

DS Construction 25-01-10
4 GMPP

Exercice 2

$$\begin{aligned}
 1) \quad y(t) &= \mu B_3^2(t) + \mu B_3^2(t) \\
 &= \mu + 3t(1-t)^2 + \mu \times 3t^2(1-t) \\
 &= 3\mu t(1-t) = -3\mu t^2 + 3\mu t
 \end{aligned}$$

$$\begin{aligned}
 2) \quad x(t) &= (2t-1)(at^2+bt+c) \\
 &= 2at^3 + (2b-a)t^2 + (2c-b)t - c \\
 &= -1 + 3(1+\mu)t - 3(1+3\mu)t^2 + 2(1+3\mu)t^3
 \end{aligned}$$

$$\Rightarrow \begin{cases} c=1 \\ 2c-b=3(1+\mu) \\ 2b-a=-3(1+3\mu) \end{cases} \Rightarrow \begin{cases} a=1+3\mu \\ b=-1-3\mu \\ c=1 \end{cases}$$

$$a = 1 + 3\mu$$

$$\Rightarrow x(t) = (2t-1) \underbrace{\left[(1+3\mu)t^2 - (1+3\mu)t + 1 \right]}_{h(t)}$$

$$3) \quad x(t) = 0 \Rightarrow t = \frac{1}{2} \text{ ou } h(t) = 0$$

$$h(t) = At^2 + Bt + C$$

$$\Delta = B^2 - 4AC = (1+3\mu)^2 - 4(1+3\mu)$$

$$= (1+3\mu)(1+3\mu-4)$$

$$= 3(1+3\mu)(\mu-1) \quad \mu > 0$$

$$\Delta > 0 \text{ pour } \mu > 1$$

2 racines t_1 et t_2 pour $\mu > 1$

$$\begin{cases} t_1 + t_2 = -\frac{B}{A} = \frac{1+3\mu}{1+3\mu} = 1 \\ t_1 t_2 = \frac{C}{A} = \frac{1}{1+3\mu} \end{cases} \Rightarrow t_1 \text{ et } t_2 > 0$$

$$t_1 + t_2 = 1 \Rightarrow t_1 \text{ et } t_2 < 1$$

Point double sur l'axe $y \Rightarrow$

$$x(t_1) = x(t_2) = 0$$

Et

$$y(t_1) = 3\mu t_1 (1-t_1) = 3\mu t_1 t_2$$

$$= 3\mu t_2 (1-t_2) = y(t_2) \quad \text{CQFD}$$

4) $\mu = 2$

$$\begin{aligned} x'(t) &= 3 [2t^2(1+3\mu) - 2(1+3\mu)t + 1 + \mu] \\ &= 3 i(t) \end{aligned}$$

$$i(t) = At^2 + Bt + C$$

$$\Delta = B^2 - 4AC = 4(2+3\mu)(\mu-1)$$

2 racines pour $\mu > 1$

$$t_1 + t_2 = 1$$

$$t_1 = \frac{2(2+3\mu) - \sqrt{\Delta}}{4(2+3\mu)}$$

$$\text{Pour } \mu = 2 \quad t_1 = \frac{14 - \sqrt{18}}{4 \times 7} \approx 0,31$$

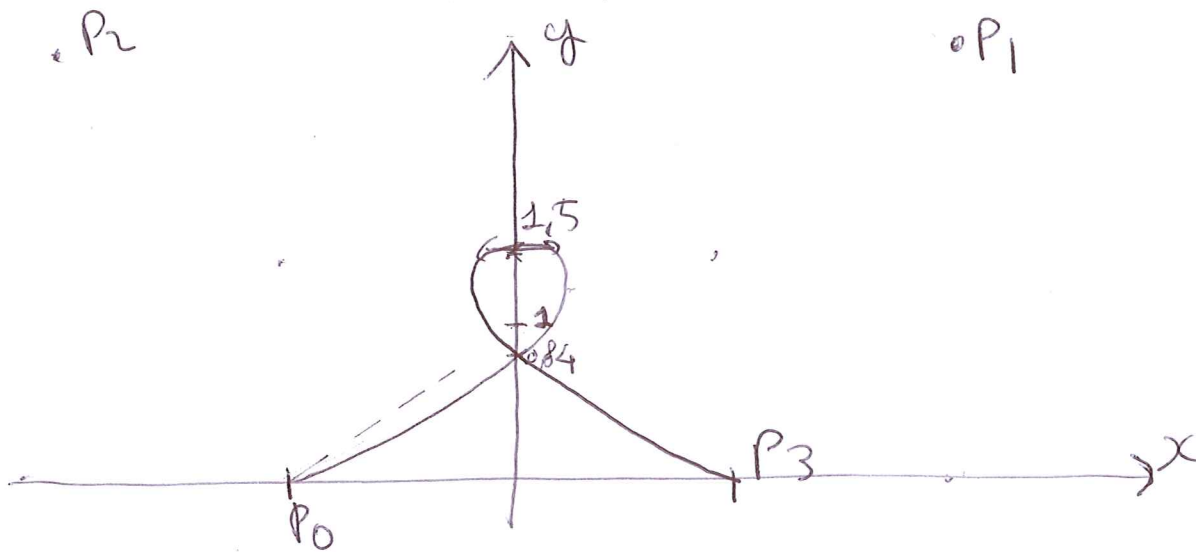
$$x'\left(\frac{1}{2}\right) = -\frac{3}{2}$$

$$t_2 = \frac{7 - \sqrt{21}}{2 \times 7} \approx 0,17$$

$$y'\left(\frac{1}{2}\right) = 0$$

$x'(t) > 0$ à l'extérieur des racines

	0	t_2	t_2	$\frac{1}{2}$
$x'(t)$	+	+	0	-
$y'(t)$	+	+	+	0
$x(t)$	-1	0	0,19	0
$y(t)$	0	0,84	1,28	$\frac{3}{2}$



$$5) \mu = 2$$

$$\left. \begin{array}{l} x'(\frac{1}{2}) = 0 \\ y'(\frac{1}{2}) = 0 \end{array} \right\}$$

$$\left. \begin{array}{l} x''(t) = 6(2+3\mu)(2t-1) \\ y''(t) = -6\mu \end{array} \right\}$$

$$\left. \begin{array}{l} x^{(3)} = 12(2+3\mu) \\ y^{(3)} = 0 \end{array} \right\}$$

$$\vec{\pi}''(\frac{1}{2}) = \vec{0}$$

$$\vec{\pi}''(\frac{1}{2}) \neq \vec{0} \quad \vec{\pi}^{(3)}(\frac{1}{2}) \neq \vec{0} \text{ et colinéaire à } \vec{\pi}''(\frac{1}{2})$$

$$\Rightarrow \left. \begin{array}{l} p=2 \\ q=3 \end{array} \right\}$$

π pair q impair \Rightarrow Point de rebroussement de 2^{ème} espèce

6) $\tilde{\pi}^{-1}(t)$ zéro nul

Pour avoir un point d'inflexion, il faut avoir

$\tilde{\pi}^{-1}(t)$ colinéaire à $\tilde{\pi}'^{-1}(t) \Rightarrow \tilde{\pi}^{-1}(t) \wedge \tilde{\pi}'^{-1}(t) = \vec{0}$

$$x^{-1}(t) = 3 \quad x(t)$$

$$x(t) = 2t^2(1+3\mu) - 2(1+3\mu)t + (1+\mu)$$

$$\tilde{\pi}^{-1}(t) \wedge \tilde{\pi}'^{-1}(t) = \begin{pmatrix} 3 \quad x(t) \\ 3\mu(1-2t) \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 6(1+3\mu)(2t-1) \\ -6\mu \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -18\mu x(t) + 18\mu(1+3\mu)(2t-1)^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 18\mu y(t) \end{pmatrix}$$

$$\begin{aligned} y(t) &= (2t-1)^2(1+3\mu) - [2t^2(1+3\mu) - 2(1+3\mu)t + 1+\mu] \\ &= 2[t^2(1+3\mu) - (1+3\mu)t + \mu] \\ &= 2R(t) \end{aligned}$$

$$R(t) = 0 = A''t^2 + B''t + C''$$

$$\begin{aligned} \Delta'' &= (1+3\mu)^2 - 4\mu(1+3\mu) \\ &= (1+3\mu)(1-\mu) \end{aligned}$$

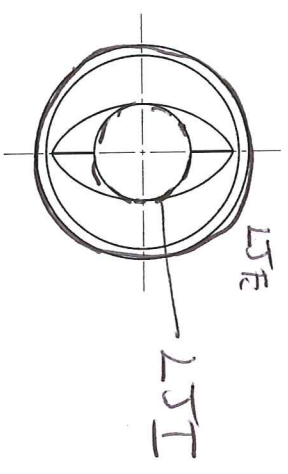
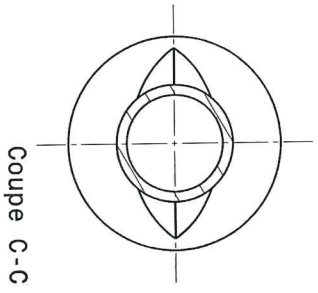
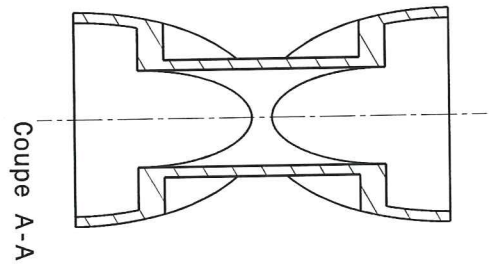
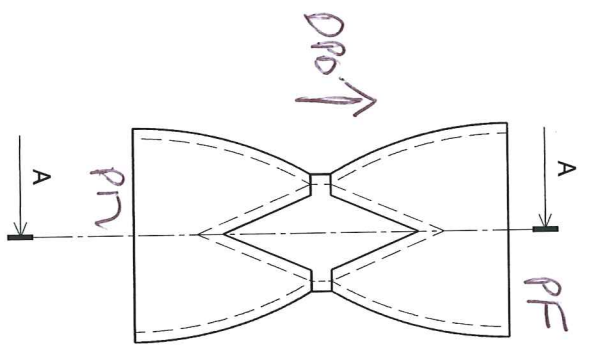
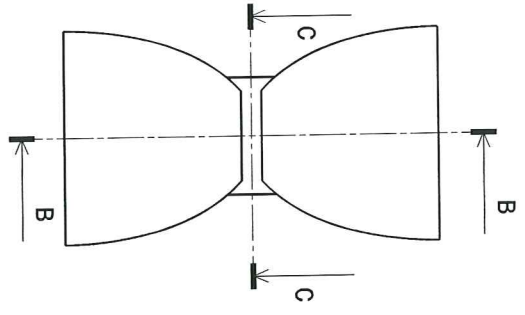
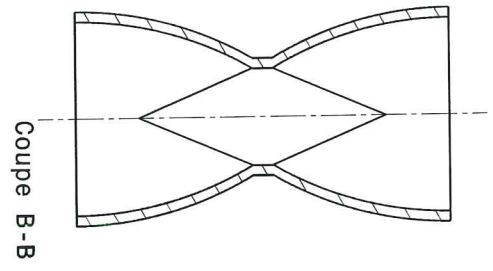
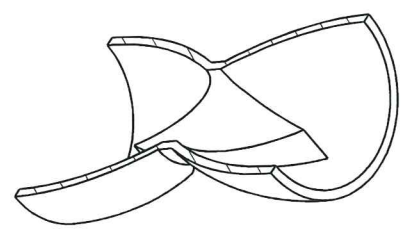
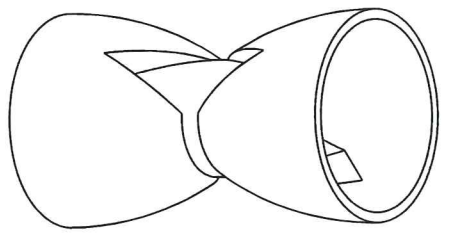
$\Delta'' > 0$ pour $\mu < 1$

Racines de $R(t) = t_1''$ et t_2''

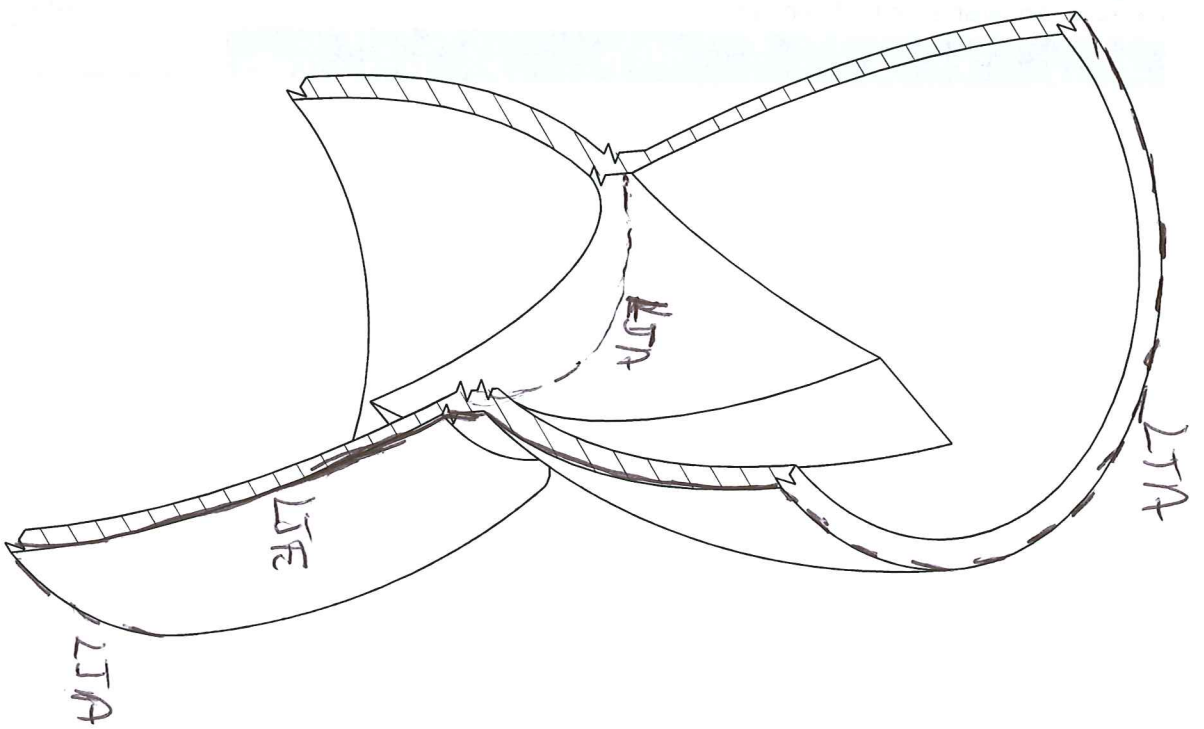
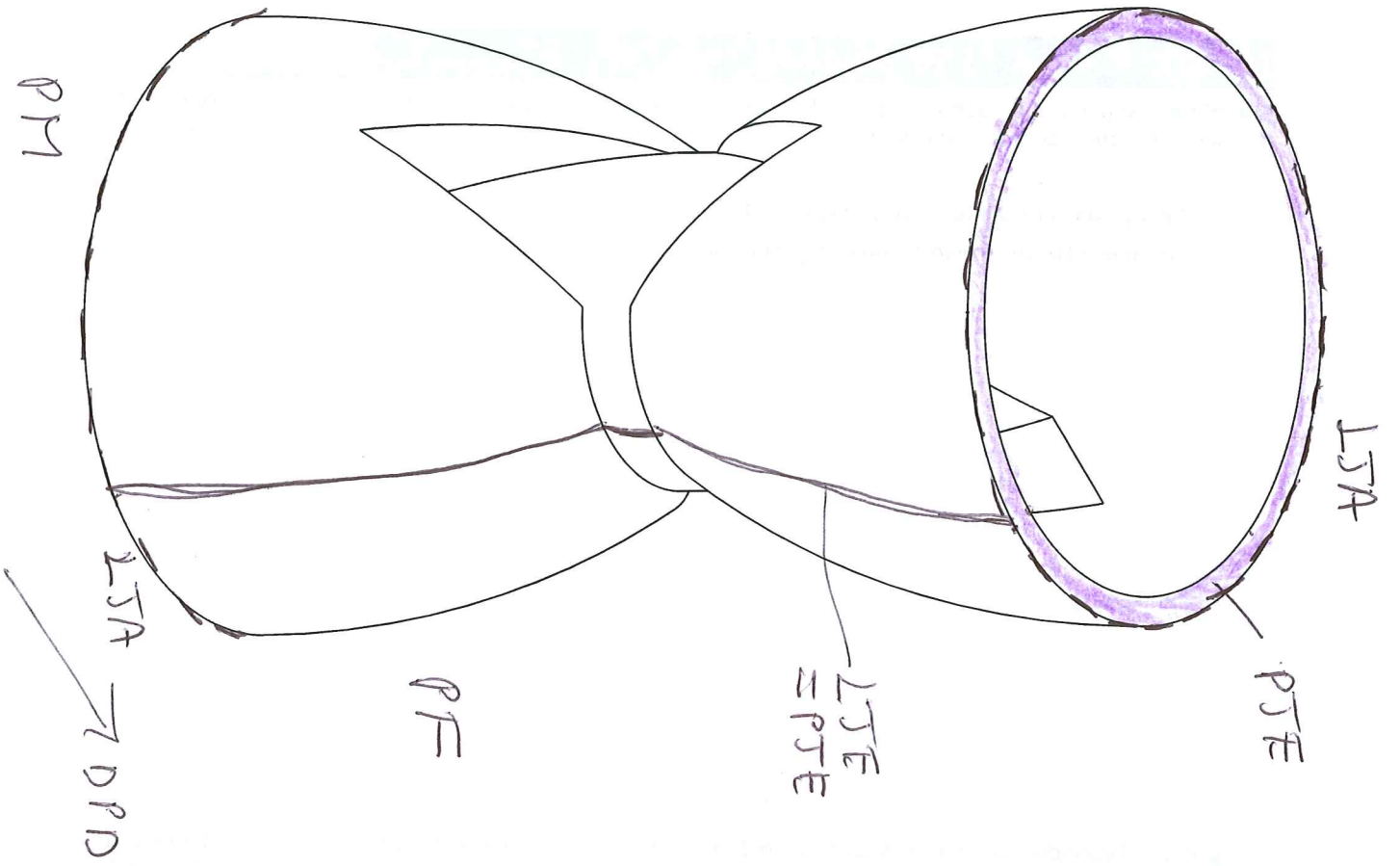
$$t_1'' = \frac{1+3\mu - \sqrt{\Delta''}}{2(1+3\mu)}$$

$$t_2'' = \frac{1+3\mu + \sqrt{\Delta''}}{2(1+3\mu)}$$

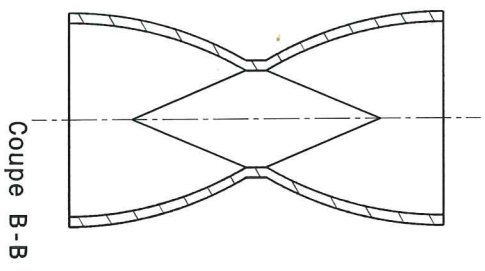
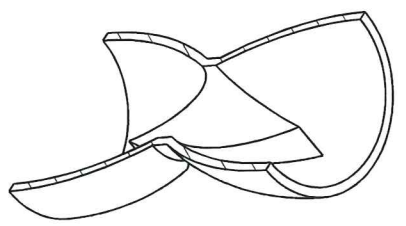
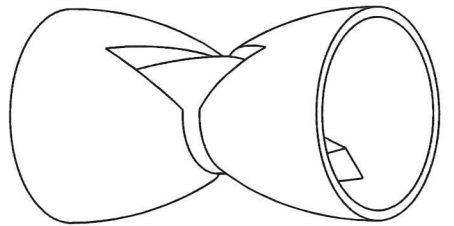
CHORDA



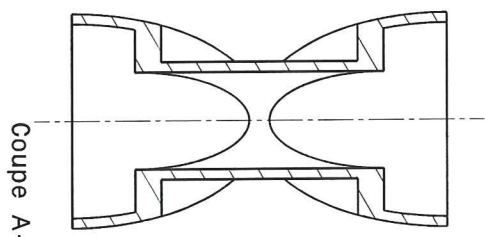
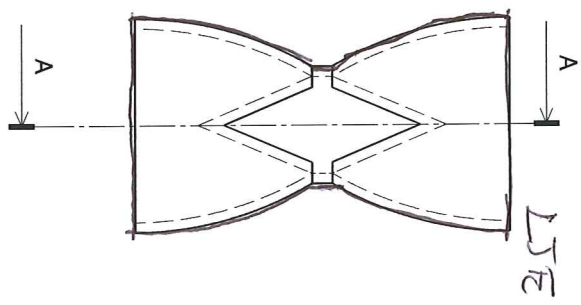
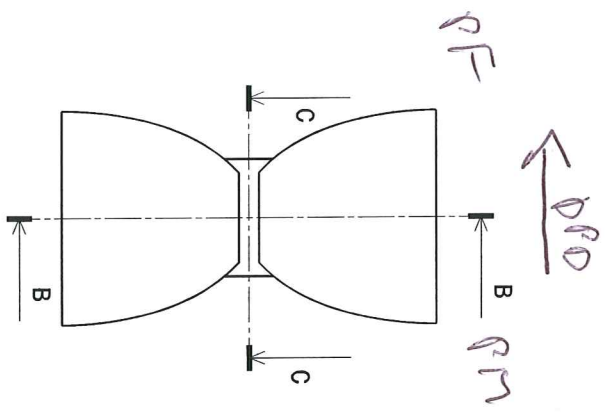
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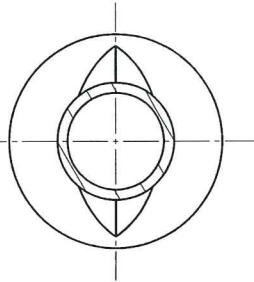
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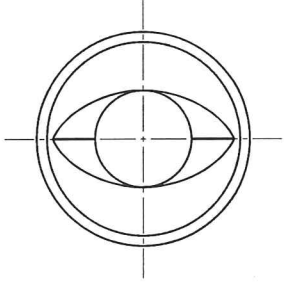
Coupe B-B



Coupe A-A



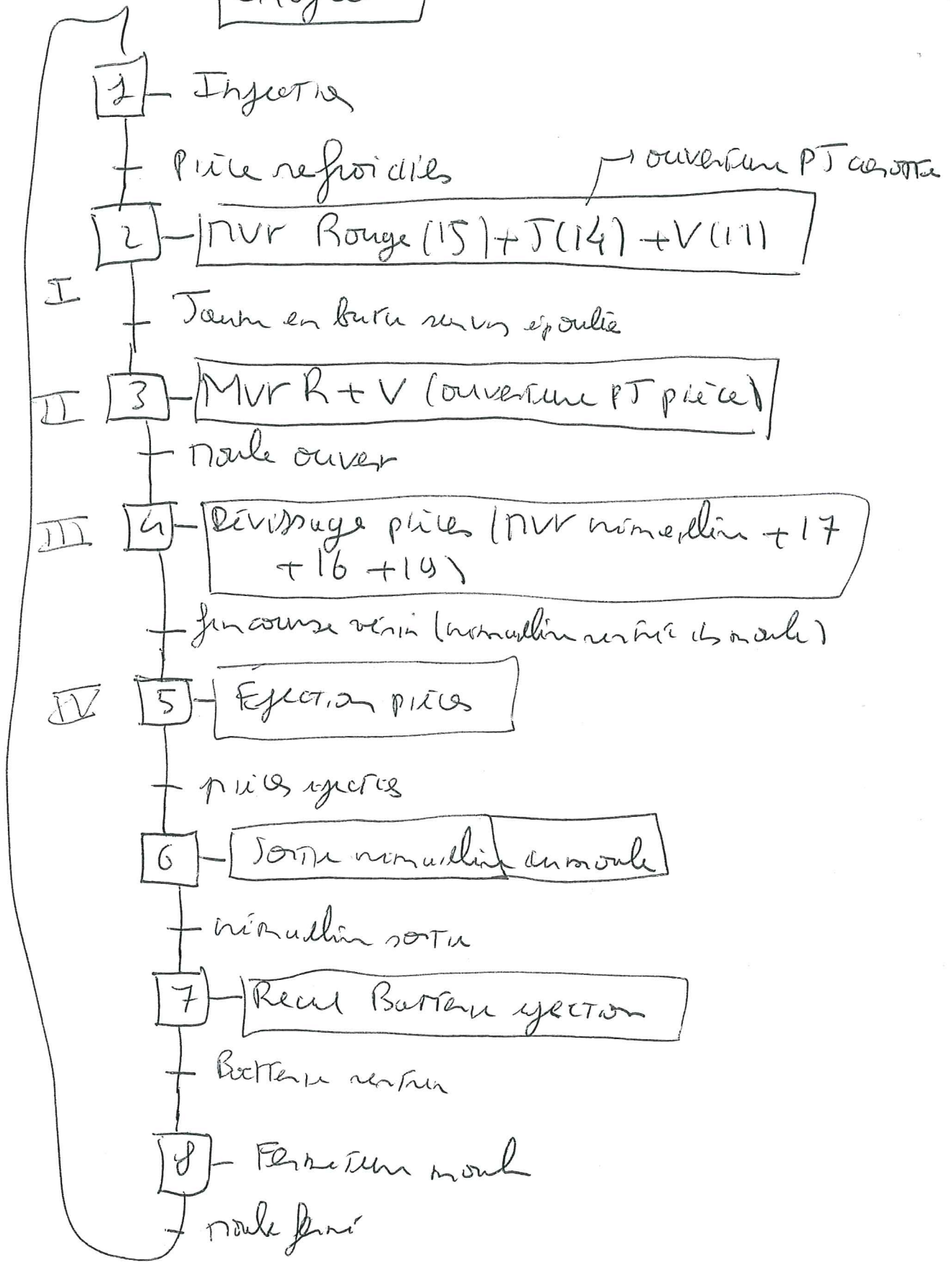
Coupe C-C



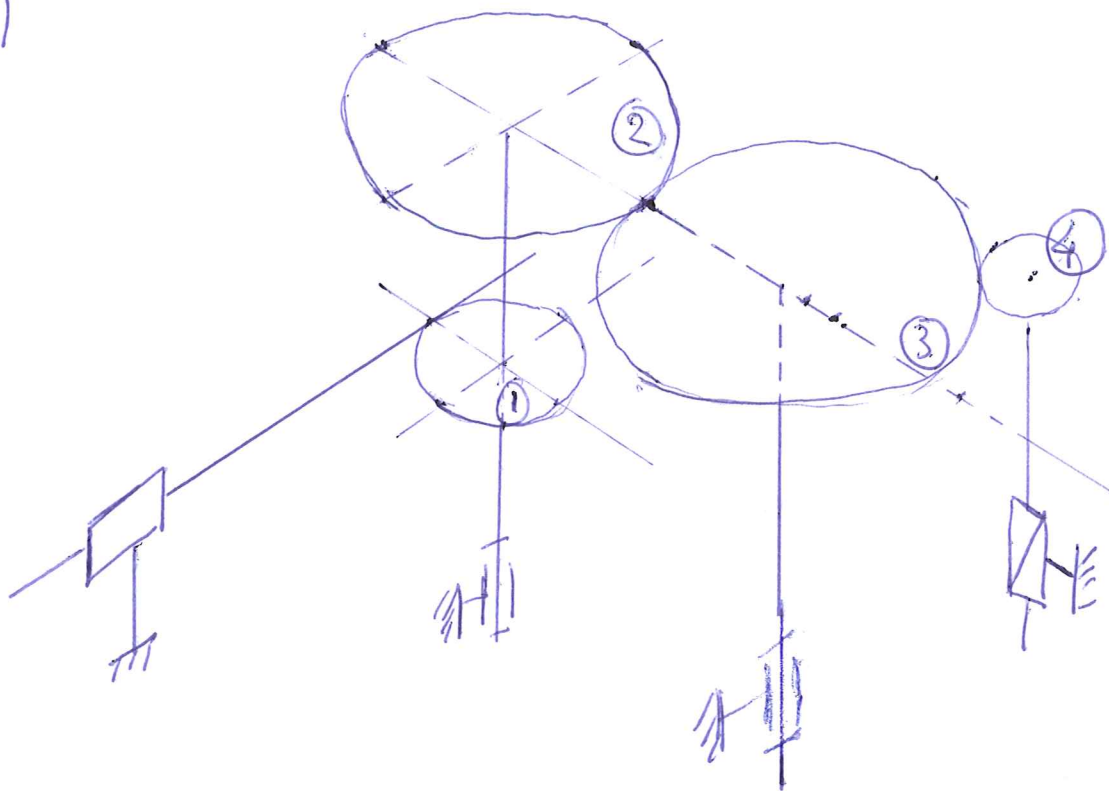
2)

HT

Groffler



3)



4) Course $C = \frac{D_1}{2} \theta_1$

$$\theta_1 = \theta_2 \quad \frac{\theta_2}{\theta_3} = \frac{Z_3}{Z_2} \quad \frac{\theta_3}{\theta_4} = \frac{Z_4}{Z_3}$$

$$\Rightarrow \frac{\theta_1}{\theta_4} = \frac{Z_3 Z_4}{Z_2 Z_3} = \frac{Z_4}{Z_2}$$

$$\Rightarrow C = \frac{D_1}{2} \theta_4 \frac{Z_4}{Z_2}$$

$$\left\{ \begin{array}{l} \theta_4 = n_4 \times 2\pi \\ \text{rad} \quad \text{rev} \end{array} \right. \Rightarrow \theta_4 = \frac{h \times 2\pi}{n}$$

$$\left\{ \begin{array}{l} x_4 = r n_4 = h \\ \text{mm} \quad \text{mm} \end{array} \right.$$

D'où $C = \frac{D_1}{2} \frac{h \times 2\pi}{n} \frac{Z_4}{Z_2}$