INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Building Energy Simulation

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Lecture 1: Introduction

Building Simulation

- Introduction
  - Concepts
  - Heat and temperature
- Thermal Analysis
  - Energy conservation
  - Constitutive laws
  - Framework
  - Dynamic models
- Conduction
- Convection
- Radiation
- Coupled Transfer

Curricula

2 x 4h Lectures
- Conduction
- Convection
- Radiation
- Coupled heat transfer

2 x 4h Tutorials and project
- Model your own SmartHome
- Simulate and discuss

1 x 2h Defend your project

1 x 2h Written exam

Prerequisites

Calculus
Linear algebra
Thermodynamics
Heat and mass trans

Introduction

Concepts

Science: models of nature with testable explanations and predictions

Causality: causal relations; the cause must precede the effect

Relations = physical laws

Conservation (operation: addition $x_1 + x_2 = x_3$)

- Energy-mass ⇔ time symmetry (invariance)
- Linear momentum ⇔ translation symmetry (invariance)
- Angular momentum ⇔ rotation symmetry (invariance)
- …

Universal laws (operation: multiplication $x = a\ y$)

- Universal attraction (Newton) $F_1 = F_2 = G\frac{m_1 m_2}{d^2}$
- Plank-Einstein relation $E = h\ \nu$
- Thermal energy $E_{thermal} = k\ T$
- …

Phenomenological laws

- Ohm's law $u = R\ i$
- Hooke's law $F = k\ \Delta l$

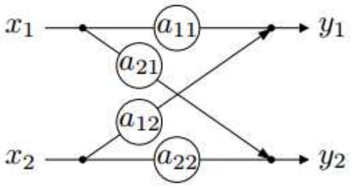
Introduction

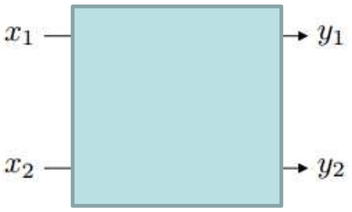
Concepts

Physical system: elements connected through conserved quantities

$$\begin{cases} a_{11}x_1 + a_{22}x_2 = y_1 \\ a_{21}x_1 + a_{22}x_2 = y_2 \end{cases}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

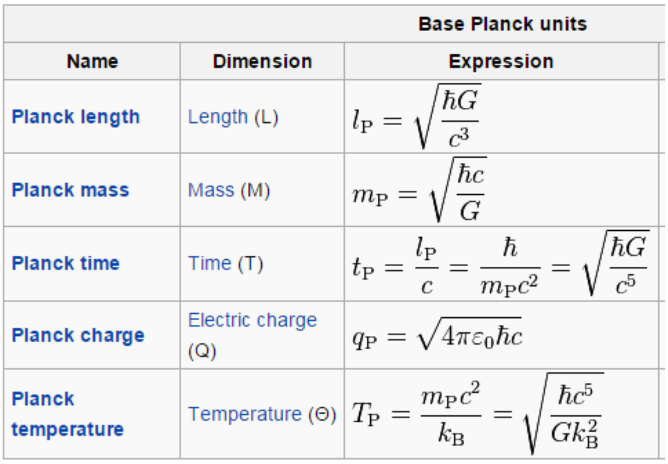




Introduction
Concepts

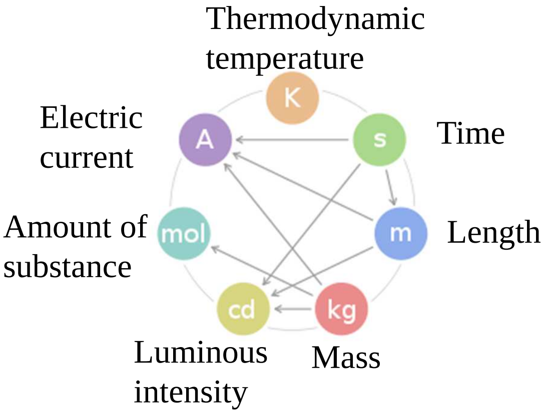
Relations between quantities → limited number of independent units

SI: system of measurable quantities + relations between quantities



Base Planck units

Name	Dimension	Expression
Planck length	Length (L)	$l_\text{P} = \sqrt{\frac{\hbar G}{c^3}}$
Planck mass	Mass (M)	$m_\text{P} = \sqrt{\frac{\hbar c}{G}}$
Planck time	Time (T)	$t_\text{P} = \frac{l_\text{P}}{c} = \frac{\hbar}{m_\text{P} c^2} = \sqrt{\frac{\hbar G}{c^5}}$
Planck charge	Electric charge (Q)	$q_\text{P} = \sqrt{4\pi\varepsilon_0 \hbar c}$
Planck temperature	Temperature (Θ)	$T_\text{P} = \frac{m_\text{P} c^2}{k_\text{B}} = \sqrt{\frac{\hbar c^5}{G k_\text{B}^2}}$



Introduction
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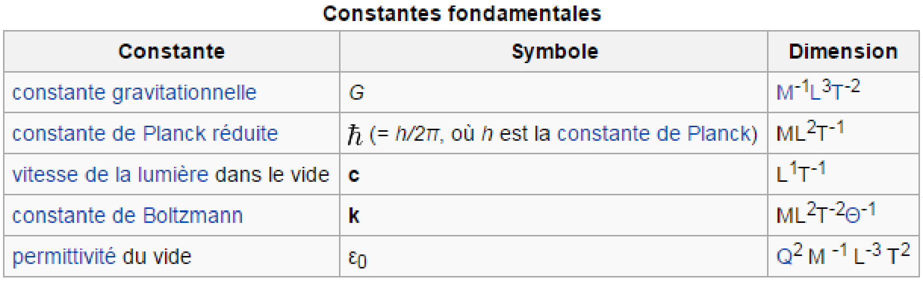


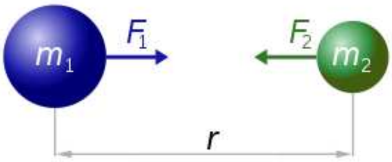
Constantes fondamentales

Constante	Symbole	Dimension
constante gravitationnelle	G	$M^{-1}L^{3}T^{-2}$
constante de Planck réduite	$\hbar$ (= $h/2\pi$, où h est la constante de Planck)	$ML^{2}T^{-1}$
vitesse de la lumière dans le vide	**c**	$L^{1}T^{-1}$
constante de Boltzmann	**k**	$ML^{2}T^{-2}\Theta^{-1}$
permittivité du vide	ε_0	$Q^{2} M^{-1} L^{-3} T^{2}$

m, kg, s

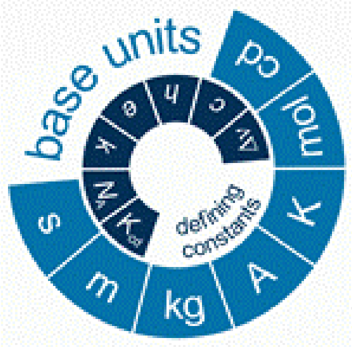
K

A



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

$$E = h\nu \qquad \textit{pour un photon}$$



$$E_{thermal} = kT$$

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Heat and temperature

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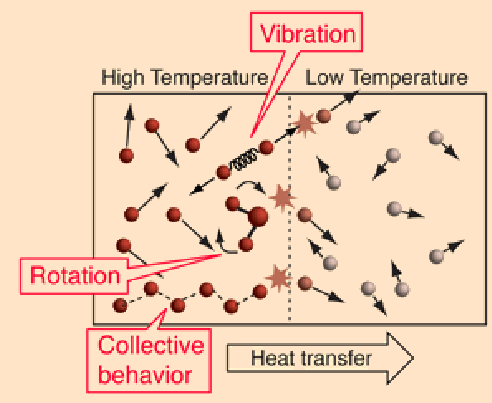
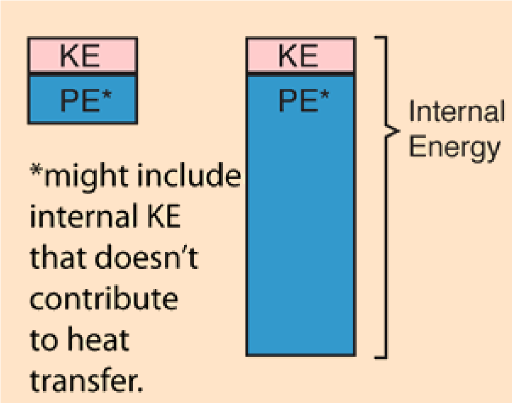
$$pV = nRT$$
$$pV = NkT$$
$$k = \frac{R}{N_A}$$

Boltzmann constant: relates thermal energy to temperature

$$E_{thermal} = kT$$

SI definition: 1 K the variation of temperature that changes the thermal energy by 1,380 648 8 $\times 10^{-23}$J





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Heat and temperature

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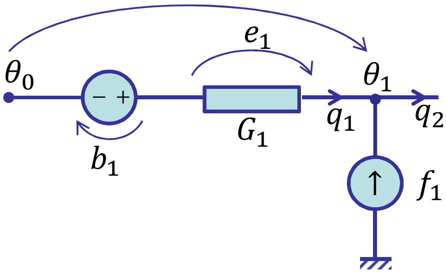
Heat transfer (or heat) is thermal energy in transit due to temperature difference

0th **principle :** temperature scales ($e_1 = \theta_0 - \theta_1 + b$)
1st **principle :** energy conservation ($q_1 - q_2 = -f$)
2nd **principle** and **constitutive laws:** direction / value of heat ($q_1 = G_1 e_1$)



Introduction

Heat and temperature

Quantity	Meaning	Symbol	Units
Thermal energy	Energy of matter at microscopic level	U	$\mathrm{J = kg\,m^2 s^{-2}}$
Temperature	Indirect measurement of stored thermal energy	$T\ or\ \theta$	$\mathrm{K}\ or\ \mathrm{°C}$
Heat transfer	Thermal energy transport due to temperature difference		
Heat	Amount of thermal energy transferred	Q	$\mathrm{J = kg\,m^2 s^{-2}}$
Heat rate	Heat transferred per unit time	$\dot{Q} \equiv q, \Phi$	$\mathrm{W = kg\,m^2 s^{-3}}$
Heat flux	Heat rate per unit surface area	$\varphi = \dfrac{dq}{dA}$	$\mathrm{W/m^2 = kg\,s^{-3}}$

Introduction

Heat and temperature

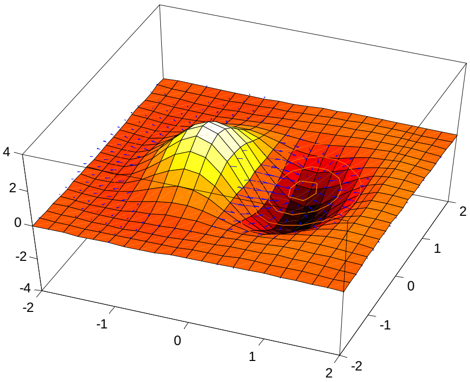
$$\begin{tikzcd} u \arrow[d, "\text{Differences}"'] & f \\ e \arrow[r, "\text{Constitutive law}"] & w \arrow[u, "\text{Balance}"'] \end{tikzcd}$$

u potential
e potential difference
w flow rate
f external flow rate

- **Heat transfer**
- **Mass transfer**
- **Momentum transfer**
- **Electrical conduction**

Transfer: irreversible statistical phenomena

Space inhomogeneity of an intensive quantity → transport of a physical quantity



Introduction
Heat and temperature

Modes of heat transfer

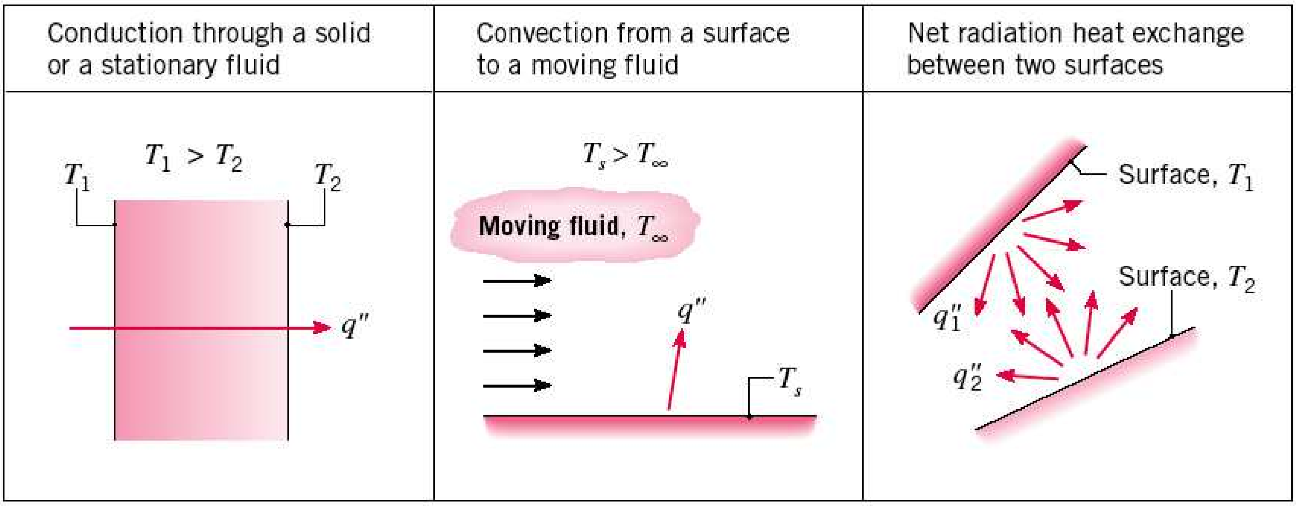
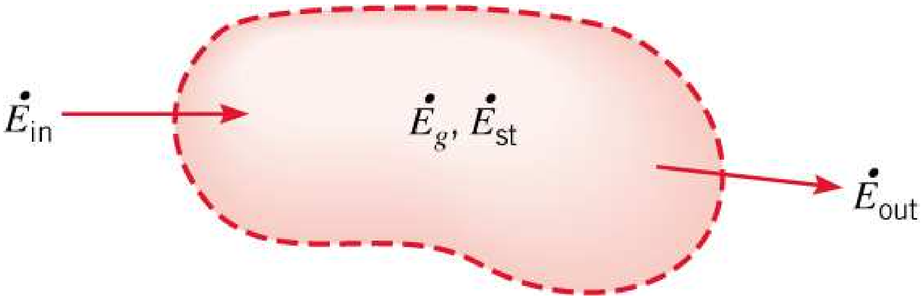


FIGURE 1.1 Conduction, convection, and radiation heat transfer modes.

Thermal analysis
Energy conservation

Energy conservation for a control volume



$$\Delta E_{st} = E_{in} - E_{out} + E_g \text{ [J]}$$

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \quad \text{[W]}$$

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Energy conservation

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- **Volume phenomena**
  - Sensible heat $$Q = mc(\theta_2 - \theta_1)$$
  - Latent heat $$Q = ml$$
  - Generated heat: thermal ⇔ other form (e.g. electrical, chemical, nuclear)

- **Surface phenomena**
  - Energy in
  - Energy out

Thermal analysis
Constitutive laws

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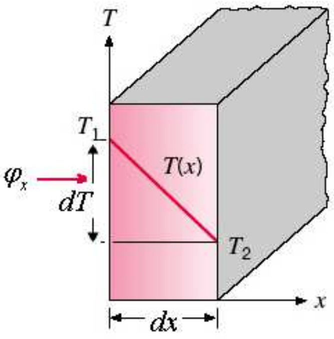
= relation between two physical quantities that is specific to a material

- **Conduction : Fourier law**

$$\varphi_n = -\lambda \frac{d\theta}{dx}$$

$$q = \frac{\lambda A}{\Delta x}(\theta_1 - \theta_2)$$



Valeurs typiques des propriétés des matériaux (Lefebvre, 1994)

Matériau	**Masse volumique, ρ** $[\mathrm{kg \cdot m^{-3}}]$	**Capacité thermique massique, c** $[\mathrm{J \cdot kg^{-1} \cdot K^{-1}}]$	**Conductivité thermique, λ** $[\mathrm{W \cdot m^{-1} \cdot K^{-1}}]$
Isolants	50 à 200	700	0.004
Bois	500	1250	0.2
Verre	1000	1000	1.2
Béton	1000 à 2000	1000	1.7
Pierre	2000	1000	2.0

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Constitutive laws

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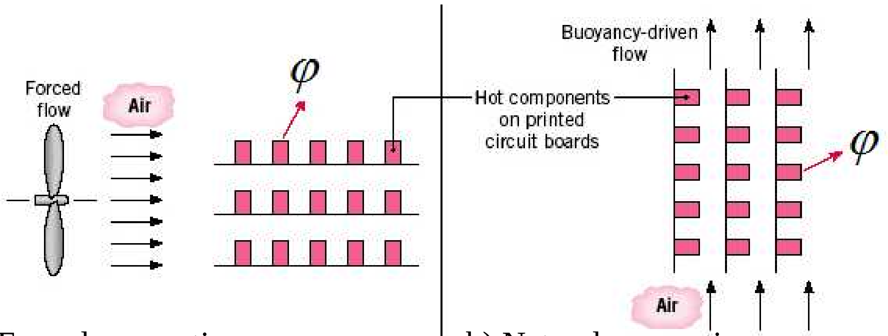
- **Convection :** Newton law

$$\varphi = h_c(\theta_S - \theta_\infty)$$

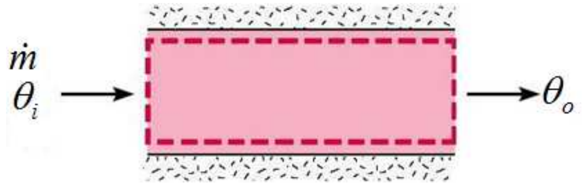
$$q = h_c A(\theta_S - \theta_\infty)$$



a) Forced convection b) Natural convection

- **Advection**

$$q = \dot{m}c_p(\theta_i - \theta_o)$$



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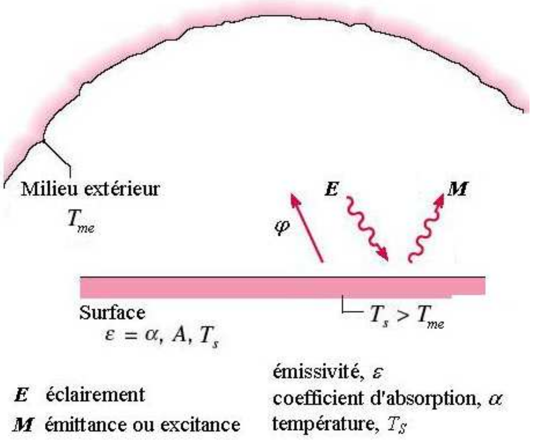
- **Typical values of convection heat coefficient**

Convection	**Heat coefficient** $[\mathrm{W/m^2K}]$
Natural	
Gas	2 -- 25
Liquid	50 -- 1000
Forced	
Gas	25 -- 250
Liquid	100 -- 20000

Thermal analysis
Constitutive laws

$$M^\circ = \sigma T_S^4$$



Valeurs typiques de l'émissivité

Matériau	**Emissivité, ε**
Aluminium	0.06
Zinc galvanisé	0.20 – 0.30
Bois	0.75 – 0.95
Brique ordinaire	0.93

Thermal analysis
Constitutive laws

- **Radiation**

$$\varphi_{abs} = \alpha E \qquad \varphi_{em} \equiv M = \varepsilon \sigma T_S^4$$

$$\alpha = \varepsilon$$

$$\varphi = \varphi_{em} - \varphi_{abs} = \varepsilon M^\circ - \alpha E$$

$$= \varepsilon \sigma (T_S^4 - \overline{T}_e^4)$$

$$q = h_r A(\theta_S - \overline{\theta}_e)$$

$$h_r \equiv \varepsilon \sigma (T_S + \overline{T}_e)(T_S^2 + \overline{T}_e^2)$$

Thermal analysis
Constitutive laws

- **Radiation**

$$T_S \cong \overline{T}_e$$

$$T_S^4\Big|_{T_S \cong \overline{T}_e} = \overline{T}_e^{\,4} + 4\overline{T}_e^{\,3}(T_S - \overline{T}_e)$$

$$T_S^4 - \overline{T}_e^{\,4} = 4\overline{T}_e^{\,3}(T_S - \overline{T}_e)$$

$$q = h_r A(\theta_S - \overline{\theta}_e) \qquad h_r = 4\varepsilon\sigma\overline{T}_e^{\,3}$$

Thermal analysis
Constitutive laws

- **Convection and radiation**

$$\varphi = \varphi_{cv} + \varphi_r$$

$$= h_c(T_S - T_\infty) + \varepsilon\sigma(T_S^4 - T_{me}^4)$$

$$T_S \cong T_{me} \cong T_S$$

$$q = h_t A(\theta_S - \overline{\theta}_e)$$

$$h_t = h_c + h_r$$

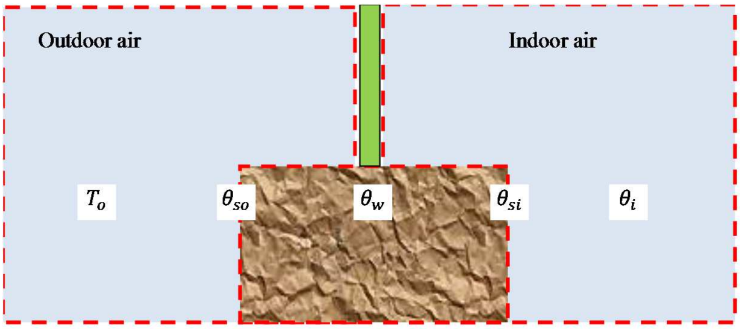
Thermal analysis
Constitutive laws

- **Conduction, convection, rayonnement**

Mode	**Mécanisme**	**Loi**	**Coefficient caractéristique**
Conduction	Diffusion	Fourier $\varphi_n = -\lambda \frac{dT}{dx}$	Conductivité thermique λ [W/m · K]
Convection	Diffusion et trnasport de masse	Newton $\varphi = h_c(T_S - T_\infty)$	Coefficient d'échange convectif h_c [W/m^2 · K]
Rayonnement	Ondes électromagnétiques	Dérivée de Stefan-Boltzmann $\varphi = \varepsilon\sigma(T_S^4 - T_{me}^4)$ $\varphi = h_r(T_S - T_{me})$	Emissivité $\varepsilon\,[-]$ Coefficient d'échange radiatif h_r [W/m^2 · K]
Advection	Transport de masse	$\dot{Q} = \dot{m}c_p(T_o - T_i)$	$\dot{m}c_p$

Thermal analysis
Framework for thermal analysis

1) On a schematic, represent the control surfaces. Consider the volumes that have the same temperature



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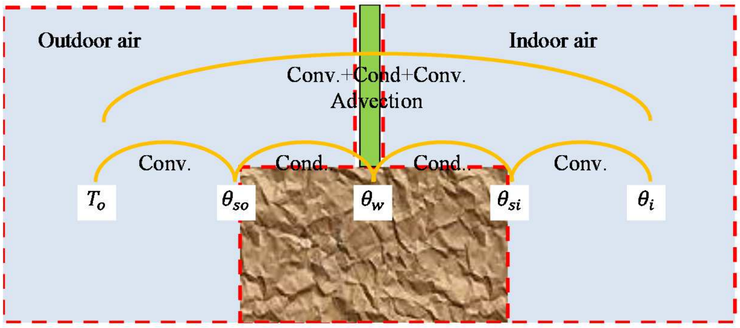
2) Identify the relevant heat transfers



Legend

Temperature source

T_o outdoor temperature.

Heat flow sources

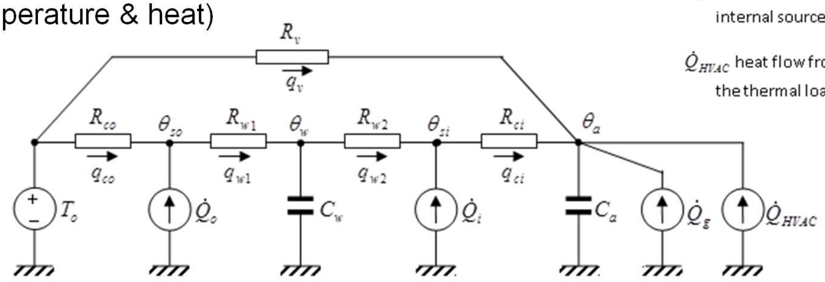
$\dot{Q}_o$ absorbed incident solar and long wave radiation on the outdoor surface;

$\dot{Q}_i$ short wave radiation from lights, transmitted solar, long wave radiation exchange with other surfaces, and long wave radiation from internal sources on the indoor surface;

$\dot{Q}_g$ heat flow gained by convection from internal sources;

$\dot{Q}_{HVAC}$ heat flow from the HVAC system, i.e. the thermal load.

3) Connect the temperatures by resistances along the heat transfer paths. Add storage and sources (temperature & heat)



(a)

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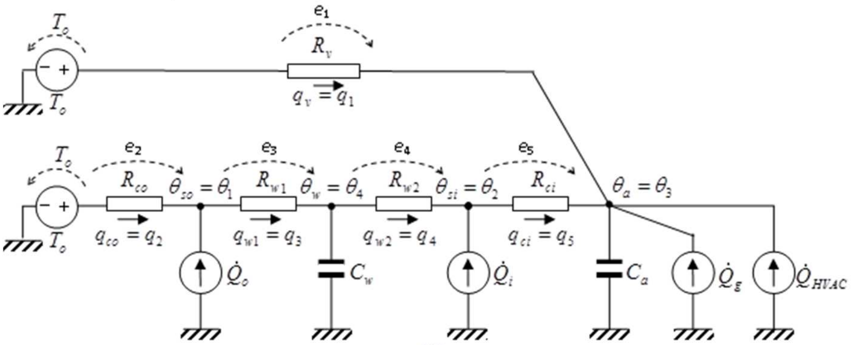
4) Solve the problem:

- temperature differences for each resistance



(b)

$$\begin{cases} e_1 = T_o - \theta_a \\ e_2 = T_o - \theta_{so} \\ e_3 = \theta_{so} - \theta_w \\ e_4 = \theta_w - \theta_{si} \\ e_5 = \theta_{si} - \theta_a \end{cases}$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = -\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{so} \\ \theta_{si} \\ \theta_a \\ \theta_w \end{bmatrix} + \begin{bmatrix} T_o \\ T_o \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{e} = [e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5]^T$$

$$\mathbf{b} = [T_o \quad T_o \quad 0 \quad 0 \quad 0]^T$$

$$\boldsymbol{\theta} = [\theta_{so} \quad \theta_{si} \quad \theta_a \quad \theta_w]^T$$

$$\mathbf{e} = -\mathbf{A}\boldsymbol{\theta} + \mathbf{b}$$

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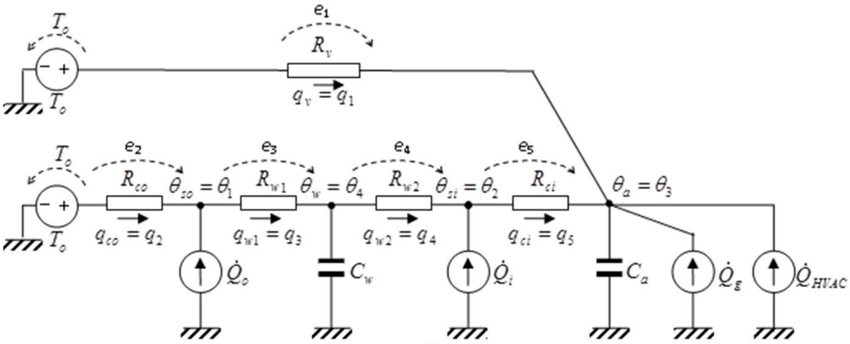
4) Solve the problem:

- heat rates for each branch



(b)

$$\begin{cases} q_1 \equiv q_v = R_v^{-1} e_1 \\ q_2 \equiv q_{co} = R_{co}^{-1} e_2 \\ q_3 \equiv q_{w1} = R_{w1}^{-1} e_3 \\ q_4 \equiv q_{w2} = R_{w2}^{-1} e_4 \\ q_5 \equiv q_{ci} = R_{ci}^{-1} e_2 \end{cases} \quad \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} R_v^{-1} & 0 & 0 & 0 & 0 \\ 0 & R_{co}^{-1} & 0 & 0 & 0 \\ 0 & 0 & R_{w1}^{-1} & 0 & 0 \\ 0 & 0 & 0 & R_{w2}^{-1} & 0 \\ 0 & 0 & 0 & 0 & R_{ci}^{-1} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix}$$

$$\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5]^T$$

$$\mathbf{G} = \begin{bmatrix} R_v^{-1} & 0 & 0 & 0 & 0 \\ 0 & R_{co}^{-1} & 0 & 0 & 0 \\ 0 & 0 & R_{w1}^{-1} & 0 & 0 \\ 0 & 0 & 0 & R_{w2}^{-1} & 0 \\ 0 & 0 & 0 & 0 & R_{ci}^{-1} \end{bmatrix}$$

$$\mathbf{q} = \mathbf{Ge}$$

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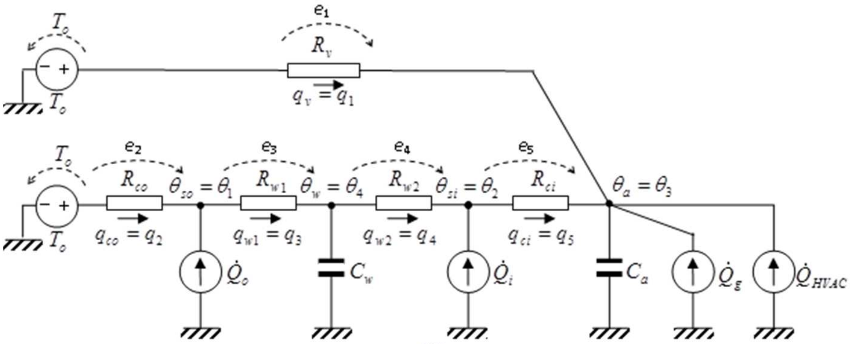
4) Solve the problem:

- energy balance for each node



(b)

$$\begin{cases} 0 = q_2 - q_3 + \dot{Q}_o \\ 0 = q_4 - q_5 + \dot{Q}_i \\ C_a \dot{\theta}_a = q_1 + q_5 + \dot{Q}_g + \dot{Q}_{HVAC} \\ C_w \dot{\theta}_w = q_3 - q_4 \end{cases} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C_a & 0 \\ 0 & 0 & 0 & C_w \end{bmatrix} \begin{bmatrix} \theta_{so} \\ \theta_{si} \\ \dot{\theta}_a \\ \dot{\theta}_w \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix} + \begin{bmatrix} \dot{Q}_o \\ \dot{Q}_i \\ \dot{Q}_g + \dot{Q}_{HVAC} \\ 0 \end{bmatrix}$$

$$\mathbf{C\dot{\theta}} = \mathbf{A}^T \mathbf{q} + \mathbf{f}$$

Thermal analysis
Framework for thermal analysis

4) Solve the problem:

$$\begin{cases} \mathbf{e} = -\mathbf{A}\boldsymbol{\theta} + \mathbf{b} \\ \mathbf{q} = \mathbf{G}\mathbf{e} \\ \mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{A}^T\mathbf{q} + \mathbf{f} \end{cases} \qquad \begin{cases} \mathbf{G}^{-1}\mathbf{q} + \mathbf{A}\boldsymbol{\theta} = \mathbf{b} \\ -\mathbf{A}^T\mathbf{q} + s\mathbf{C}\boldsymbol{\theta} = \mathbf{f} \end{cases} \qquad \begin{bmatrix} \mathbf{G}^{-1} & \mathbf{A} \\ -\mathbf{A}^T & s\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{f} \end{bmatrix}$$

- solution: system of algebraic differential equations

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f}$$

$$\begin{tikzcd}
\boldsymbol{\theta} \arrow[d, "-\mathbf{A}", "\text{Connexions}"'] & \mathbf{f} \\
\mathbf{-b+e} \arrow[r, "\text{Loi constitutive}", "\mathbf{G}"'] & \mathbf{q} \arrow[u, "\text{Bilans}", "-\mathbf{A}^T"']
\end{tikzcd}$$

$\boldsymbol{\theta}$	node temperatures
$\mathbf{e}$	temperature differences over resistances
$\mathbf{q}$	heat flux through resistances
$\mathbf{f}$	external fluxes

Thermal analysis
Framework for thermal analysis

4) Solve the problem

$$\begin{array}{c c} & \begin{array}{cccc} \theta_{so} & \theta_{si} & \theta_a & \theta_w \end{array} \\ \begin{array}{c} R_v \\ R_{co} \\ R_{w1} \\ R_{w2} \\ R_{ci} \end{array} & \mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \end{array} \qquad \mathbf{G} = \begin{bmatrix} R_v^{-1} & 0 & 0 & 0 & 0 \\ 0 & R_{co}^{-1} & 0 & 0 & 0 \\ 0 & 0 & R_{w1}^{-1} & 0 & 0 \\ 0 & 0 & 0 & R_{w2}^{-1} & 0 \\ 0 & 0 & 0 & 0 & R_{ci}^{-1} \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} T_o \\ T_o \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C_a & 0 \\ 0 & 0 & 0 & C_w \end{bmatrix}$$

$$\mathbf{f} = [\dot{Q}_o \quad \dot{Q}_i \quad \dot{Q}_g + \dot{Q}_{HVAC} \quad 0]^T$$

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f} \qquad \begin{bmatrix} \mathbf{G}^{-1} & \mathbf{A} \\ -\mathbf{A}^T & s\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{f} \end{bmatrix}$$

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algebraic differential equations to state-space representation

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f}$$

$$\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}_S\boldsymbol{\theta}_C + \mathbf{D}_S\mathbf{u}$$

$$\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{K}\boldsymbol{\theta} + \mathbf{K}_b\mathbf{b} + \mathbf{f}$$

$$\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_C\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}_0\\ \dot{\boldsymbol{\theta}}_C\end{bmatrix} = \begin{bmatrix}\mathbf{K}_{11} & \mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_0\\ \boldsymbol{\theta}_C\end{bmatrix} + \begin{bmatrix}\mathbf{K}_{b1}\\ \mathbf{K}_{b2}\end{bmatrix}\mathbf{b} + \begin{bmatrix}\mathbf{I}_{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_C\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}_0\\ \dot{\boldsymbol{\theta}}_C\end{bmatrix} = \begin{bmatrix}-\mathbf{K}_{21} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_0\\ \boldsymbol{\theta}_C\end{bmatrix} + \begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1}\\ \mathbf{K}_{b2}\end{bmatrix}\mathbf{b} + \begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{0} & \mathbf{C}_C\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}_0\\ \dot{\boldsymbol{\theta}}_C\end{bmatrix} = \begin{bmatrix}\mathbf{0} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_0\\ \boldsymbol{\theta}_C\end{bmatrix} + (-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2})\mathbf{b} + \begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}$$

$$\mathbf{C}_C\dot{\boldsymbol{\theta}}_C = (-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22})\boldsymbol{\theta}_C + (-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2})\mathbf{b} + \begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}$$

$$\dot{\boldsymbol{\theta}}_C = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22})\boldsymbol{\theta}_C + \mathbf{C}_C^{-1}\begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{b}\\ \mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}$$

$$\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u} \qquad \mathbf{A}_S = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22})$$

$$\mathbf{B}_S = \mathbf{C}_C^{-1}\begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{I}_{22}\end{bmatrix}$$

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From algebraic differential equations …

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f}$$

$$\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_C\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}_0\\ \dot{\boldsymbol{\theta}}_C\end{bmatrix} = \begin{bmatrix}\mathbf{K}_{11} & \mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_0\\ \boldsymbol{\theta}_C\end{bmatrix} + \begin{bmatrix}\mathbf{K}_{b1}\\ \mathbf{K}_{b2}\end{bmatrix}\mathbf{b} + \begin{bmatrix}\mathbf{I}_{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}$$

… to state-space representation

$$\begin{cases}\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u}\\ \boldsymbol{\theta}_0 = \mathbf{C}_S\boldsymbol{\theta}_C + \mathbf{D}_S\mathbf{u}\end{cases}$$

$$\mathbf{A}_S = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22})$$

$$\mathbf{B}_S = \mathbf{C}_C^{-1}\begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{I}_{22}\end{bmatrix}$$

$$\mathbf{C}_S = -\mathbf{K}_{11}^{-1}\mathbf{K}_{12}$$

$$\mathbf{D}_S = -\mathbf{K}_{11}^{-1}\begin{bmatrix}\mathbf{K}_{b1} & \mathbf{I}_{11} & \mathbf{0}\end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix}\mathbf{b} & \mathbf{f}_0 & \mathbf{f}_C\end{bmatrix}^T$$

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From state-space representation …

$$\begin{cases} \dot{\boldsymbol{\theta}}_C = \mathbf{A}_S \boldsymbol{\theta}_C + \mathbf{B}_S \mathbf{u} \\ \boldsymbol{\theta}_0 = \mathbf{C}_S \boldsymbol{\theta}_C + \mathbf{D}_S \mathbf{u} \end{cases}$$

… to transfer function

$$\theta_a = [\mathbf{C}_S (s\mathbf{I} - \mathbf{A}_S)^{-1} \mathbf{B}_S + \mathbf{D}_S]\mathbf{u} \qquad \theta_a = \mathbf{H}\mathbf{u}$$

$$\mathbf{H} = \mathbf{C}_S (s\mathbf{I} - \mathbf{A}_S)^{-1} \mathbf{B}_S + \mathbf{D}_S$$