+ P$$

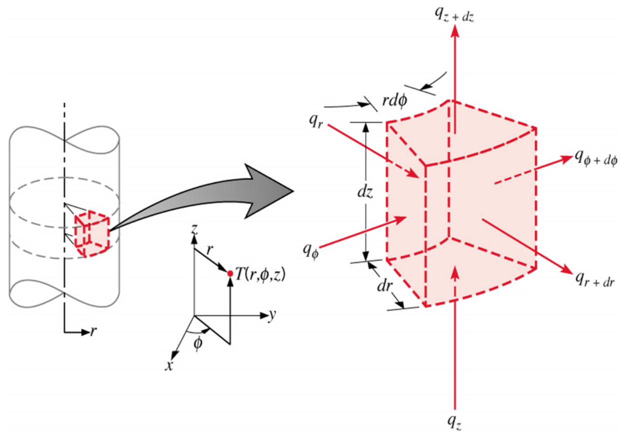




$$\rho\, c\, dx\, dy\, dz\, \frac{\partial T}{\partial t} = +p\, dx\, dy\, dz + (-q_{r+dr} + q_{r}) + (-q_{\phi+d\phi} + q_{\phi}) + (-q_{z+dz} + q_{z})$$

$$q_r = -\lambda \left(\frac{\partial T}{\partial r}\right)_r r\, d\phi\, dz$$

$$q_{r+dr} = -\lambda \left(\frac{\partial T}{\partial r}\right)_{r+dr} (r+dr)\, d\phi\, dz$$

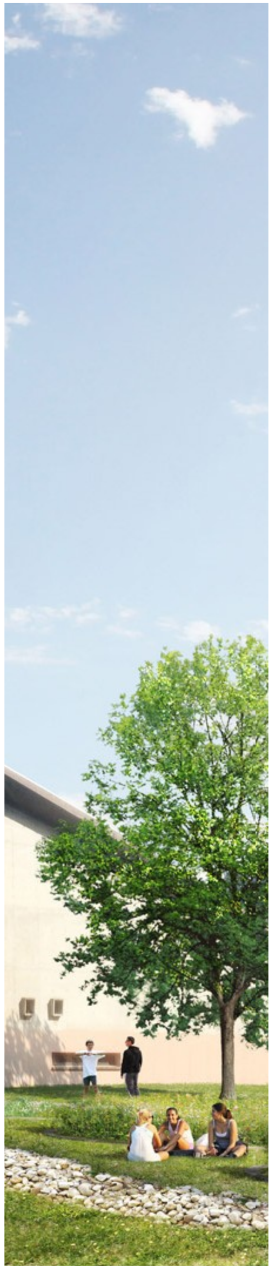


$$q_\phi = -\lambda \left(\frac{1}{r}\frac{\partial T}{\partial \phi}\right)_\phi dr\, dz$$

$$q_{\phi+d\phi} = -\lambda \left(\frac{1}{r}\frac{\partial T}{\partial \phi}\right)_{\phi+d\phi} dz \cdot dr$$

$$q_{z+dz} = -\lambda \left(\frac{\partial T}{\partial z}\right)_{z+dz} r\, d\phi\, dr$$

$$q_z = -\lambda \left(\frac{\partial T}{\partial z}\right)_z r\, d\phi\, dz$$



$$\rho c \frac{\partial T}{\partial t} = p + \frac{1}{r\,dr}\left[\left(r\lambda\frac{\partial T}{\partial r}\right)_{r+dr} - \left(r\,\lambda\frac{\partial T}{\partial r}\right)_{r}\right]$$

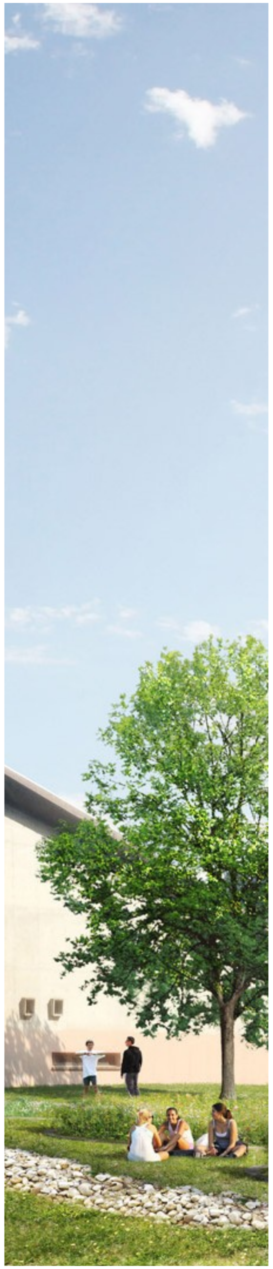
$$+\frac{1}{r^2 d\phi}\left[\left(\lambda\,\frac{\partial T}{\partial \phi}\right)_{\phi+d\phi} - \left(\lambda\,\frac{\partial T}{\partial \phi}\right)_{\phi}\right]$$

$$+\frac{1}{dz}\left[\lambda\left(\frac{\partial T}{\partial z}\right)_{z+dz} - \left(\lambda\,\frac{\partial T}{\partial z}\right)_{z}\right]$$

$dr, d\phi, dz \rightarrow 0$

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial r}\left(\lambda \cdot r\,\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\lambda\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) + p$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\lambda r\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \phi}(\lambda\frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z}(\lambda\frac{\partial T}{\partial z}) + p$$



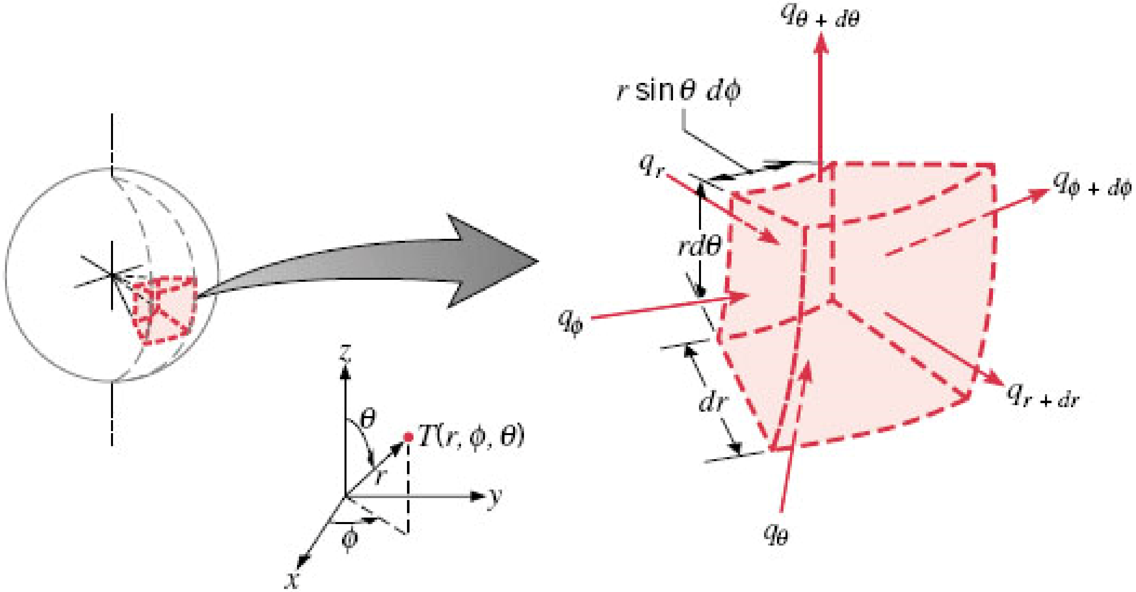


FIGURE 2.13 Differential control volume, $dr \cdot r \sin\theta \, d\phi \cdot r \, d\theta$, for conduction analysis in spherical coordinates (r, ϕ, θ).

Figures 2.11, 2.12 et 2.13 empruntées à :
P.INCROPERA, D. DEWITT, Fundamentals of Heat and Mass Transfer, Fifth Edition, WILEY
https://epdf.pub/fundamentals-of-heat-and-mass-transfer.html