Building Energy Simulation

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Lecture 3: Radiation

Building Simulation

- Introduction
- Thermal Analysis
- Conduction
- Convection
- **Radiation**
  - Physical quantities
  - Radiation laws
  - Radiation exchange
    - ems-recep
    - view factor
    - black
    - gray
- Coupled Transfer

Curricula

2 x 4h Lectures
- Conduction
- Convection
- Radiation
- Coupled heat transfer

2 x 4h Tutorials and project
- Model your own SmartHome
- Simulate and discuss

1 x 2h Defend your project

1 x 2h Written exam

Prerequisites
- Calculus
- Linear algebra
- Thermodynamics
- Heat and mass transfer

Radiation
Physical quantities

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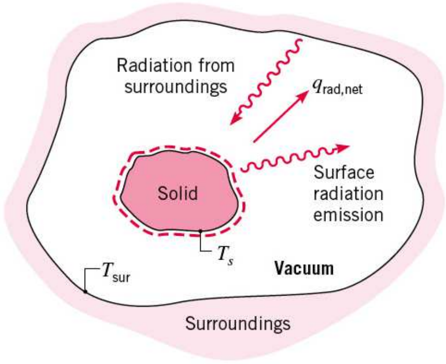

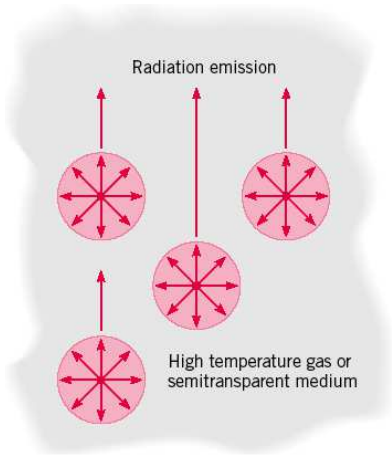

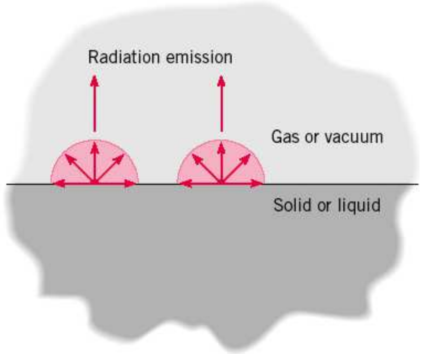

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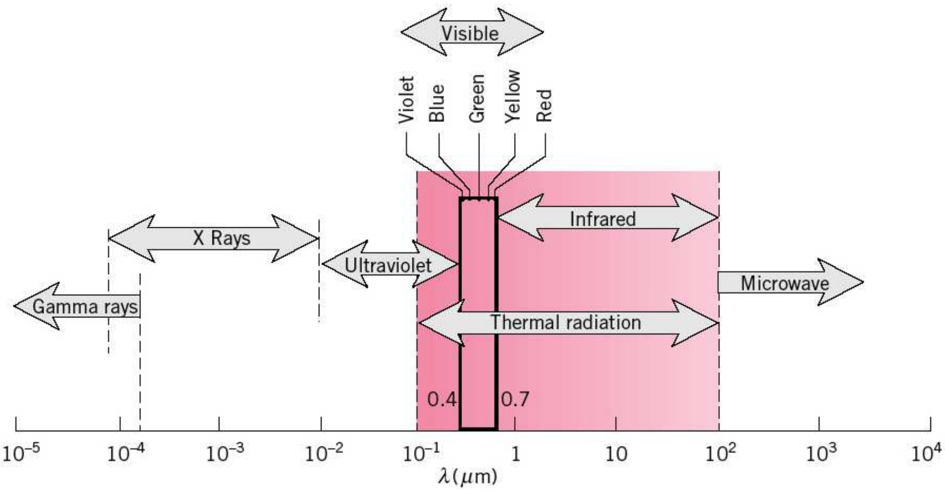

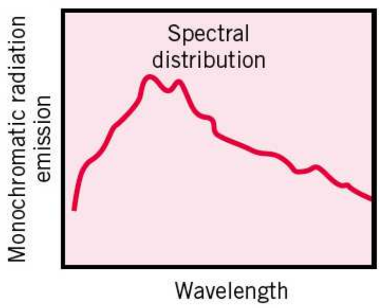

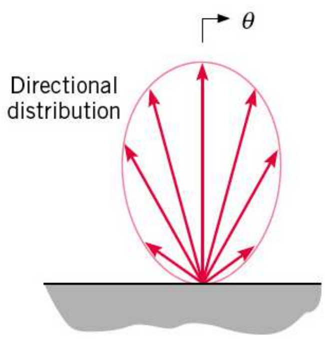

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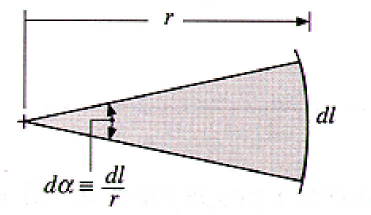
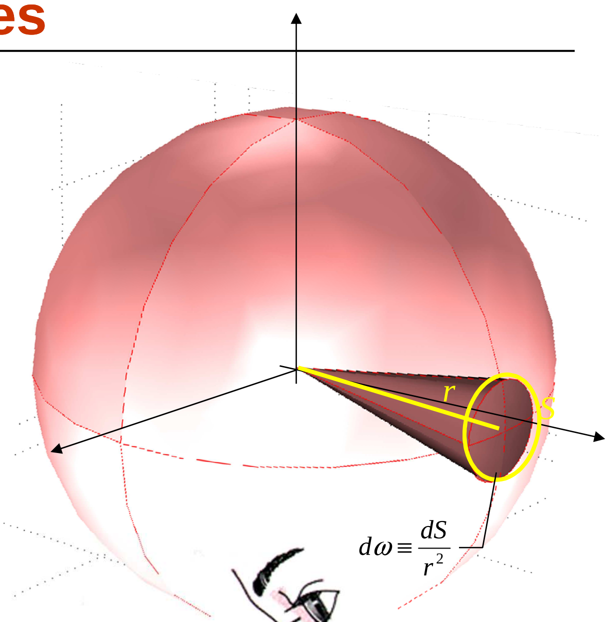
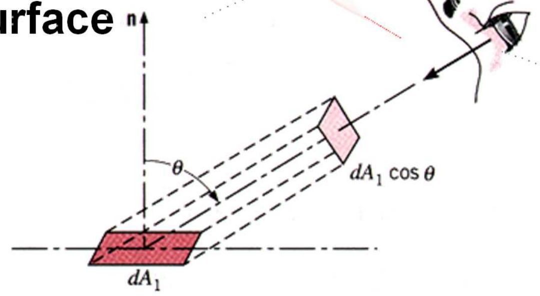
- **Solid angle**





- **Viewed surface**



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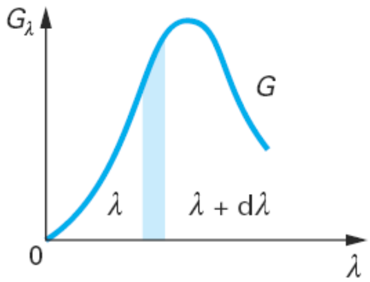
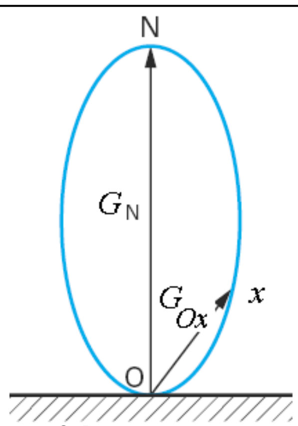
- **Physical quantities**





Function of wavelength

"spectral"

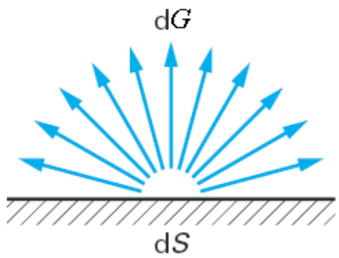
$$G_\lambda = \frac{dG}{d\lambda}$$

"total"

$$G = \int_0^\infty G_\lambda d\lambda$$

Function of direction

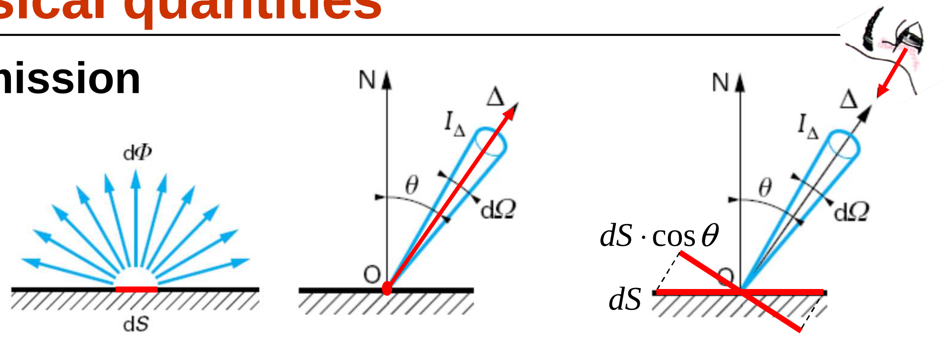
"directional"



"hemispheric"

Radiation
Physical quantities

- **Emission**



Flux dans tout l'espace	Emissive power	Intensity	Luminance of the viewed surface in a direction

  - **total**

$\Phi\,[\mathrm{W}]$ $\quad M = \dfrac{d\Phi}{dS}\left[\dfrac{\mathrm{W}}{\mathrm{m}^2}\right]$ $\quad I_\Delta = \dfrac{d\Phi_\Delta}{d\Omega}\left[\dfrac{\mathrm{W}}{\mathrm{sr}}\right]$

$$L_\Delta = \frac{dI_\Delta}{dS\cos\theta} = \frac{d^2\Phi_\Delta}{d\Omega dS\cos\theta}\left[\frac{\mathrm{W}}{\mathrm{sr}\cdot\mathrm{m}^2}\right]$$

  - **spectral**

$\Phi = \int_0^\infty \Phi_\lambda \,.d\lambda$ $\quad M_\lambda = \dfrac{d\Phi_\lambda}{dS}$ $\quad I_{\Delta\lambda} = \dfrac{d\Phi_\lambda}{d\Omega}$ $\quad L_{\Delta\lambda} = \dfrac{d^2\Phi_\lambda}{d\Omega\, dS\, \cos\theta}$

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- **Reception**
  - **total illuminance**

$$E = \frac{d\Phi}{dS}\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

  - **spectral illuminance**

$$E_\lambda = \frac{d\Phi_\lambda}{dS}$$

Physical quantities

- **Relation emissive power - illuminance**

Heat Flux directional

$$d^2\Phi_{Ox} = L_{Ox} \cdot dS \cos\theta \cdot d\Omega$$

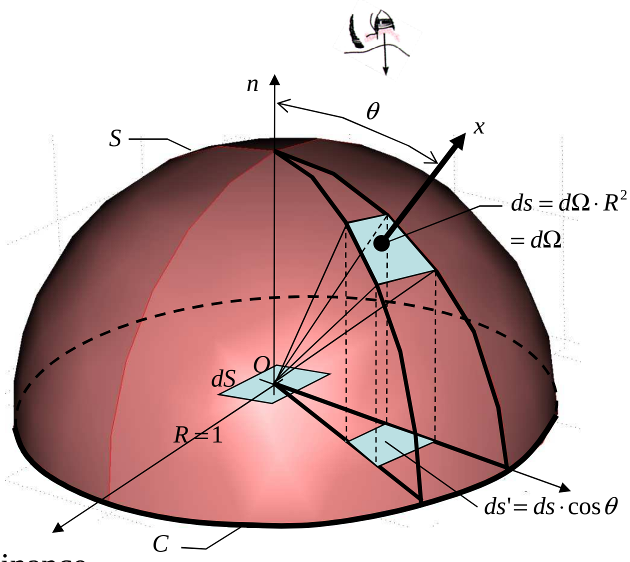
Hemispheric flux

$$d\Phi = L \cdot dS \int_S \cos\theta \cdot d\Omega$$

$$= L \cdot dS \int_C ds'$$

$$= L \cdot dS \cdot \pi$$

$$\frac{d\Phi}{dS} = L \cdot \pi$$



Relation emissive power - illuminance

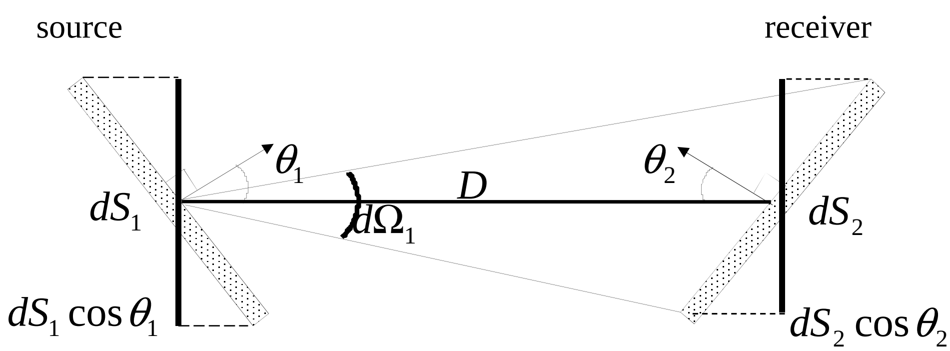
$$M = \pi L$$

Radiation

Physical quantities

- **Relation between physical quantities:**

de Bouguer



$$d^2\Phi_{12} = L_1 \cdot dS_1 \cos\theta_1 \cdot d\Omega_1 \qquad d\Omega_1 = \frac{dS_2 \cos\theta_2}{D^2}$$

$$dE = \frac{d^2\Phi_{12}}{dS_2} = L_1 \frac{\cos\theta_1 \cdot \cos\theta_2}{D^2} dS_1$$

Radiation

Radiation laws

Emissive power of black body

spectral

- **Plank law**
  1. **emissive power as a function of temperature and wavelength**
- **Wien laws**
  2. **maximum of spectral emissive power**
  3. **maximum as a function of temperature**

total

- **Stefan – Boltzmann law**
  4. **emissive power as a function of temperature**

Gray body

- **Kirchhoff law**
  5. **relation absorption - emission**

Radiation

Radiation laws

Black body :

- **absorbs all incident radiation** (regardless of wavelength or direction)
- **emits the maximum energy** for a given temperature and wavelength
- **is a diffuse emitter** (but in function of temperature and wavelength) **according to Lambert law**

$$L^0 = \frac{M^0}{\pi} \qquad L_\lambda^0 = \frac{M_\lambda^0}{\pi}$$

Black body is:

- **reference for real bodies : "yardstick" for radiation**
- **perfect "source" and "absorber"**
- **characterized by hemispheric physical quantities (noted °)**

Radiation
Radiation laws

- **Plank law**

$$M^{\circ}_{\lambda,T} = \frac{C_1 \lambda^{-5}}{e^{\frac{C_2}{\lambda . T}} - 1}$$

$$C_1 = 2\pi h c^2 = 3{,}74 \cdot 10^{-16}\ [\mathrm{W \cdot m^2}]$$
$$C_2 = hc/k = 1{,}4 \cdot 10^{-2}\ [\mathrm{m \cdot K}]$$

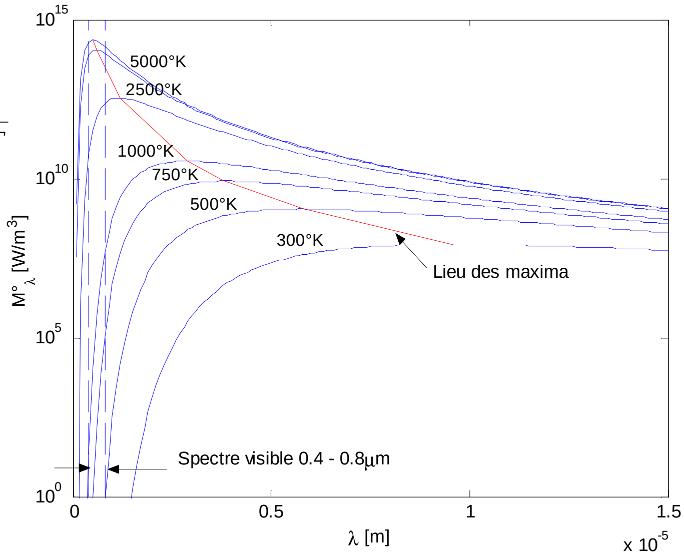
c speed of light
h Plank constant
k Boltzmann constant

shortwave (SW)

$$M^{\circ}_{\lambda,T} = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda . T}}$$

longwave (LW)

$$M^{\circ}_{\lambda,T} = \frac{C_1 T}{C_2 \lambda^4}$$



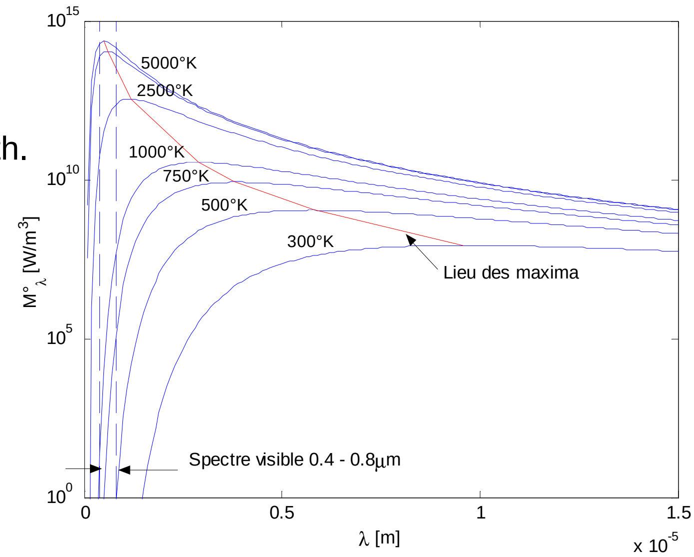
Radiation
Radiation laws

- **Plank law**

1. Emissive power varies with wavelength.

2. Emissive power increases with the temperature of the source for every wavelength.

3. Visible spectrum contains a large majority of solar radiation.



4. Effective emission band depends on source temperature: higher the temperature, higher the frequency radiation (Wien laws, separation SW / LW).

Radiation laws

- **Wien laws**

1. λ for maximum spectral power emission

$$\frac{dM^0_{\lambda T}}{d\lambda} = 0 \Rightarrow \lambda_M T = 2.897 \cdot 10^{-3}\,[\mathrm{m \cdot K}]$$

2. maximum spectral power emission

$$M^0_{\lambda_M T} = B \cdot T^5\,;\; B = 1.286 \cdot 10^5\,[\mathrm{W \cdot m^{-3} \cdot K^{-5}}]$$

- **Stefan – Boltzmann law**

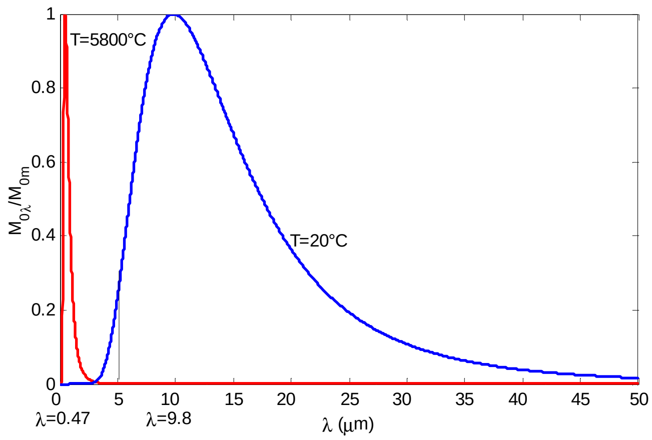
$$M^0 = \int_0^\infty M^0_{\lambda T}\, d\lambda = \sigma_0 T^4 \qquad M^0 = 5.68\left(\frac{T}{100}\right)^4$$

$$\sigma_0 = 5.68 \cdot 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$$

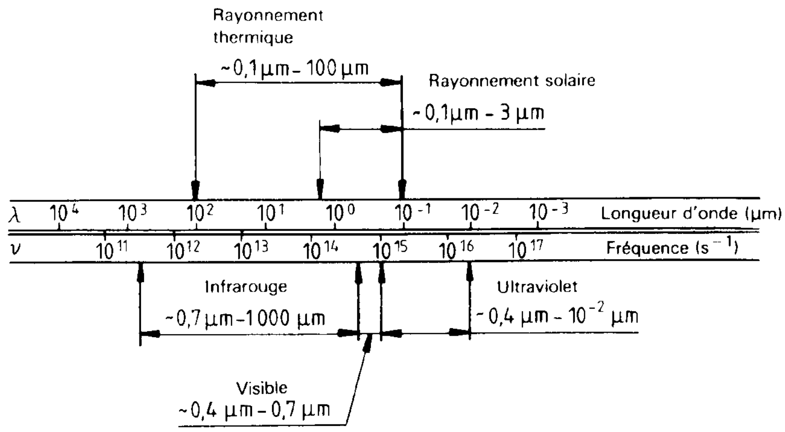
Radiation laws

Effective emission band

- **Fraction of total emission in a band** $F_{\lambda_1-\lambda_2} = \dfrac{\int_{\lambda_1}^{\lambda_2} M^0_{\lambda T}\, d\lambda}{M^0}$
- **Effective spectral band** $F_{0.5\lambda-5\lambda} = 0.956$
- **solar emission:** 50% visible, 40% IR, 8% UV*
- **SW and LW radiation**



Radiation



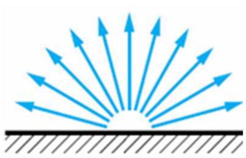
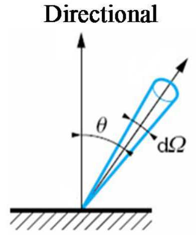
Emissive power of black body

Temperature		Emissive power	Max. wavelength	Spectral band
Absolue, T [K]	Celsius, θ [°C]	M^0 [W/cm^2]	λ_M [μm]	$0.5\lambda_M - 5\lambda_M$ [μm]
300	27	0.05	9.6	4.8 - 41
500	227	0.36	5.7	3.0 - 25
750	477	1.80	3.8	2.0 - 16
1000	727	5.70	2.9	1.5 - 12
1200	927	11.82	2.4	1.2 - 11
1500	1227	28.90	1.9	1.0 – 8
2000	1727	91.00	1.4	0.7 - 6
3000	2727	462.00	0.96	0.5 - 4
5790	5517	6383.6	0.50	0.25 – 2.5

Radiation

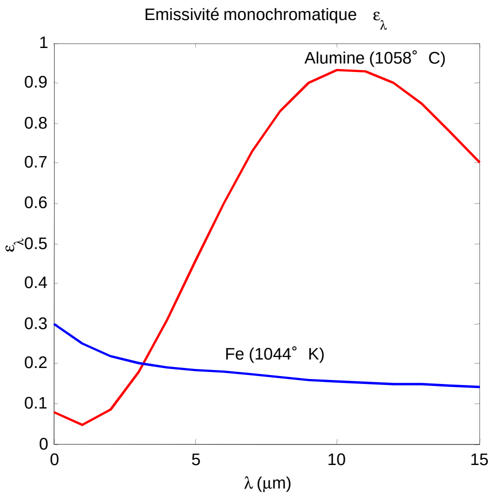
Radiation laws

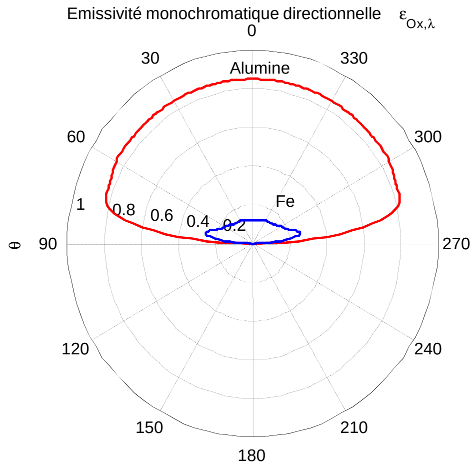
Emission from real surfaces: comparison with a black body for the same temperature and wavelength = **emissivity**

	Hemispheric	Directional
		
Spectral	$\varepsilon_\lambda = \dfrac{M_\lambda}{M_\lambda^0}$	$\varepsilon_{Ox,\lambda} = \dfrac{L_{Ox,\lambda}}{L_\lambda^0} = \dfrac{L_{Ox,\lambda}}{M_\lambda^0 / \pi}$
Total	$\varepsilon = \dfrac{M}{M^0} = \dfrac{\int_0^\infty \varepsilon_\lambda M_\lambda^0 d\lambda}{\sigma T^4}$	$\varepsilon_{Ox} = \dfrac{\int_0^\infty \varepsilon_{Ox,\lambda} L_\lambda^0 d\lambda}{\int_0^\infty L_\lambda^0 d\lambda} = \dfrac{\int_0^\infty \varepsilon_{Ox,\lambda} M_\lambda^0 d\lambda}{\sigma T^4}$

Radiation
Radiation laws

- **Emissivity**





spectral, hemispherical

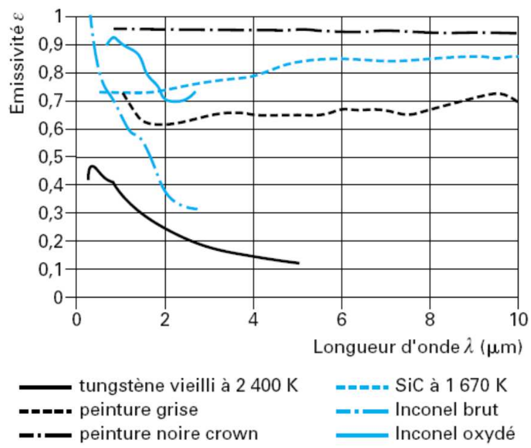
$$\varepsilon_\lambda = \frac{M_\lambda}{M_\lambda^0}$$

spectral, directional

$$\varepsilon_{Ox,\lambda} = \frac{L_{Ox,\lambda}}{L_\lambda^0} = \frac{L_{Ox,\lambda}}{M_\lambda^0/\pi}$$

Radiation
Radiation laws

- **Emissivity: particular cases**
  - Gray body $\varepsilon_{Ox,\lambda} \to \varepsilon_{Ox}\,;\varepsilon_\lambda \to \varepsilon$
  - Diffuse (isotropic) emission $\varepsilon_{Ox,\lambda} \to \varepsilon_\lambda;\ \varepsilon_{Ox} \to \varepsilon$
  - Gray and diffuse $\varepsilon_{Ox,\lambda} \to \varepsilon$



Example of emissivity

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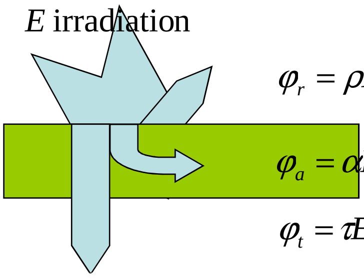
- **Radiation reception**

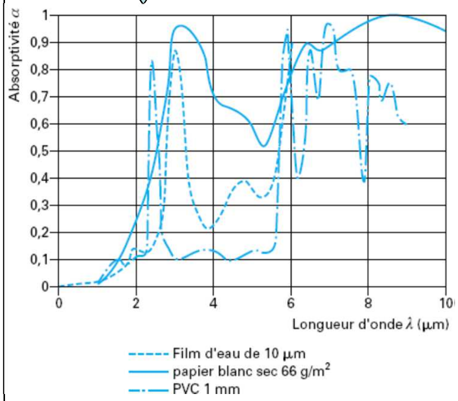


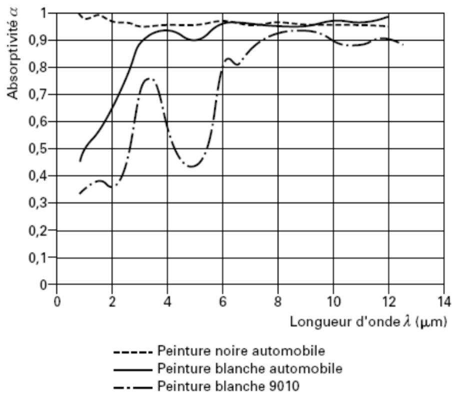
$\varphi_r = \rho E$; reflected

$\varphi_a = \alpha E$; absorbed

$\varphi_t = \tau E$; transmitted

$\alpha + \tau + \rho = 1$





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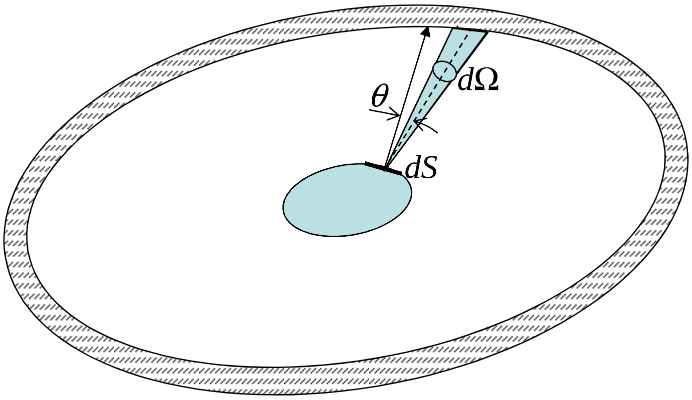
- **Kirchhoff law:** relation emissivity and absorptivity

$$\varepsilon_{Ox,\lambda} = \alpha_{Ox,\lambda}$$



emission :

$$\left(d^2\Phi_{Ox,\lambda}\right)_e = \varepsilon_{Ox,\lambda}\ L^0_\lambda\ dS \cos\theta\ d\Omega$$

absorption :

$$\left(d^2\Phi_{Ox,\lambda}\right)_a = \alpha_{Ox,\lambda}\ L^0_\lambda\ dS \cos\theta\ d\Omega$$

Radiation
Radiation laws

- **Kirchhoff law** $\varepsilon_{Ox,\lambda} = \alpha_{Ox,\lambda}$
  - diffuse emission $\varepsilon_\lambda = \alpha_\lambda$
  - in general $\varepsilon \neq \alpha$

$$\varepsilon(T) = \frac{M(T)}{M^0(T)} = \frac{\int_0^\infty \varepsilon_\lambda M_\lambda^0(T)d\lambda}{\int_0^\infty M_\lambda^0(T)d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda M_\lambda^0(T)d\lambda}{\sigma T^4} \quad \text{(own temp.)}$$

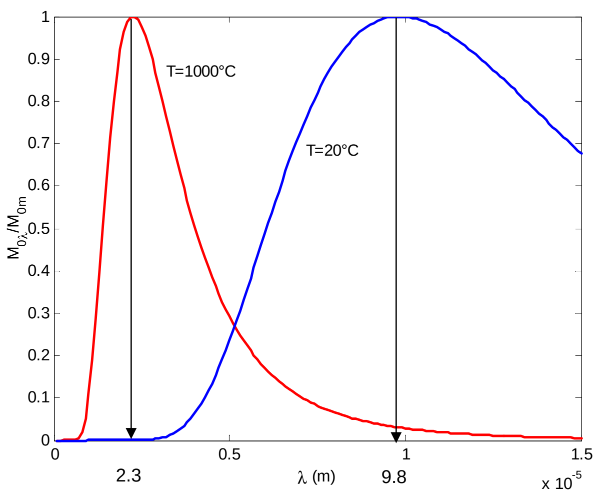
$$\alpha = \frac{\varphi_a}{E} = \frac{\int_0^\infty \alpha_\lambda E_\lambda d\lambda}{\int_0^\infty E_\lambda d\lambda} \quad \text{(depends on received radiation)}$$

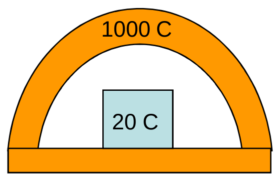
  - exceptions
    - *gray bodies* $\varepsilon = \alpha$
    - *black body* $\varepsilon = \alpha = 1$

Radiation
Radiation laws

- **Practical consequences: radiative heating**

Radiation in different spectral bands $F_{0.5\lambda-5\lambda} = 0.956$





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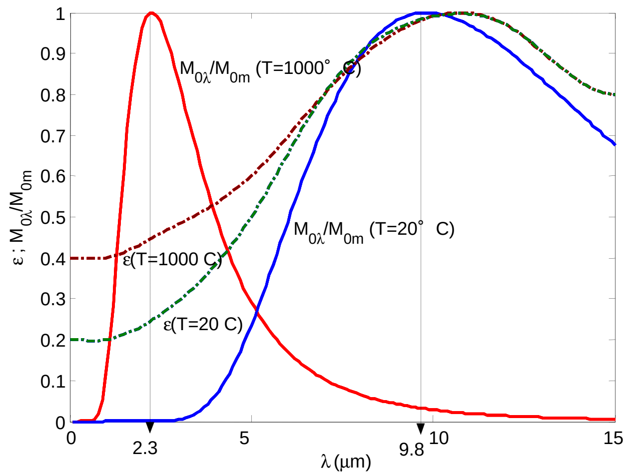
- **Practical consequences: radiative heating**

for every surface, ε corresponding to a spectral band

$$\varepsilon_{20°C} = \frac{1}{50-5}\int_{5}^{50} \varepsilon_\lambda \cdot d\lambda = 0.9 \qquad \varepsilon_{1000°C} = \frac{1}{5-1.25}\int_{1.25}^{5} \varepsilon_\lambda \cdot d\lambda = 0.43$$



	5790K	300K
Matériau	a	ε
Carton goudronné noir	0,82	0,91
Brique rouge	0,75	0,93
Blanc de zinc	0,22	0,92
Neige propre	0,20...0,35	0,95
Chrome poli	0,40	0,07
Or poli	0,29	0,026
cuivre poli	0,18	0,03
cuivre, oxydé	0,70	0,45

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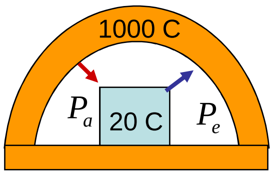
- **Practical consequences: radiative heating**



$$P_a = \varepsilon_{1000°C} M^0 = 0.43\sigma(1273)^4 = 6.7 \cdot 10^4 \text{ W/m}^2$$
$$P_e = \varepsilon_{20°C} M^0 = 0.9\sigma(293)^4 = 3.76 \cdot 10^2 \text{ W/m}^2$$

$P_a > P_e \Rightarrow$ body is heating

Temperature

- source of radiation
- own

Radiation laws

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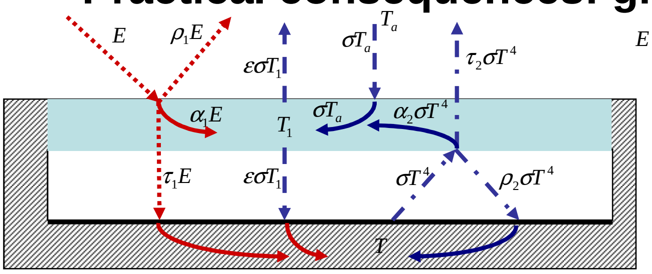
- **Practical consequences: greenhouse effect**



$E = 1000\ \mathrm{W/m^2}$

CLO $(0.25 \le \lambda \le 2.5\,\mu\mathrm{m})$, $T = 5780$ K $(\lambda_M = 0.5\,\mu\mathrm{m})$

GLO $(3 \le \lambda \le 30\,\mu\mathrm{m})$, $T = 450$ K $(\lambda_M = 6.1\,\mu\mathrm{m})$

GLO $(5 \le \lambda \le 50\,\mu\mathrm{m})$, $T = 300$ K $(\lambda_M = 9.8\,\mu\mathrm{m})$

Bilan sur la vitre $\quad 2\varepsilon \cdot \sigma T_1^4 = \alpha_1 E + \alpha_2 \sigma T^4 + \alpha_3 \sigma T_a^4$

Rayonnement absorbé :

- $\alpha_1 E$ - solaire
- $\alpha_2 \sigma T^4$ - émis par la surface noire
- $\alpha_3 \sigma T_a^4$ - émis par l'environnement

Rayonnement émis :

- $2\varepsilon \cdot \sigma T_1^4$ - les deux surfaces de la vitre

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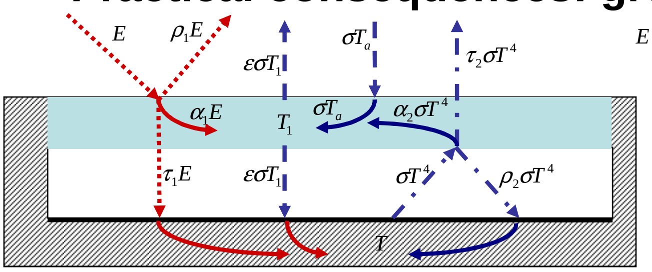
- **Practical consequences: greenhouse effect**



$E = 1000\ \mathrm{W/m^2}$

CLO $(0.25 \le \lambda \le 2.5\,\mu\mathrm{m})$, $T = 5780$ K $(\lambda_M = 0.5\,\mu\mathrm{m})$

GLO $(3 \le \lambda \le 30\,\mu\mathrm{m})$, $T = 450$ K $(\lambda_M = 6.1\,\mu\mathrm{m})$

GLO $(5 \le \lambda \le 50\,\mu\mathrm{m})$, $T = 300$ K $(\lambda_M = 9.8\,\mu\mathrm{m})$

Balance on black absorber $\quad \sigma T^4 = \tau_1 E + \varepsilon \sigma T_1^4 + \rho_2 \sigma T^4$

Absorbed radiation :

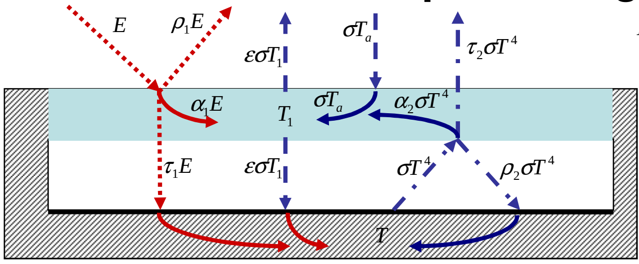
- $\tau_1 E$ transmitted solar radiation through the glass
- $\varepsilon \sigma T_1^4$ emmitted by the glass
- $\rho_2 \sigma T^4$ reflected by the glass

Emmitted radiation :

- σT^4 - towards the glass

Radiation laws

- **Practical consequences: greenhouse effect**



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GLO $(5 \le \lambda \le 50\,\mu\text{m})$, $T = 300$ K $(\lambda_M = 9.8\,\mu\text{m})$

$E = 1000\ \text{W/m}^2$

$$2\varepsilon\sigma T_1^4 = \alpha_1 E + \alpha_2\sigma T^4 + \alpha_3\sigma T_a^4$$
$$\sigma T^4 = \tau_1 E + \rho_2\sigma T^4 + \varepsilon\sigma T_1^4$$

$$\Rightarrow T = 460\ \text{K} = 187\ ^\circ\text{C}$$

Radiative properties of glass

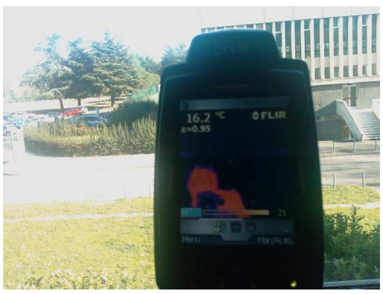
	Spectral band	Temperature	α	ρ	τ
1	$0.25 \le \lambda \le 2.5\,\mu\text{m}$	5780 K ($\lambda = 0.5\,\mu\text{m}$)	0	0.05	0.95
2	$3 \le \lambda \le 30\,\mu\text{m}$	450 K ($\lambda = 6.1\,\mu\text{m}$)	0.65	0.30	0.05
3	$5 \le \lambda \le 50\,\mu\text{m}$	300 K ($\lambda = 9.8\,\mu\text{m}$)	1.00	0	0

Radiation laws

- **Practical consequences: greenhouse effect**

Propriétés radiatives du verre

	Bande spectrale	Température de rayonnement	α	ρ	τ
1	$0.25 \le \lambda \le 2.5\ \mu\text{m}$	5780 K ($\lambda = 0.5\ \mu\text{m}$)	0	0.05	0.95
2	$5 \le \lambda \le 50\ \mu\text{m}$	300 K ($\lambda = 9.8\ \mu\text{m}$)	1.00	0	0



Radiation

Radiation exchange: emission-reception

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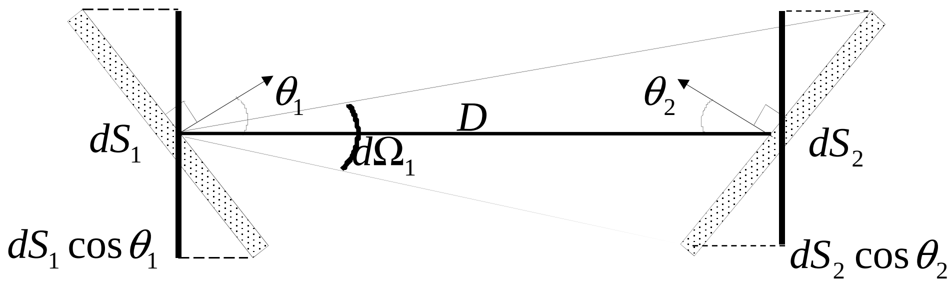
- **Bouguer relation**



$$d^2\Phi_{12} = L_1 \cdot dS_1 \cos\theta_1 \cdot d\Omega_1 \qquad d\Omega_1 = \frac{dS_2 \cos\theta_2}{D^2}$$

$$d^2\Phi_{12} = \frac{M_1^0}{\pi} \cdot dS_1 \cos\theta_1 \cdot \frac{dS_2 \cos\theta_2}{D^2}$$

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Radiation exchange: emission-reception

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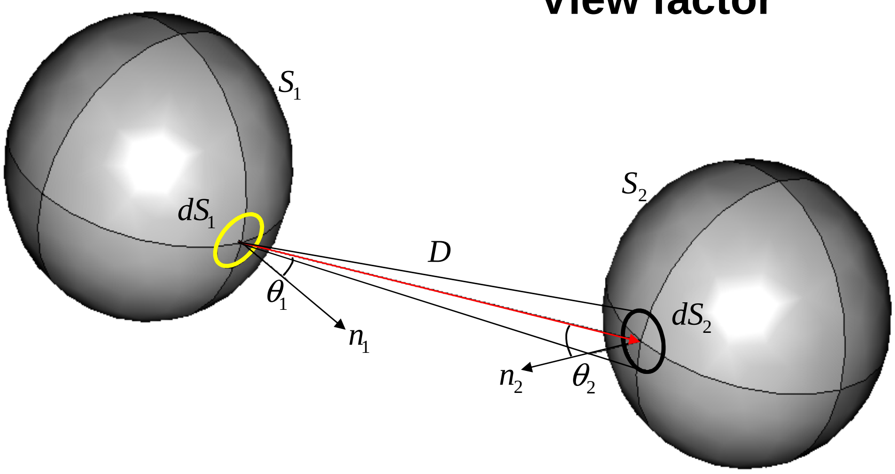
View factor



$$d^2\Phi_{12} = L_1^0 \cdot dS_1 \cos\theta_1 \cdot d\Omega_1 = \frac{M_1^0}{\pi} \cdot dS_1 \cos\theta_1 \cdot \frac{dS_2 \cos\theta_2}{D^2}$$

Radiation

Radiation exchange: emission-reception

$$\Phi_{12} = M_1^0 \int_{S_1}\int_{S_2} \frac{dS_1 \cos\theta_1 \cdot dS_2 \cos\theta_2}{\pi \cdot D^2}$$

View factor

$$F_{12} \equiv \frac{\Phi_{12}}{\Phi_1} = \frac{\Phi_{12}}{M_1^0 S_1} = \frac{1}{\pi S_1}\int_{S_1}\int_{S_2} \frac{dS_1 \cos\theta_1 \cdot dS_2 \cos\theta_2}{D^2}$$

$$F_{21} \equiv \frac{\Phi_{21}}{\Phi_2} = \frac{1}{\pi S_2}\int_{S_1}\int_{S_2} \frac{dS_1 \cos\theta_1 \cdot dS_2 \cos\theta_2}{D^2}$$

Radiation

Radiation exchange: view factors

- **Relations between view factors**
  - reciprocity $S_1 F_{12} = S_2 F_{21}$
  - complementarity (closed enclosure) $\sum_{j=1}^{n} F_{ij} = 1$

$$\Phi_i = \sum_{j=1}^{n} \Phi_{ij}$$

$$\Phi_i = \sum_{j=1}^{n} F_{ij} \Phi_i$$

Radiation exchange: view factors

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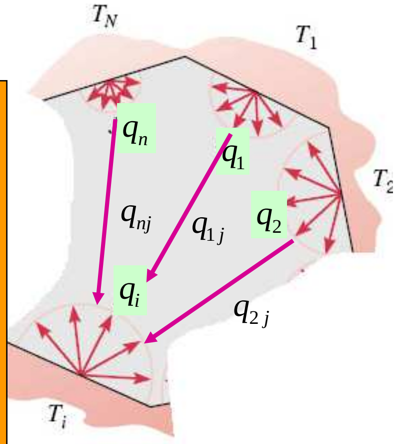
- **Relations between view factors**

$$S_1 F_{12} = S_2 F_{21}$$

$$\mathbf{F} = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & F_{22} & \cdots & F_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ F_{n1} & F_{n2} & \cdots & F_{nn} \end{bmatrix}$$

$$\rightarrow \sum_{j=1}^{n} F_{1j} = 1 \qquad \rightarrow \sum_{j=1}^{n} F_{2j} = 1 \qquad \rightarrow \sum_{j=1}^{n} F_{nj} = 1$$



$n(n-1)/2$ facteurs de forme à calculer

$F_{jj} = 0$ si la surface est plane ou concave

Radiation exchange: view factors

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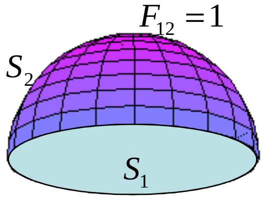
- **Exemples of view factors**

$F_{12} = F_{21} = 1$

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$F_{12} = 1$ $\quad S_1 F_{12} = S_2 F_{21}$ $\quad F_{21} + F_{22} = 1$

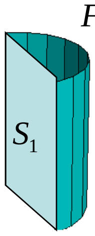
$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$$



$$F_{21} = \frac{S_1}{S_2} = \frac{\pi R^2}{\frac{1}{2} 4\pi R^2} = \frac{1}{2} \qquad F_{22} = \frac{1}{2}$$

$F_{12} = 1$ $\quad S_1 F_{12} = S_2 F_{21}$ $\quad F_{21} + F_{22} = 1$

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ 2/\pi & 1 - 2/\pi \end{bmatrix}$$



$$F_{21} = \frac{S_1}{S_2} = \frac{2Rh}{\pi Rh} = \frac{2}{\pi} \qquad F_{22} = 1 - \frac{2}{\pi}$$

Radiation exchange: view factors

- **Two small surfaces**

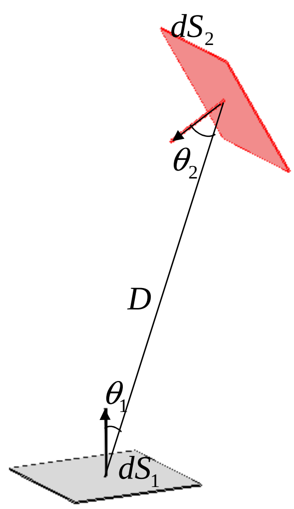
$$d^2\Phi_{12} = L_1^0 \cdot dS_1 \cos\theta_1 \cdot d\Omega_1 = \frac{M_1^0}{\pi} \cdot dS_1 \cos\theta_1 \cdot \frac{dS_2 \cos\theta_2}{D^2}$$

$$\Phi_{12} = M_1^0 \frac{dS_1 \cos\theta_1 \cdot dS_2 \cos\theta_2}{\pi D^2} \qquad \Phi_1 = M_1^0 \cdot dS_1$$

$$F_{12} \equiv \frac{\Phi_{12}}{\Phi_1} = \frac{\Phi_{12}}{M_1^0 \cdot dS_1} = \frac{\cos\theta_1 \cdot \cos\theta_2}{\pi D^2} dS_2$$

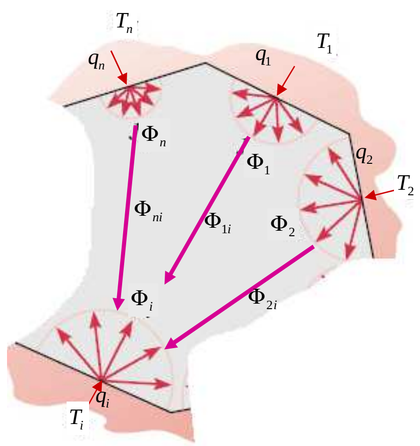
$$F_{21} = \frac{\cos\theta_1 \cdot \cos\theta_2}{\pi D^2} dS_1$$

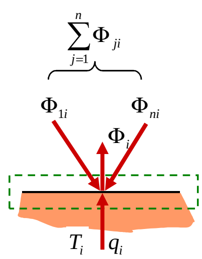
$$\mathbf{F} = \begin{bmatrix} 0 & \frac{\cos\theta_1 \cdot \cos\theta_2}{\pi D^2} dS_2 \\ \frac{\cos\theta_1 \cdot \cos\theta_2}{\pi D^2} dS_1 & 0 \end{bmatrix}$$



Radiation exchange: black surfaces

- **Net radiative heat exchange**
  - exchange in a closed black enclosure





$$q_i \equiv \underbrace{\Phi_{i,net}}_{\text{net}} = \underbrace{\Phi_i}_{\text{sent}} - \underbrace{\sum_{j=1}^{n} \Phi_{ji}}_{\text{reçeived}} = \Phi_i - \sum_{j=1}^{n} F_{ji}\Phi_j = S_i M_i^0 - \sum_{j=1}^{n} S_j F_{ji} M_j^0$$

Black surfaces

Radiation exchange: black surfaces

- **Net heat**
  - black, closed enclosure

$$S_i F_{ij} = S_j F_{ji} \Rightarrow \underbrace{\Phi_{i,net}}_{\text{net}} = \underbrace{S_i M_i^0}_{\text{émis}} - \underbrace{\sum_{j=1}^{n} S_j F_{ji} M_j^0}_{\text{reçu}} = S_i M_i^0 - S_i \sum_{j=1}^{n} F_{ij} M_j^0$$

Réciprocité

$$\sum_{j=1}^{n} F_{ij} = 1 \quad \Rightarrow \Phi_{i,net} = S_i M_i^0 - S_i \sum_{j=1}^{n} F_{ij} M_j^0 = S_i M_i^0 \sum_{j=1}^{n} F_{ij} - S_i \sum_{j=1}^{n} F_{ij} M_j^0$$

Complémentarité

$$\Phi_{i,net} = \underbrace{\sum_{j=1}^{n} S_i F_{ij} (M_i^0 - M_j^0)}$$

flux échangé entre les surfaces noires S_i → S_j

$$\Phi_{i,net} = \sum_{j=1}^{n} S_i F_{ij} (M_i^0 - M_j^0) = \sum_{j=1}^{n} (\underbrace{S_i F_{ij} M_i^0}_{\text{émis}} - \underbrace{S_j F_{ji} M_j^0}_{\text{reçu}}) = \sum_{j=1}^{n} \underbrace{\Phi_{ij,net}}_{\text{net}}$$

Radiation exchange: black surfaces

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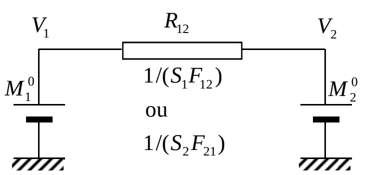
- **Thermal circuit**



$$\underbrace{q_{ij}}_{q} = \Phi_{ij,net} = \underbrace{S_i F_{ij}}_{R^{-1}} \underbrace{(M_i^0 - M_j^0)}_{(\theta_i - \theta_j)}$$

$$\Phi_{12,net} = S_1 F_{12} (M_1^0 - M_2^0) = S_2 F_{21} (M_1^0 - M_2^0)$$

(1/R)

S_1 → S_2

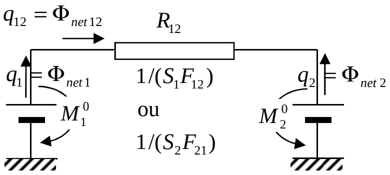
$$\Phi_{21,net} = S_2 F_{21} (M_2^0 - M_1^0) = S_1 F_{12} (M_2^0 - M_1^0)$$

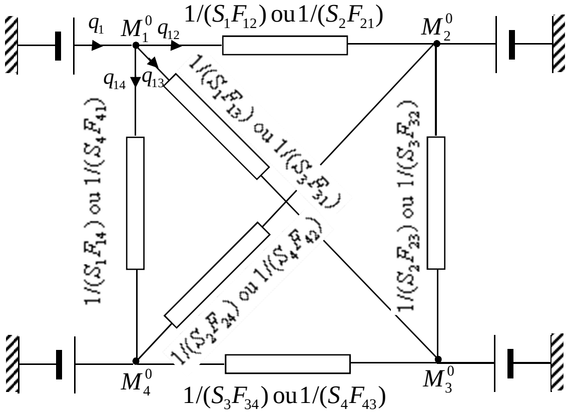
S_2 → S_1

Radiation exchange: black surfaces

- **Thermal circuit**

$$q_{ij} \equiv \underbrace{\Phi_{net\,ij}}_{q} = \underbrace{S_i F_{ij}}_{R^{-1}} \underbrace{(M_i^0 - M_j^0)}_{(\theta_i - \theta_j)}$$



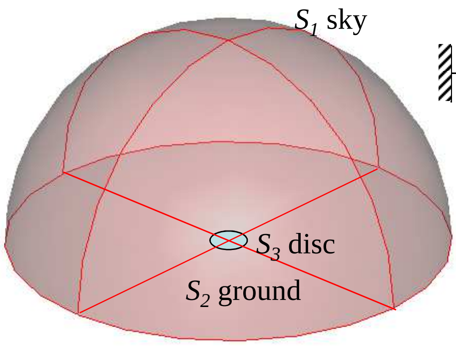


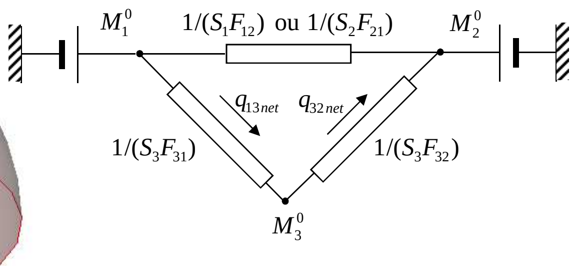
$$q_{12} \equiv \Phi_{net\,12} = S_1 F_{12}(M_1^0 - M_2^0)$$

$$q_1 \equiv \Phi_{net\,1} = \sum_{j=2}^{4} S_1 F_{1j}(M_1^0 - M_j^0) = \sum_{j=2}^{4} \Phi_{net\,1j} = \sum_{j=2}^{4} q_{1j}$$

Radiation exchange: black surfaces

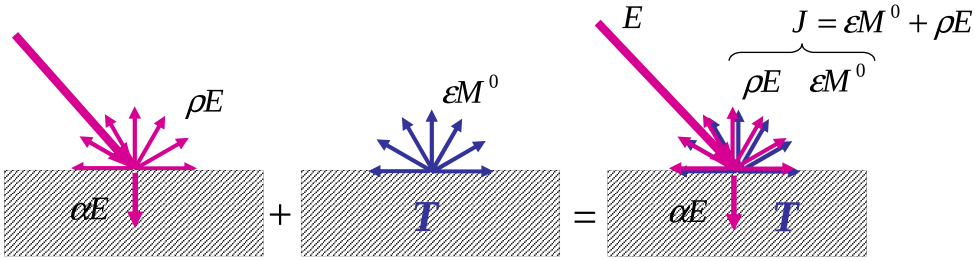
- **Thermal circuit**





Radiation
Radiation exchange: gray surfaces

- **Radiosité**



$$J = \varepsilon M^0 + \rho E$$

Opaque surface $\tau = 0$

$$\Rightarrow \rho = 1 - \alpha = 1 - \varepsilon$$

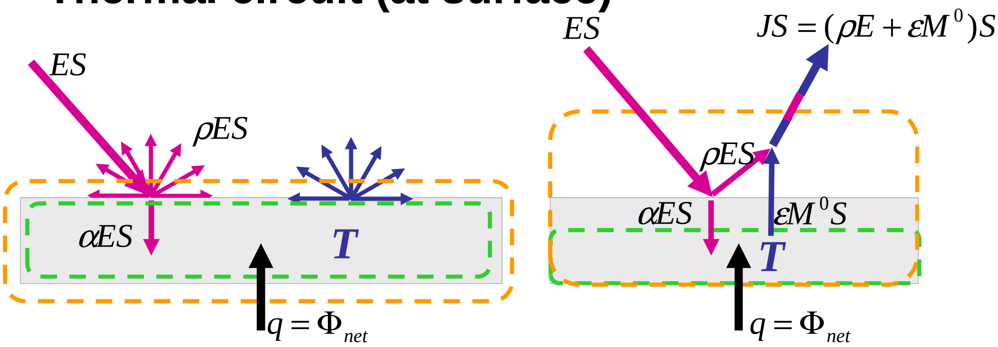
$$\Rightarrow J = \varepsilon M^0 + (1 - \varepsilon) E$$

equal if the same wavelength!

or gray body

Radiation
Radiation exchange: gray surfaces

- **Thermal circuit (at surface)**



$$\Phi_{net} = \varepsilon M^0 S - \alpha ES$$

$$\Phi_{net} = JS - ES$$

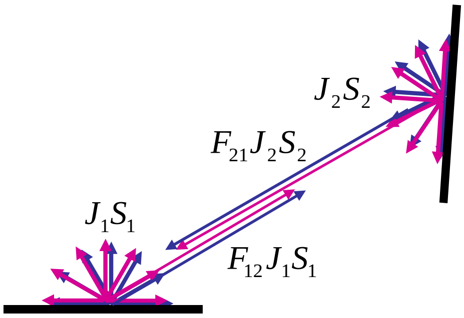
$$= (\rho E + \varepsilon M^0)S - ES$$

$$= \varepsilon M^0 S - (1 - \rho)ES$$

$$= \varepsilon M^0 S - \alpha ES$$

Radiation exchange: gray surfaces

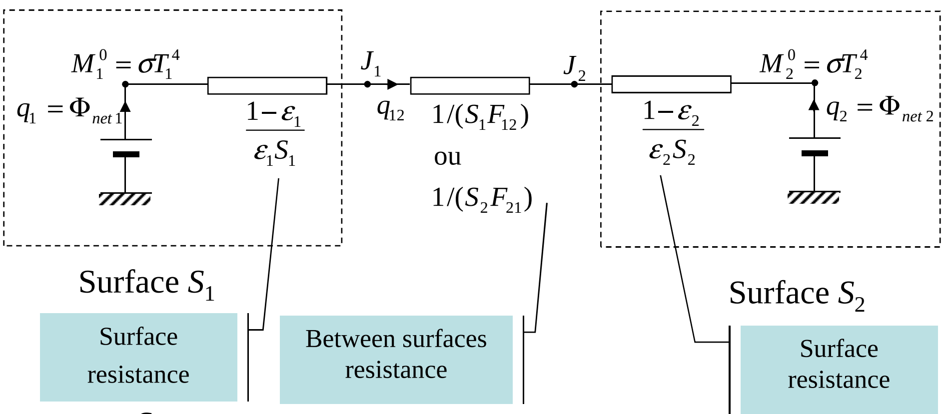
- **Thermal circuit (between surfaces)**



$$\underbrace{\Phi_{12,net}}_{\text{net}} = \underbrace{S_1F_{12}J_1}_{\text{émis}} - \underbrace{S_2F_{21}J_2}_{\text{reçu}} = S_1F_{12}(J_1 - J_2) = S_2F_{21}(J_1 - J_2)$$

Radiation exchange: gray surfaces

- **Circuit thermique**



$$q_1 \equiv \Phi_{1,net} = \frac{\varepsilon_1 S_1}{1-\varepsilon_1}(M_1^0 - J_1)$$

$$q_{12} \equiv \Phi_{12,net} = S_1F_{12}(J_1^0 - J_2^0) = S_2F_{21}(J_1^0 - J_2^0)$$

$$q_2 \equiv \Phi_{2,net} = \frac{\varepsilon_2 S_2}{1-\varepsilon_2}(M_2^0 - J_2)$$

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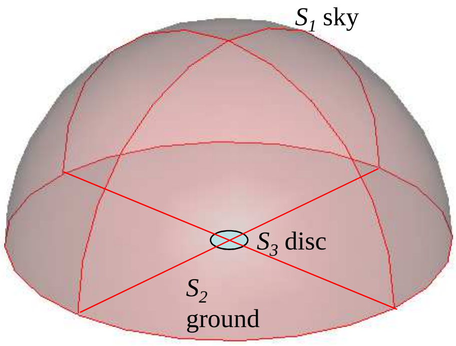
- **Thermal circuit**



$$\text{Energybalance}\ \Phi_{3,net} = \Phi_{3,émis} - \Phi_{3,abs} = 0$$

$$\Phi_{3,net} = \varepsilon_3 M_3^0 S_3 - \alpha_3 E_3 S_3 = 0$$

$$\Phi_{3,net} = \frac{\varepsilon}{1-\varepsilon}(M_3^0 - J_3) S_3 = 0$$

$$\Rightarrow M_3^0 = J_3$$