### Combinatorial Circuits

Lecturer: Guillaume Beslon (Lecture adapted from Lionel Morel)

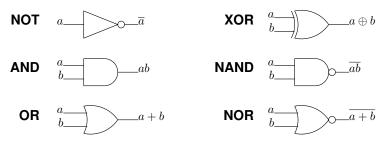
Computer Science and Information Technologies - INSA Lyon

Fall 2024

- Until now we have seen how to compute logical functions using Boole algebra
- Now, we will show how to implement these logical functions into digital circuits

## Logical gates and circuits

**Logical gates** are the basic building blocks of digital circuits :



A **logical signal** is a physical mean of transmitting a truth value from one place to another. We represent them as wires.

From the outside, a **logical circuit** shows input and output signals: **every output signal is a function of the input signals (possibly a subset of)**.

## Assembly rules - combinational circuits

Combinatorial Logical Circuits (CLC) can be defined recursively:

- as a gate
- as a wire
- as a side-by-side juxtaposition of 2 CLCs
- by connecting the outputs of a CLC to inputs of another CLC
- by connecting inputs of a CLC together.

#### This definition forbids:

 to make cycles, because they introduce undefined behaviors, eg





to connect outputs with each other (what if an output is 1 and the other is 0?)

# Blackboard Example

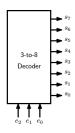
#### Decoder

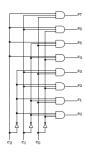
A decoder n to  $2^n$  is a circuit with:

- ▶ *n* inputs  $e_i$ , encoding an integer  $(e_{n-1} \dots e_0)_2$ ;
- ▶  $2^n$  outputs  $s_i$ , indexed from 0 à  $2^n 1$ .

The only active output line is  $s_{(e_{n-1}...e_0)_2}$ .

E.g., a 3-to-8 decoder





Blackboard example: Building a 2-to-4 decoder from its truth table.

### Multiplexer

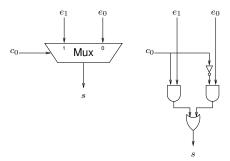
A 2<sup>n</sup> to 1 multiplexer is a circuit with:

- ▶  $2^n$  inputs  $e_i$  indexed from 0 to  $2^n 1$ ;
- ▶ *n* selection lines, encoding the integer  $(c_{n-1} \dots c_0)_2$ ;
- ▶ 1 output s.

When selection lines for the value  $(c_{n-1} \dots c_0)_2$ ,

$$s = e_{(c_{n-1}...c_0)_2}.$$

**E.g.**, a 2<sup>1</sup> to 1 multiplexer



Blackboard example: Building the 2 to 1 multiplexer from its truth table.

## Blackboard example: 1 to 2 demultiplexer

## (Another) Multiplexer

An k-bits  $2^n$ -to-1 multiplexer is a circuit with:

- ▶  $k \cdot 2^n$  inputs and n selection lines;
- k output signals

It selects k signals among  $k \cdot 2^n$  input signals

Ex: 8-bits 2-to-1 multiplexer:

