Combinatorial Circuits

Lecturer: Guillaume Beslon (Lecture adapted from Lionel Morel)

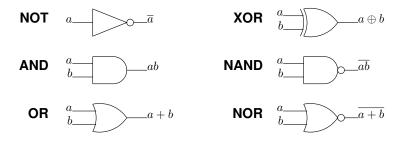
Computer Science and Information Technologies - INSA Lyon

Fall 2023

- Until now we have seen how to compute logical functions using Boole algebra
- Now, we will show how to implement these logical functions into digital circuits

Logical gates and circuits

Logical gates are the basic building blocks of digital circuits :



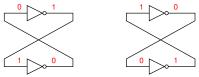
A **logical signal** is a physical mean of transmitting a truth value from one place to another. We represent them as wires.

From the outside, a **logical circuit** shows input and output signals: **every output signal is a function of the input signals (possibly a subset of)**.

Assembly rules - combinational circuits

Combinatorial Logical Circuits (CLC) can be defined recursively:

- as a gate
- as a wire
- as a side-by-side juxtaposition of 2 CLCs
- by connecting the outputs of a CLC to inputs of another CLC
- by connecting inputs of a CLC together.
- This definition forbids:
 - to make cycles, because they introduce undefined behaviors, eg



to connect outputs with each other (what if an output is 1 and the other is 0?)

Blackboard Example

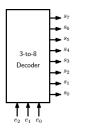
Decoder

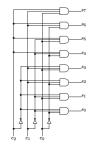
A decoder n to 2^n is a circuit with:

- *n* inputs e_i , encoding an integer $(e_{n-1} \dots e_0)_2$;
- ▶ 2^n outputs s_i , indexed from 0 à $2^n 1$.

The only active output line is $s_{(e_{n-1}...e_0)_2}$.

E.g., a 3-to-8 decoder





Blackboard example: Building a 2-to-4 decoder from its truth table.

Multiplexer

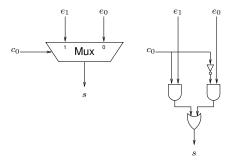
A 2^n to 1 *multiplexer* is a circuit with:

- 2^n inputs e_i indexed from 0 to $2^n 1$;
- *n* selection lines, encoding the integer $(c_{n-1} \dots c_0)_2$;
- 1 output s.

When selection lines for the value $(c_{n-1} \dots c_0)_2$,

$$s=e_{(c_{n-1}\ldots c_0)_2}.$$

E.g., a 2¹ to 1 multiplexer



Blackboard example: Building the 2 to 1 multiplexer from its truth table.

Blackboard example: 1 to 2 demultiplexer

(Another) Multiplexer

An *k-bits* 2^{*n*}-*to*-1 *multiplexer* is a circuit with:

- $k \cdot 2^n$ inputs and *n* selection lines;
- k output signals

It selects k signals among $k \cdot 2^n$ input signals

Ex: 8-bits 2-to-1 multiplexer:

