

Combinatorial Circuits

Lecturer: Guillaume Beslon
(Lecture adapted from Lionel Morel)

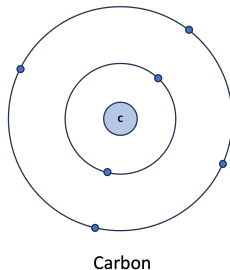
Computer Science and Information Technologies - INSA Lyon

Fall 2026

- ▶ Until now we have seen how to compute logical functions using Boole algebra
- ▶ Now, we will show how to implement these logical functions into digital circuits
- ▶ Let's start from “simple” electric wires...

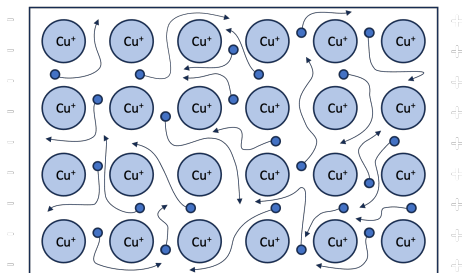
From conductors to semiconductors

- ▶ Wires are made of atoms...
- ▶ An atom consists of a nucleus of protons (+) and generally neutrons, and electrons (-).
- ▶ Electrons are distributed among the “electron shells”
- ▶ The number of electrons on the last shell (valence electrons) determines the electronic and chemical properties of the atom



From conductors to semiconductors

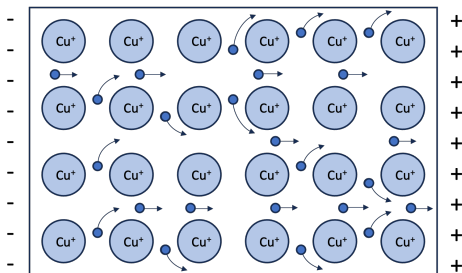
- ▶ A **conductor** is a material with few, loosely bound, valence electrons, allowing charges to flow freely (typically metals have a single valence electron $\rightarrow$ “sea of electrons”).
- ▶ A bit can be represented by a conductor at a certain voltage relative to ground (0V):
 - ▶ Near 0V $\rightarrow$ logical 0
 - ▶ Near V_{cc} $\rightarrow$ logical 1
- ▶ Intermediate voltages are invalid, and transitions between 0 and 1 must be fast ($\approx 1 \mu s$)
- ▶ Such signals are called logic/digital signals ($\neq$ analog)



0V $\rightarrow$ Logical 0

From conductors to semiconductors

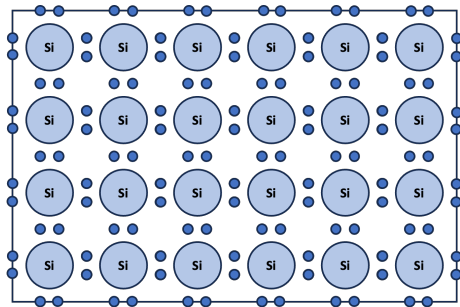
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$V_{cc} \rightarrow$ Logical 1

From conductors to semiconductors

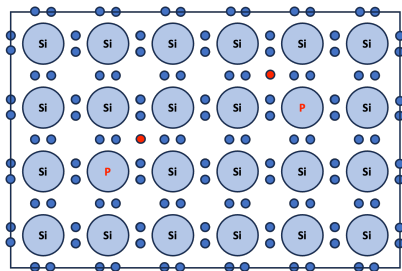
- ▶ An **insulator** is a substance or device which does not readily conduct electricity because its valence shell is full (8 valence electrons).
- ▶ A **semiconductor** is a material with electrical conductivity between that of a conductor and an insulator.
- ▶ With 4 valence electrons, **Silicon** is a semi-conductor
- ▶ But silicon is classically an insulator at room temperature



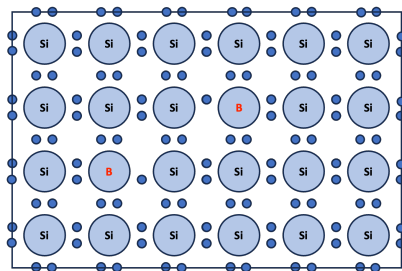
From conductors to semiconductors

- ▶ The electrical properties of silicon can be finely tuned through “**doping**”
- ▶ A small amount of *antimony*, *phosphorus*, or *arsenic* adds free electrons and increases the conductivity (→ type *n* semiconductor)
- ▶ A small amount of *boron*, *gallium* or *indium* adds “holes” and increases the conductivity (→ type *p* semiconductor)

Type *n* semiconductor
(Phosphorus doping)

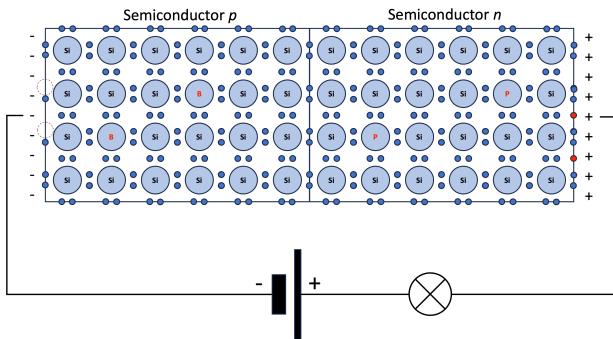


Type *p* semiconductor
(Boron doping)



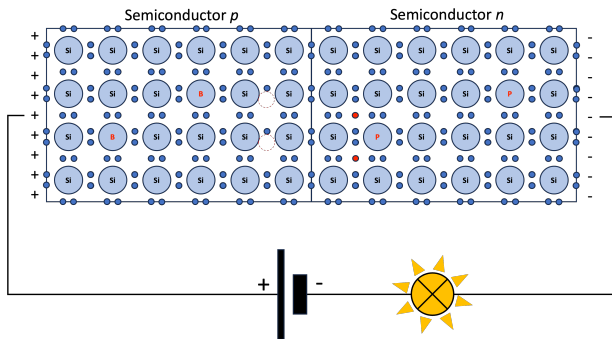
From semiconductors to electronic circuits

- ▶ Mixing n - and p -semiconductors allows creating electric circuits with complex properties: **diodes** and **transistors**
- ▶ Diodes are electronic components made of a type p semiconductor followed by a type n semiconductor. This allows the electric current to pass only in one direction



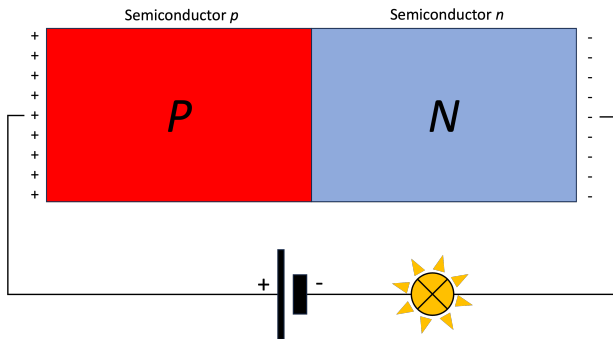
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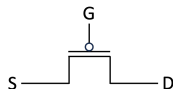
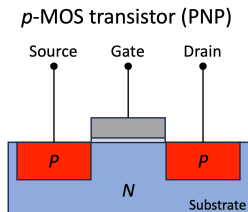
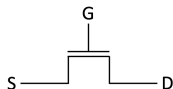
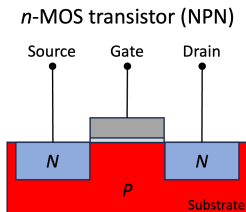
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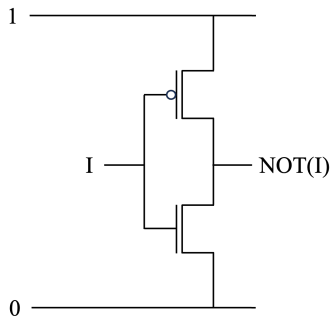
From semiconductors to transistors

- ▶ Transistors are electronic components made of three semiconductors.
- ▶ They allow controlling the connectivity between the Source (S) and the Drain (D) by applying (NPN) or not (PNP) a current on the Gate (G)
- ▶ Modern technologies (MOS – Metal Oxide Semiconductor) allow building components of less than 5 nm (Gate size) → more than 10 billions of transistors on a single chip...



From transistors to logical operations

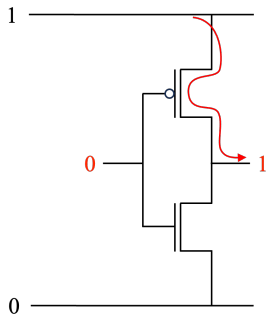
- Combining several PNP and NPN transistors, we can build electronic elements implementing Boolean operations



I	NOT(I)
0	1
1	0

From transistors to logical operations

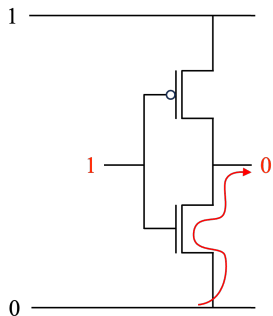
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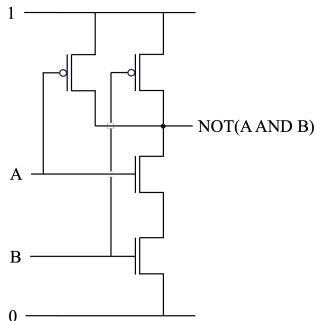
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From transistors to logical operations

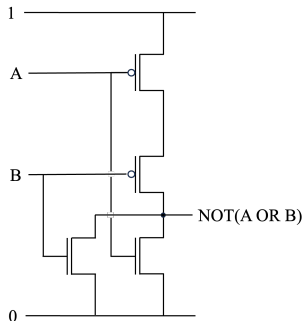
- Combining several PNP and NPN transistors, we can build electronic elements implementing Boolean operations



A	B	NOT (A AND B)
0	0	1
0	1	1
1	0	1
1	1	0

From transistors to logical operations

- ▶ Combining several PNP and NPN transistors, we can build electronic elements implementing Boolean operations



A	B	NOT (A OR B)
0	0	1
0	1	0
1	0	0
1	1	0

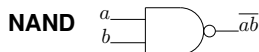
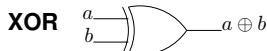
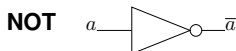
Why is it important?

Semiconductor technology is at the heart of the power of modern computers:

- ▶ Semiconductor technology has progressed continuously since the 60's.
- ▶ Fabrication technologies also improved, making it possible to integrate large numbers of transistors into a single chip.
- ▶ The level of integration has reached the nanometer scale: modern transistors are only a few nanometers in their critical dimensions.
 - ▶ Transistor size: ~ 30 nm
 - ▶ Switching time: ~ 10 -30 ps ($1\text{ps} = 10^{-12}\text{s}$)
 - ▶ Number of transistors in a modern chip: $\sim 10^{10}$ – 10^{11} Tx
- ▶ This is often summarized by **Moore's law**.
 - ▶ Moore's law is an empirical observation stating that the number of transistors on an integrated circuit tends to **double approximately every two years**.
 - ▶ It is not a physical law, but a (performative) long-term trend in semiconductor technology.

From logical operations to computation circuits

- ▶ Knowing how logical operations can be implemented, we can forget about electronics and consider **logical gates** as the basic building blocks of computation circuits:



- ▶ We can connect gates with **wires** to transmit the result of one logical operation to the input of another.
- ▶ By combining gates, we can build increasingly complex circuits, just as we combine operations in a mathematical expression.
- ▶ This leads to the idea of encapsulation: we can treat a circuit as a **black box**, hiding **how** it works internally and focusing only on **what** it computes.

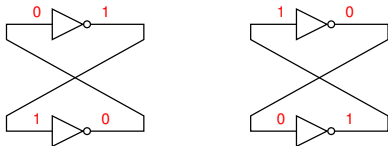
Assembly rules - combinational circuits

Combinatorial Logical Circuits (CLC) can be defined recursively:

- ▶ as a gate
- ▶ as a wire
- ▶ as a side-by-side juxtaposition of 2 CLCs
- ▶ by connecting the outputs of a CLC to inputs of **another** CLC
- ▶ by connecting inputs of a CLC together.

This definition forbids:

- ▶ to make cycles, because they introduce undefined behaviors, eg



- ▶ to connect outputs with each other (what if an output is 1 and the other is 0?)

Blackboard Example

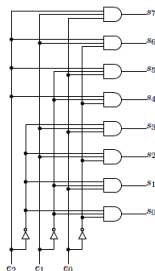
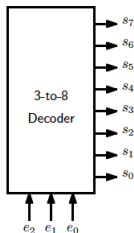
Decoder

A decoder n to 2^n is a circuit with:

- ▶ n inputs e_i , encoding an integer $(e_{n-1} \dots e_0)_2$;
- ▶ 2^n outputs s_i , indexed from 0 à $2^n - 1$.

The only active output line is $s_{(e_{n-1} \dots e_0)_2}$.

E.g., a 3-to-8 decoder



Blackboard example: Building a 2-to-4 decoder from its truth table.

Multiplexer

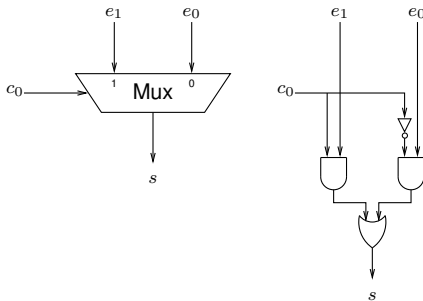
A 2^n to 1 multiplexer is a circuit with:

- ▶ 2^n inputs e_i indexed from 0 to $2^n - 1$;
- ▶ n selection lines, encoding the integer $(c_{n-1} \dots c_0)_2$;
- ▶ 1 output s .

When selection lines for the value $(c_{n-1} \dots c_0)_2$,

$$s = e_{(c_{n-1} \dots c_0)_2}.$$

E.g., a 2^1 to 1 multiplexer



Blackboard example: Building the 2 to 1 multiplexer from its truth table.

Blackboard example: 1 to 2 demultiplexer

(Another) Multiplexer

An k -bits 2^n -to-1 multiplexer is a circuit with:

- ▶ $k \cdot 2^n$ inputs and n selection lines;
- ▶ k output signals

It selects k signals among $k \cdot 2^n$ input signals

Ex : 8-bits 2-to-1 multiplexer:

