Content

Sequential behaviors

Registers

n-bit Registers

Addressable memories
Why Sequential circuits?

A combinatorial circuit implements a function (in the mathematical sense of the term):

- For a given input value, it **always** produces the same output, whatever input values it received in the past.

**Question**
Sometimes, we’d like to keep some information within a circuit, when inputs change.
Running Example

Let’s design a circuit with input $w$, and output $z$.

- $z$ becomes true whenever $w$ was true for “long enough”
- $z$ becomes true whenever $w$ was true 4 “instants” in a row

$\Rightarrow$ need memory to “remember” occurrences of $w$. 

![Circuit Diagram]

$w$  

$4\text{inarow}$  

$z$
Sequential circuits

A sequential element keeps a state. So a sequential circuit is not a function:

- for a given input value, the output value depends on:
  - the current input value
  - the sequence of input values that have been received in the past

⇒ Representing a sequential circuit with a truth table is impossible
Representing sequences of values

Two forms:

▶ time-diagrams, called **chronograms**
  → Chronograms describe graphically a **specific** sequence of input/output values
  → A chronogram is only a **representative example** of the circuit’s behavior
  → Example: 4-in-a-row

▶ State machines (FSM):
  → Describe the full system’s behavior **formally**
  → Next lecture
Clocks

- 99% of existing processors are synchronous
- A synchronous sequential circuit receives a clock signal
Synchronous circuits

- 99% of existing processors are *synchronous*
- A synchronous sequential circuit receives a *clock signal*
- Signals can then be synchronized, either on rising or falling edges of the clock
- Clock ticks are supposed to be long enough to mask propagation delays.
Content

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D-Latch (Verrou, bascule asynchrone)

- Specifications:
  - When \( \text{Keep} = 0 \), copy IN to OUT
  - When \( \text{Keep} = 1 \), stay unchanged

- Can easily be implemented using a MUX (multiplexer)

! Care you really understand the behavior of this circuit!
We need more

A latch’s output changes *whenever the keep signal is up*...

This is not accurate: signal is up for while!
We need a memory element that:

- is driven by a clock
- changes on edges

This can be implemented using TWO D-Latches...
D-Flip-flop (bascule synchrone)

- The D-FlipFlop stores the “D” (Data) bit when its CLK input goes high.
- Beware: there are various types of Flip-Flop (T-FlipFlop, RS-FlipFlop, JK-FlipFlop) which behave differently.
We need more

A D-Flip-Flop stores the D input **On each rising edge of the clock signal**...

Hmm... memory content may changes at each time step! We need a memory element that:

- is driven by a clock
- changes on edges
- can change or not depending on a WRITE-ENABLE (or ENABLE or “EN”) signal

This can be implemented by adding a MUX to the Flip-Flop...
Write-enabled flip-flop
Reset command

With the same kind of idea, one can add a RESET command to the Flip-Flop.

WARNING: This circuit implements a **Synchronous** reset but many D-FlipFlop (including the one of the Digital simulator) are **Asynchronous**.
Reset command

Note that the exact behavior of the circuit depends on the relative positions of the

Here the Reset signal is effective whatever the state of the EN signal...
Sequential behaviors

Registers

n-bit Registers

Addressable memories
n-bits register (registre n-bits)

The simplest possible n-bits register is made of $n$ D-flip-flops in parallel (all flip-flops having the same clock signal):

The input is stored at the CLK rising hedge; the output is stable during the whole CLK cycle.

Between two CLK rising hedges, all variations of the input signal are ignored.
Write-Enable n-bits register

To memorize a data over several cycles, we add a signal $WE$ (Write Enable, sometimes simply called “EN”):

At the end of each cycle:

- If $WE=1$ → the input data is stored and made available on the output;
- if $WE=0$ → the output is kept constant.
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Sequential behaviors

Registers

n-bit Registers

Addressable memories
In a von Neumann machine, memories are used to store data and instructions.
Memories contain several words (actually a lot!) identified by their **address**.
One word can be read/written at a time depending on the **control** signal.
Memory in general

A: addresses on $k$ bits for a memory of $2^k$ words

DI: Data Input, on $m$ bits (generally a multiple of 8). The input will be written to word (aka memory cell) of address A is WE is true

DO: Data Output, always outputs the $m$ bits contained in the memory cell of address A.

WE: Write Enable
Size of the memory

\( k \) specifies the number of words \((2^k)\)

\( m \) is the size of the words (in bits).

- The capacity of the memory is \( m \times 2^k \) bits
- If \( m = l \times 8 \), the capacity of the memory is \( l \times 2^k \) bytes
Memory: possible implementation
Memory as seen from Software

A memory is a Vector of $2^k$ $m$ bits elements. With $k$ address bits we have:

\[
\begin{array}{c}
\text{address} \quad \leftarrow \quad m \text{ bits} \quad \rightarrow \\
0 \\
1 \\
2 \\
3 \\
4 \\
5 \\
\ldots \\
2^k - 1
\end{array}
\]

But in practice, such memories are difficult to implement...

- Memories are generally made of 8-bits cells
- Addresses correspond to 8-bits elements but several ($l = m/8$) elements are read simultaneously
With $k$ address bits and $l = 4$:

<table>
<thead>
<tr>
<th>address</th>
<th>offset 0</th>
<th>offset 1</th>
<th>offset 2</th>
<th>offset 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^k/4 - 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$4$
Ideally one would desire an indefinitely large memory capacity such that any particular [...] word would be immediately available. [...] We are [...] forced to recognize the possibility of constructing a hierarchy of memories, each of which has greater capacity than the preceding but which is less quickly accessible.

Preliminary Discussion of the Logical Design of an Electronic Computing Instrument, 1946
Capacity/Latency trade-off

- a fast memory is small
- a big memory is slow

This is due to the laws of physics and cannot be overcomed!
Memory Hierarchy
Memory Hierarchy

<table>
<thead>
<tr>
<th>Memory technology</th>
<th>Typical access time</th>
<th>$ per GiB in 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRAM semiconductor memory</td>
<td>0.5–2.5 ns</td>
<td>$500–$1000</td>
</tr>
<tr>
<td>DRAM semiconductor memory</td>
<td>50–70 ns</td>
<td>$10–$20</td>
</tr>
<tr>
<td>Flash semiconductor memory</td>
<td>5,000–50,000 ns</td>
<td>$0.75–$1.00</td>
</tr>
<tr>
<td>Magnetic disk</td>
<td>5,000,000–20,000,000 ns</td>
<td>$0.05–$0.10</td>
</tr>
</tbody>
</table>

¹Numbers as of 2012
Latencies — order of magnitude

<table>
<thead>
<tr>
<th>Operation</th>
<th>Latency</th>
<th>Factor</th>
<th>Time Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executing one instruction</td>
<td>0.5 ns</td>
<td>×100 ~ ×200</td>
<td>Two per second (heartbeat)</td>
</tr>
<tr>
<td>Accessing DRAM</td>
<td>50 ns</td>
<td>×1K ~ ×3K</td>
<td>~ one minute</td>
</tr>
<tr>
<td>SSD I/O</td>
<td>50 μs</td>
<td>×100K ~ ×1M</td>
<td>~ half a day</td>
</tr>
<tr>
<td>HDD I/O</td>
<td>5 ms</td>
<td>×1 billion (10^9)</td>
<td>~ a few months</td>
</tr>
</tbody>
</table>

⇒ human scale
Memory hierarchies in practice

In “real computers”, a typical memory hierarchy is generally composed of 5 different levels:

1. Registers
2. Cache memories (managed by the CPU)
   - L1
     - L1-D
     - L1-Ins
   - L2
   - L3
3. Main memory (RAM)
4. Storage devices (ROM, Hard Drives, Removable Drives...)
   → Virtual memory (managed by the Operating System)
5. Network/Internet storage