Elasto Hydrodynamic Lubrication

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Outline

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- **Introduction**
- Reduced Geometry & Non-Conforming

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- EHL Characteristics

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- Moes Venner

Introduction

Three questions for successful operation and optimisation

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Three questions for successful operation and optimisation

- **Will it work?**
  - *Avoid instantaneous failure.*
- **How much does it require to work?**
  - *Avoid waste of energy/material/money.*
- **Will it last?**
  - *Avoid premature/dangerous failure.*

Examples

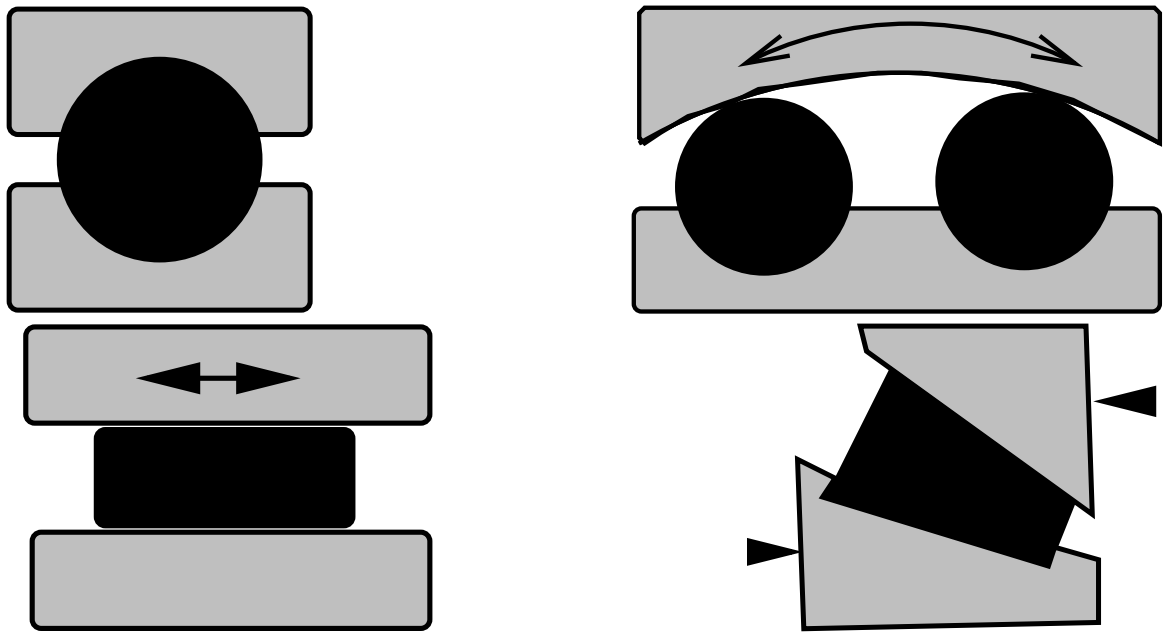


Figure *Sketch of a DGBB, SABB, CRB and TRB cross section.*

Will it Work

compute contact pressure (p_h)

as the contact area is small ($< R$)
the geometry can be approximated
by a parabola

Reduced Geometry

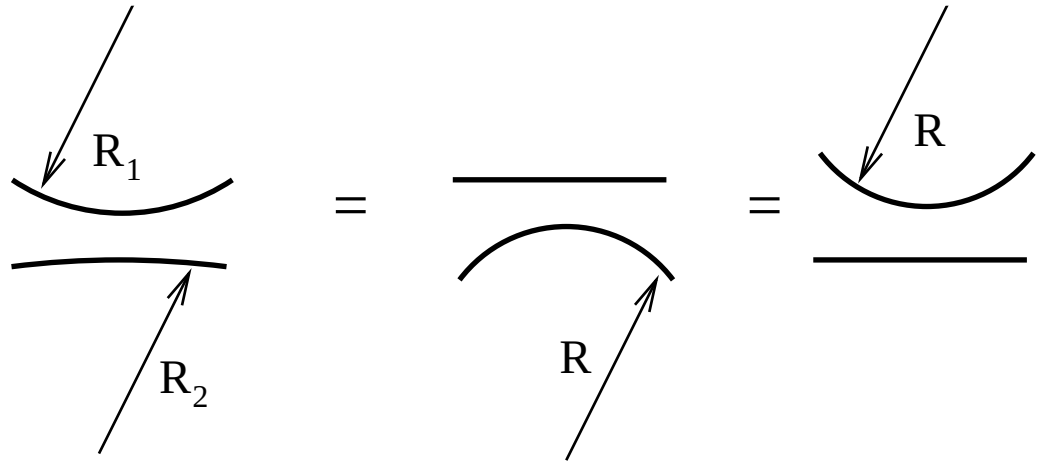


Figure *Equivalent geometries* $1/R = 1/R_1 + 1/R_2$.

$$H(X) = H_0 + \frac{X^2}{2R} + \cdots \quad \text{(parabolic approximation)}$$

examples: contact types in a car engine

conforming - non-conforming

Non Conforming Contacts

load w is important
contact area b small with respect to R
pressure p_h very high
ratio deformation to film δ/h very important

compute h, p_h, b and δ

EHL Characteristics

- important piezo-viscous effects

$\eta(p_h) >> \eta_0$

- important elastic effects

$\delta >> h_c$

- contact close to Hertzian conditions

$p(x,y) \simeq p_h \sqrt{1-(x/a)^2-(y/a)^2}$, $h(x,y) \simeq h_c$

EHL Trends

- increased load: decreased film, pressure more Hertzian

w $\nearrow$: $h \searrow$, $p \to p_h$

- decreased speed: decreased film, pressure more Hertzian

$u \searrow$: $h \searrow$, $p \to p_h$

- both effects make the contact more Hertzian

(character of equations becomes more Hertzian)

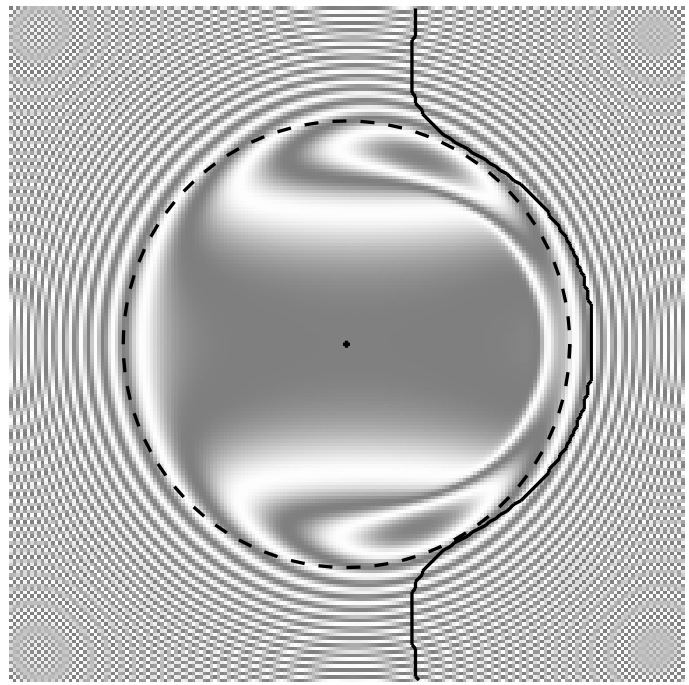
Equations

Reynolds equation:

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)}_{poiseuille}-\underbrace{\frac{\partial(u_m\rho h)}{\partial x}}_{couette}-\underbrace{\frac{\partial(\rho h)}{\partial t}}_{transient}=0$$

Cavitation

Cavitation Condition: $p \geq 0$.



Elastic Deformation

$$h(x,y) = h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + \frac{2}{\pi E'} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{p(x',y')\,dx'\,dy'}{\sqrt{(x-x')^2 + (y-y')^2}}$$

Density Pressure relation

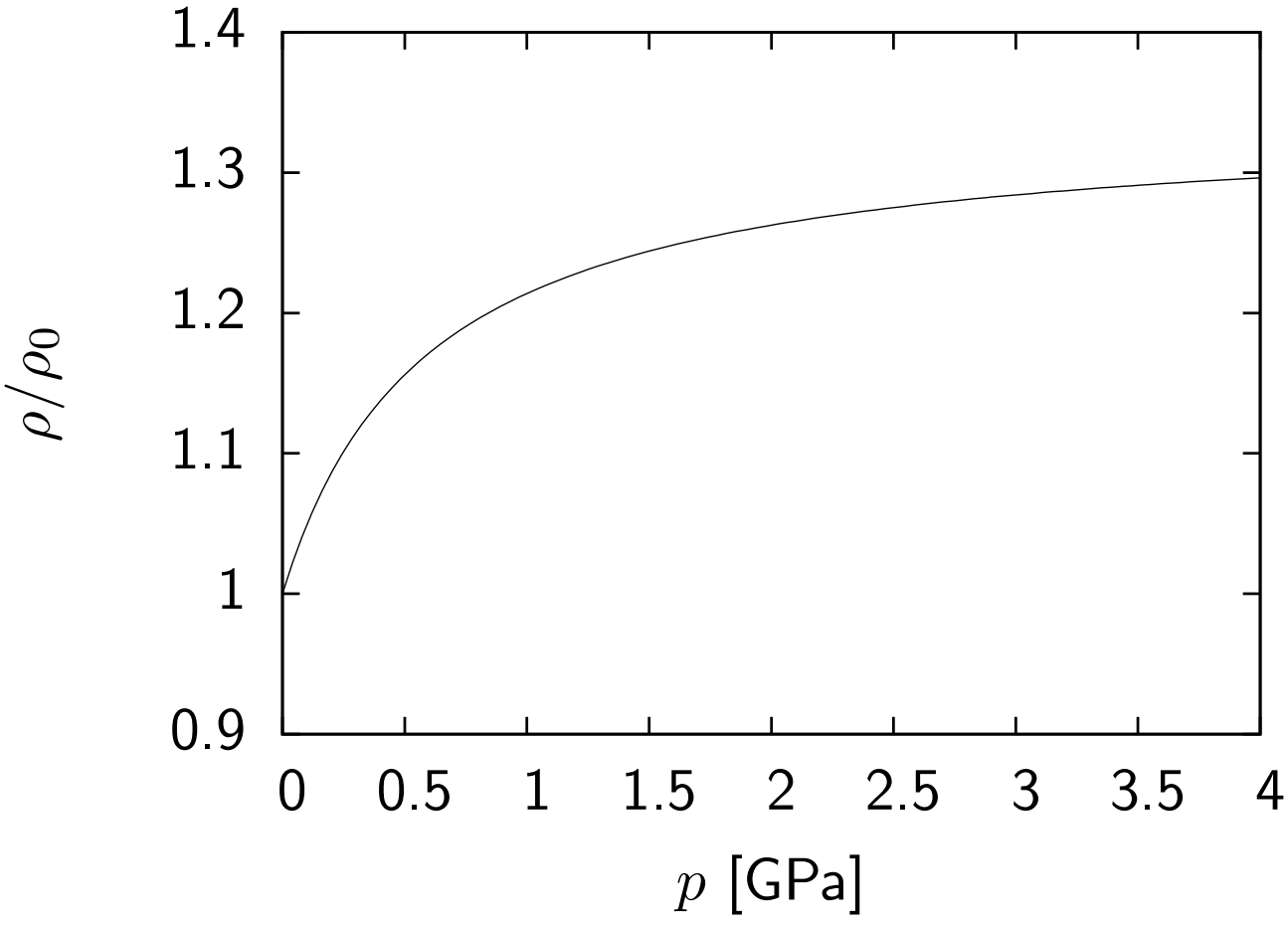


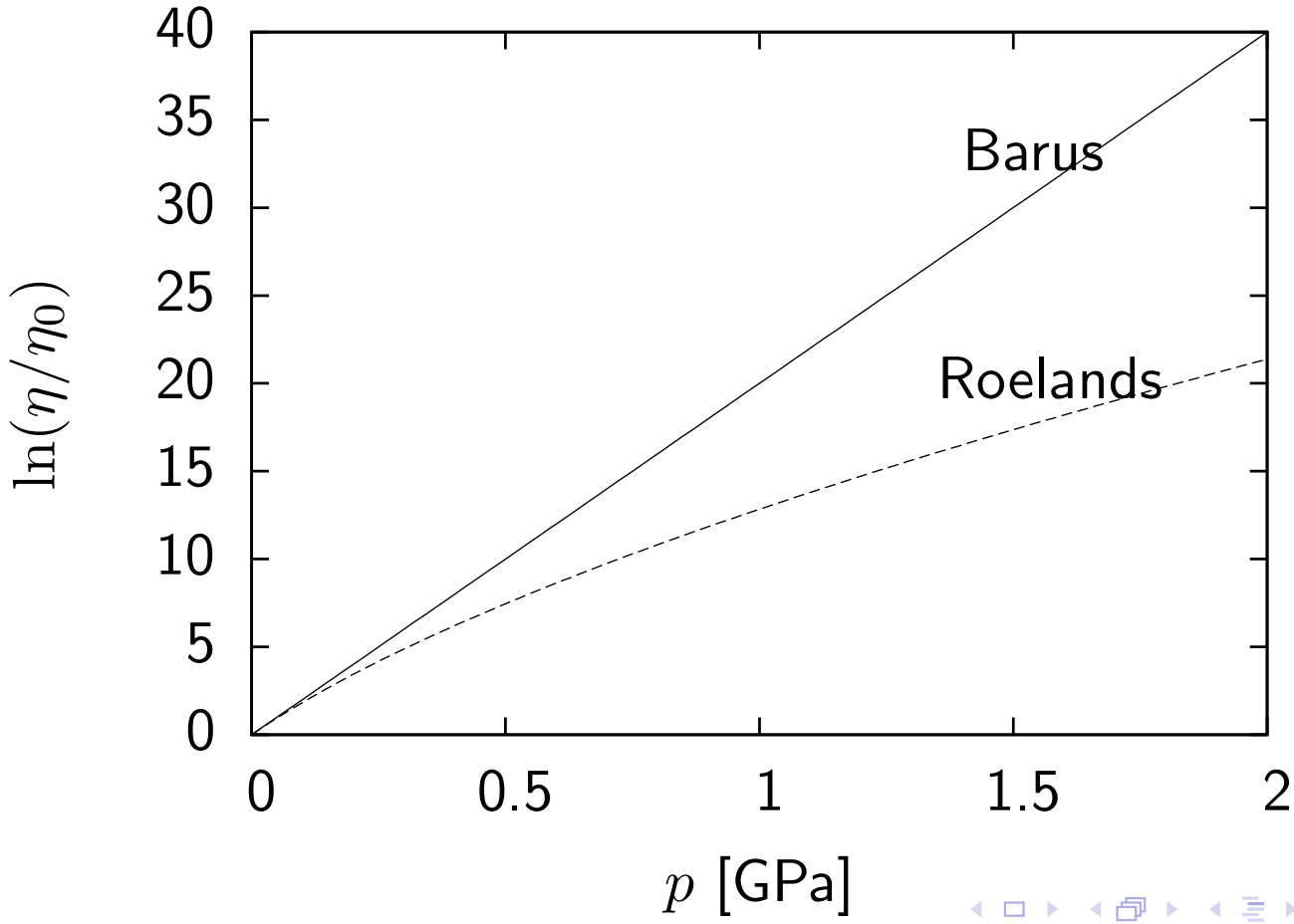
Figure *Relative density ρ/ρ_0 as a function of p.*

$$\rho(p) = \rho_0 \frac{5.9\,10^8 + 1.34\,p}{5.9\,10^8 + p}$$

Viscosity Pressure relation

$$\eta(p) = \eta_0 \exp(\alpha\, p)$$

$$\eta(p) = \eta_0 \exp\left[\ln(\eta_0) + 9.67)\{-1 + (1 + \frac{p}{p_0})^z\}\right]$$



Force Balance

$$w = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y')\, dx'\, dy'$$

Dimensionless Equations

Reynolds Equation with $P = p/p_h$, $X = x/a$, $Y = y/a$ and $H = hR_x/a^2$, based on Hertz and $T = u_m\, t/a$, $\bar{\eta} = \eta/\eta_0$ and $\bar{\rho} = \rho/\rho_0$.

$$\frac{\partial}{\partial X}(\epsilon\frac{\partial P}{\partial X}) + \frac{\partial}{\partial Y}(\epsilon\frac{\partial P}{\partial Y}) - \frac{\partial(\bar{\rho}H)}{\partial X} - \frac{\partial(\bar{\rho}H)}{\partial T} = 0$$

Where $\epsilon = (\bar{\rho}H^3)/(\bar{\eta}\lambda)$, and $\lambda = (12\eta_0 u_m R_x^2)/(a^3 p_h)$.
Film Thickness equation

$$H(X,Y) = H_0 + \frac{X^2}{2} + \frac{Y^2}{2} + \frac{2}{\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{P(X',Y')\,dX'\,dY'}{\sqrt{(X-X')^2+(Y-Y')^2}}$$

Force Balance equation

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(X',Y')\,dX'\,dY' = \frac{2\pi}{3}$$

Hertzian Solution

The Hertzian solution is the solution of the dry contact problem found by H. Hertz (1881). This approximation neglects the film thickness (h) with respect to the elastic deformation (δ). As a consequence, the solution accurately predicts the maximum (Hertzian) pressure (p_h), contact area (a and b) (and stresses τ) and deformation (δ). Obviously, the film thickness prediction is useless!

The Hertzian prediction becomes more and more accurate whenever the velocity (u) becomes small or the load (w) becomes large ($h \to 0$).

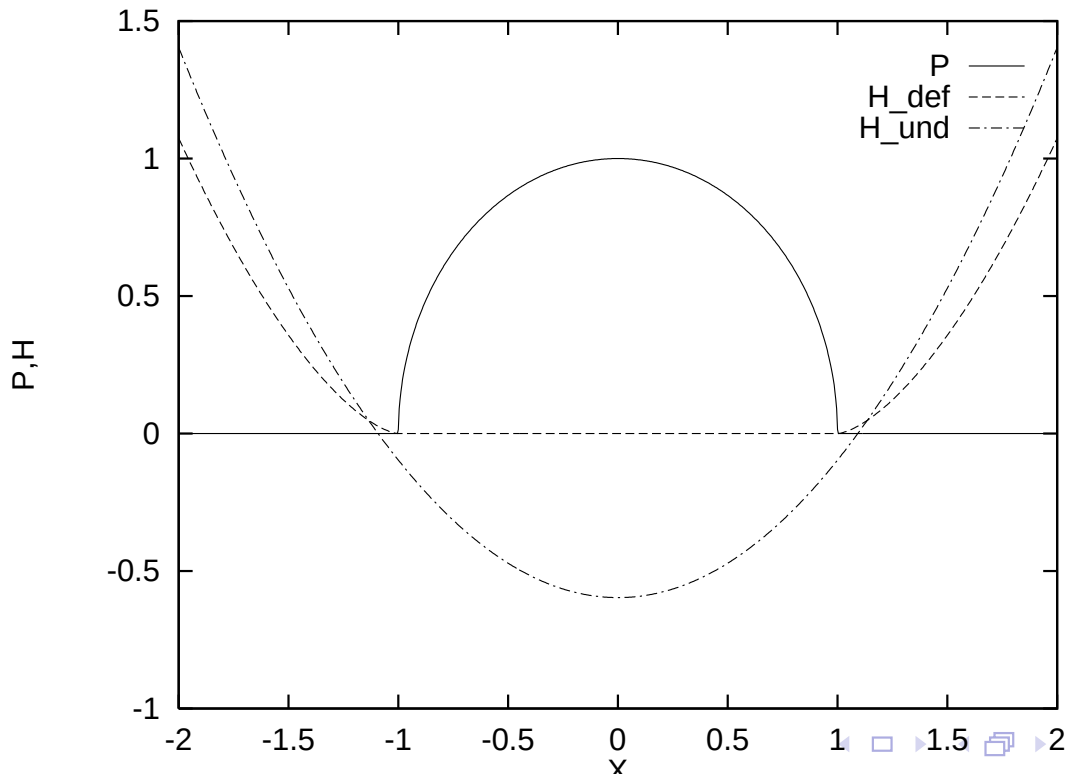
Q: what is the influence of the viscosity η_0?

Hertzian Parameters

the line contact Hertzian pressure:

$$\frac{p(x/b)}{p_h} = P(X) = \begin{cases} \sqrt{1-X^2}, & \text{if } |X| \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$p_h = \frac{2w_1}{\pi b} \quad b = \sqrt{\frac{8\,w_1\,R}{\pi\,E'}} \quad \delta = (\frac{1}{4} + \frac{1}{2}\ln 2)\frac{b^2}{R} \approx 0.596...\frac{b^2}{R}$$



Hertzian Parameters

the circular contact Hertzian pressure:

$$\frac{p(x/a, y/a)}{p_h} = P(X, Y) = \begin{cases} \sqrt{1 - X^2 - Y^2}, & \text{if } X^2 + Y^2 \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$p_h = \frac{3w}{2\pi a^2} = \frac{E'}{\pi}\sqrt[3]{\frac{3W_2}{2}} \quad a = \sqrt[3]{\frac{3wR_x}{2E'}} = R_x\sqrt[3]{\frac{3W_2}{2}}$$

$$\delta = \frac{a^2}{R_x} = \sqrt[3]{\frac{9w^2}{4E'^2R_x}} = R_x\sqrt[3]{\frac{9W_2^2}{4}}$$

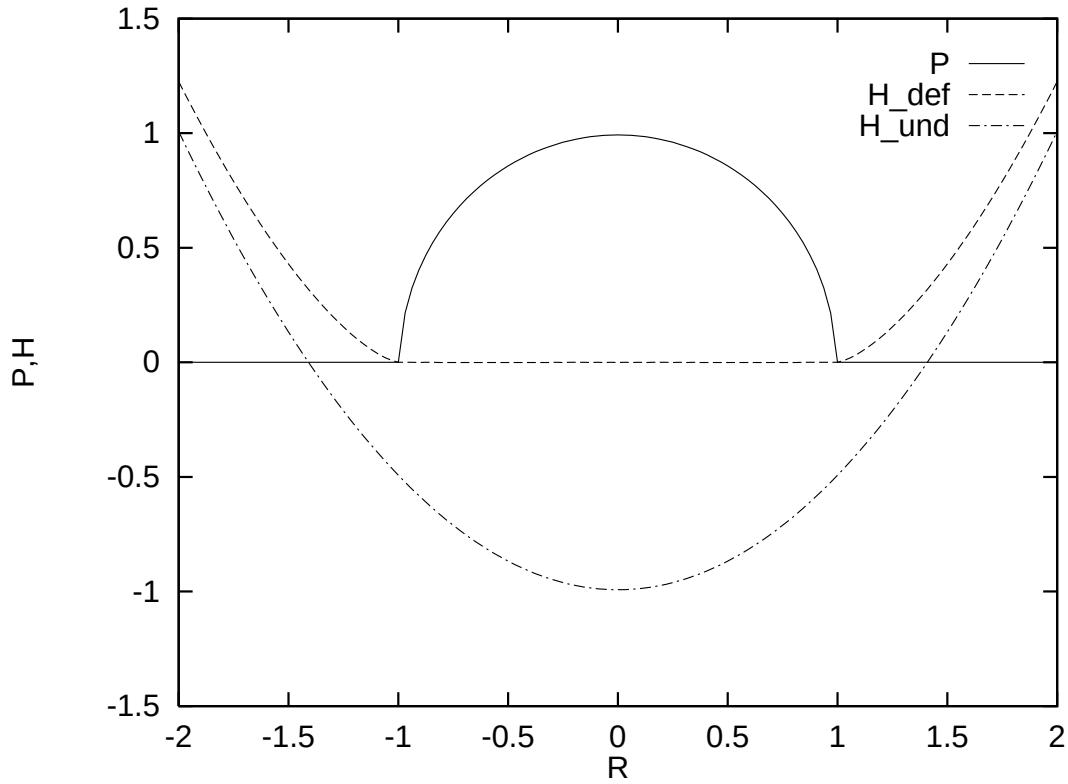


Figure *Dimensionless Hertzian contact pressure, dimensionless deformed and undeformed geometry, for the circular contact case.*

Hertzian Parameters

for the elliptical contact parameters: see course notes.

Ertel Grubin Analysis

The Ertel-Grubin analysis assumes that the lubricated geometry is equal to the Hertzian (dry contact geometry) plus a constant shift. It also assumes that the pressure reaches ∞ at $x = -b$: $(p(x \to -b) = \infty)$. In dimensionless parameters: $(P(X \to -1) = \infty)$. Finally it uses the reduced pressure q.

$$q = \frac{1}{\alpha}(1 - e^{-\alpha p})$$

Thus

$$q(X \to -1) = \frac{1}{\alpha}$$

Which gives

$$\frac{\partial q}{\partial x} = 6\eta_0(u_1 + u_2)\frac{h - h^*}{h^3}$$

Ertel Grubin Analysis

Integrating between $x = -\infty$ and $x = -b$ using the Hertzian (dry) film thickness $h_h(x)$ gives the Ertel Grubin result.
The EG predictions become more and more accurate for higher loads w and lower speeds u, why?

Ertel Grubin results

$$\frac{h^*}{R} = 1.31 \left(\frac{(\alpha E')\eta_0(u_1+u_2)}{E'R} \right)^{3/4} \left(\frac{w_1}{E'R} \right)^{-1/8}$$

Introducing the dimensionless groups: $H^* = h^*/R$, $W_1 = w_1/(E'R)$, $U = \eta_0(u_1+u_2)/(E'R)$, $G = \alpha E'$.
one obtains:

$$H^* = 1.31\,(GU)^{3/4}(W_1)^{-1/8}$$

Ertel Grubin graph

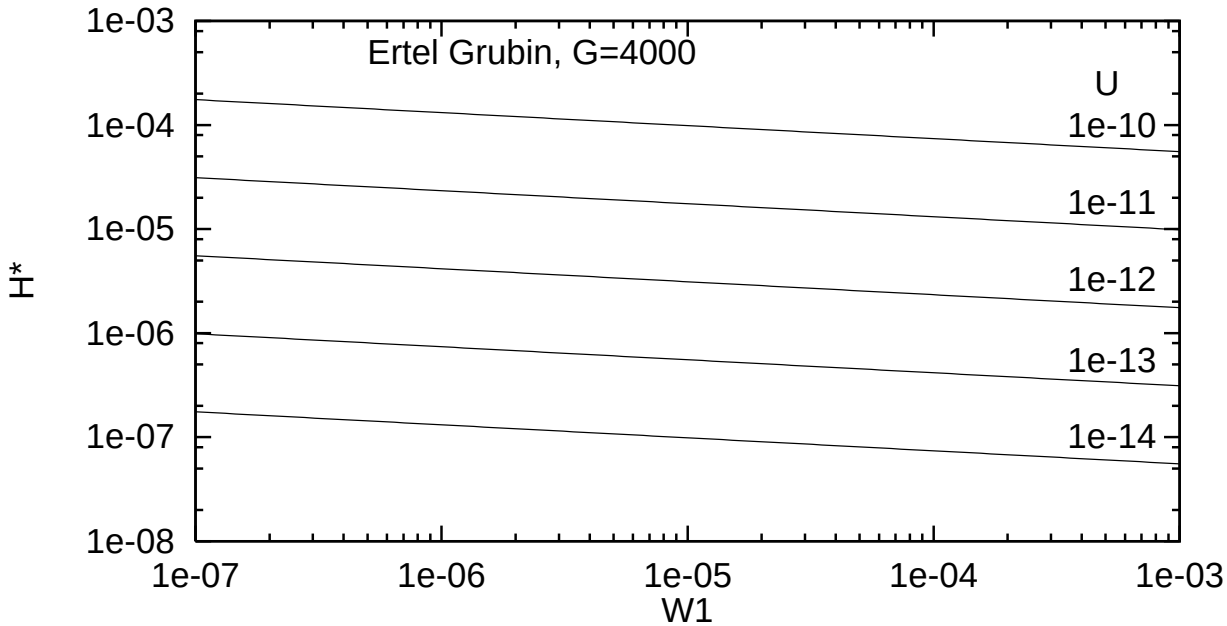


Figure *Dimensionless film thickness H^* as a function of W_1 and U for G=4000.*

Numerical Results

Since the 1960-1970, computers have become sufficiently powerful to solve the complete system of equations: Reynolds equation, film thickness equation and force balance equation. The first solutions were the 1d line contact solutions, whereas in the late 1970's the first point contact solutions were calculated. Using todays' computers one can calculate transient EHL solutions of a surface indentation passing through the contact.

Numerical Results, varying U

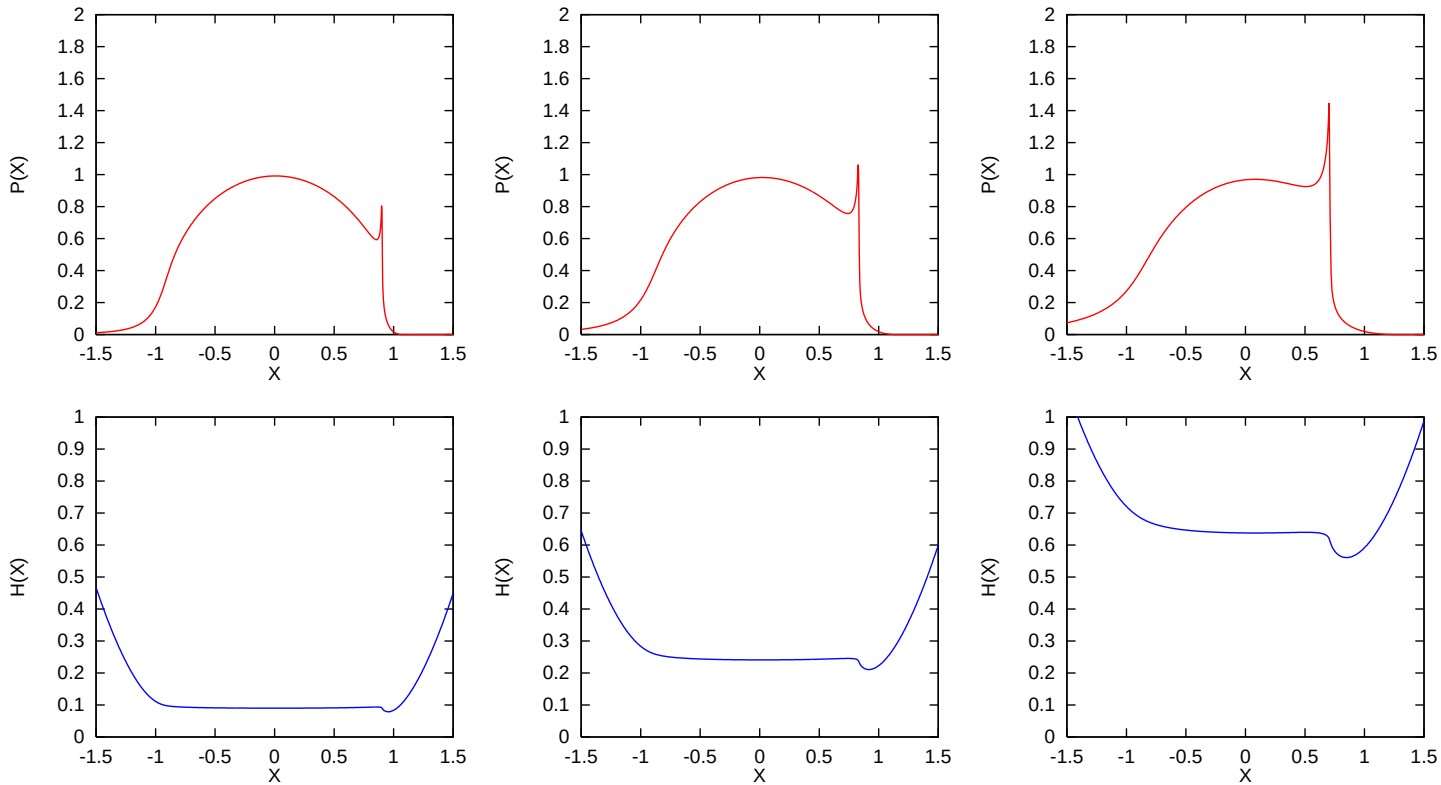


Figure *Dimensionless pressure and film thickness distribution* $W_1 = 1.53 \cdot 10^{-4}$, $U = 5.89, 23.6, 94.2 \cdot 10^{-11}$, $G = 4000$.

Numerical Results, varying U

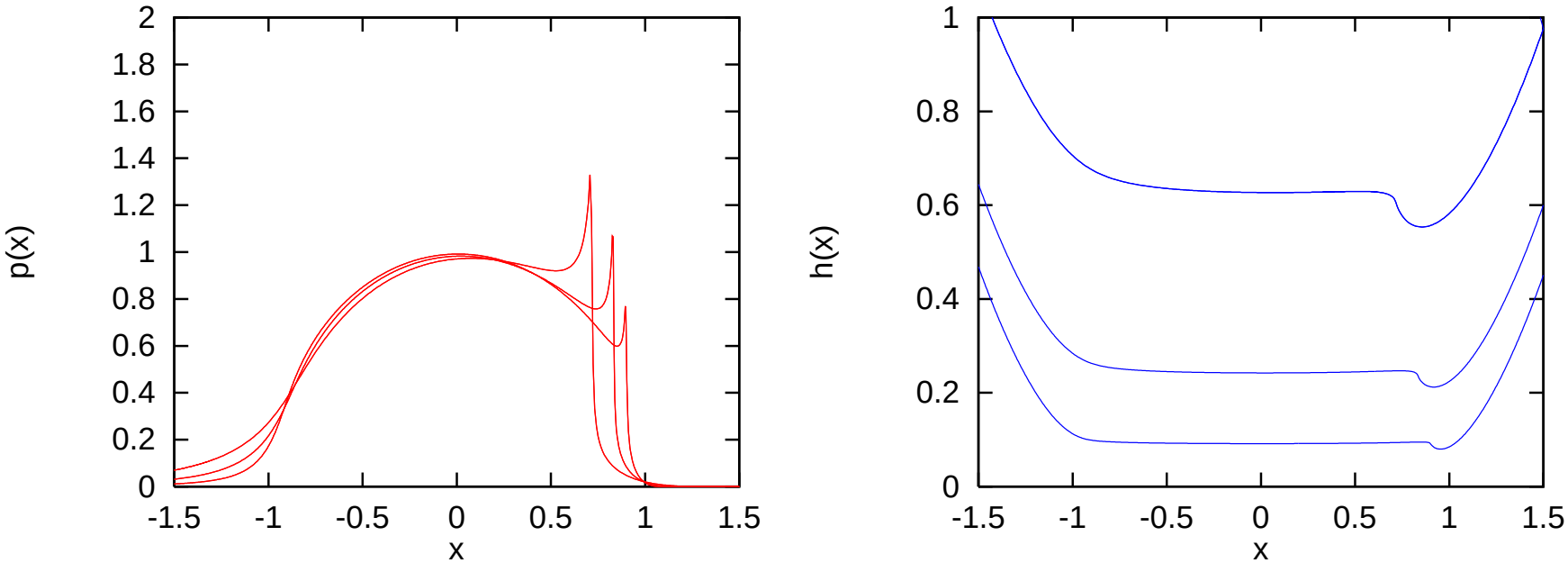


Figure *Dimensional pressure and film thickness distribution* $W_1 = 1.53 \cdot 10^{-4}$, $U = 5.89, 23.6, 94.2 \cdot 10^{-11}$, $G = 4000$.

Numerical Results, varying W_1

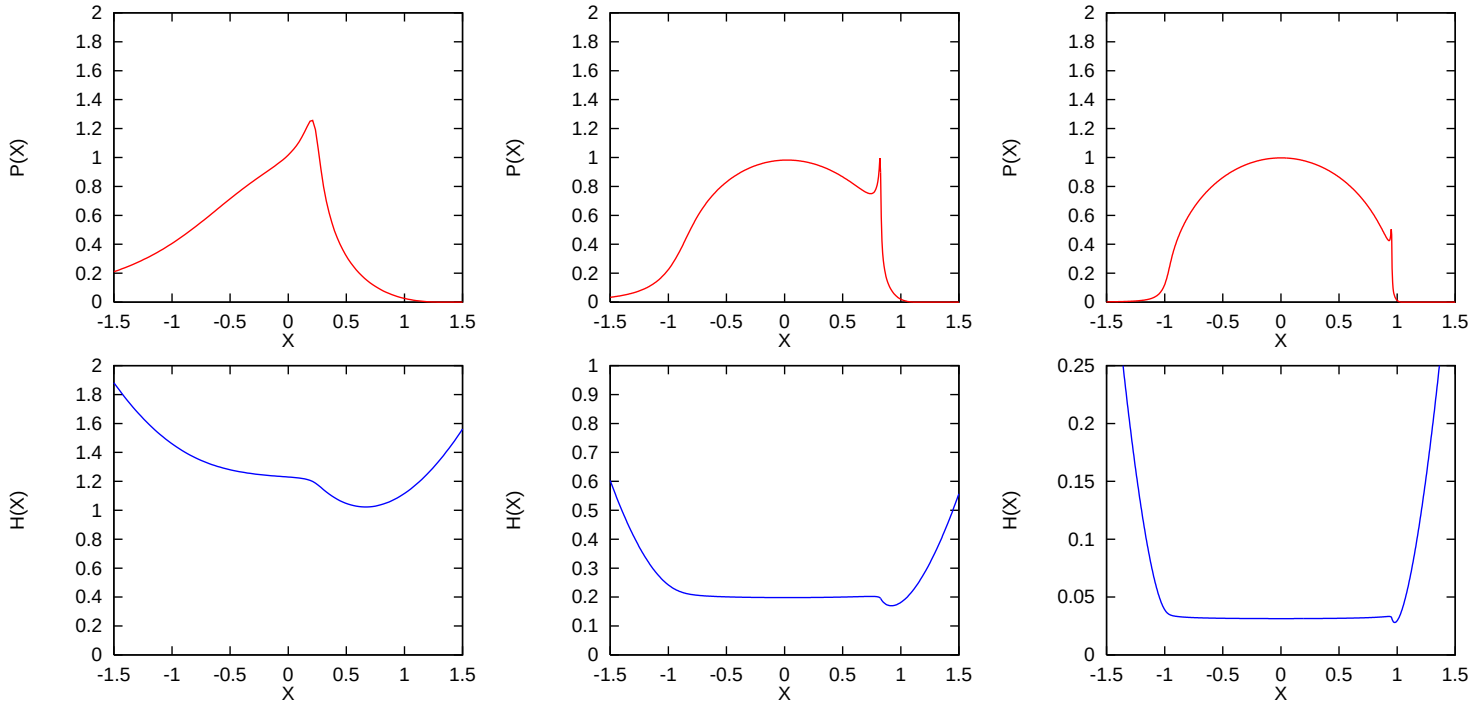


Figure *Dimensionless pressure and film thickness distribution* $W_1 = 1.53, 7.67, 38.4 \cdot 10^{-5}$, $U = 5.89 \cdot 10^{-11}$, $G = 4000$.

Numerical Results, varying W_1

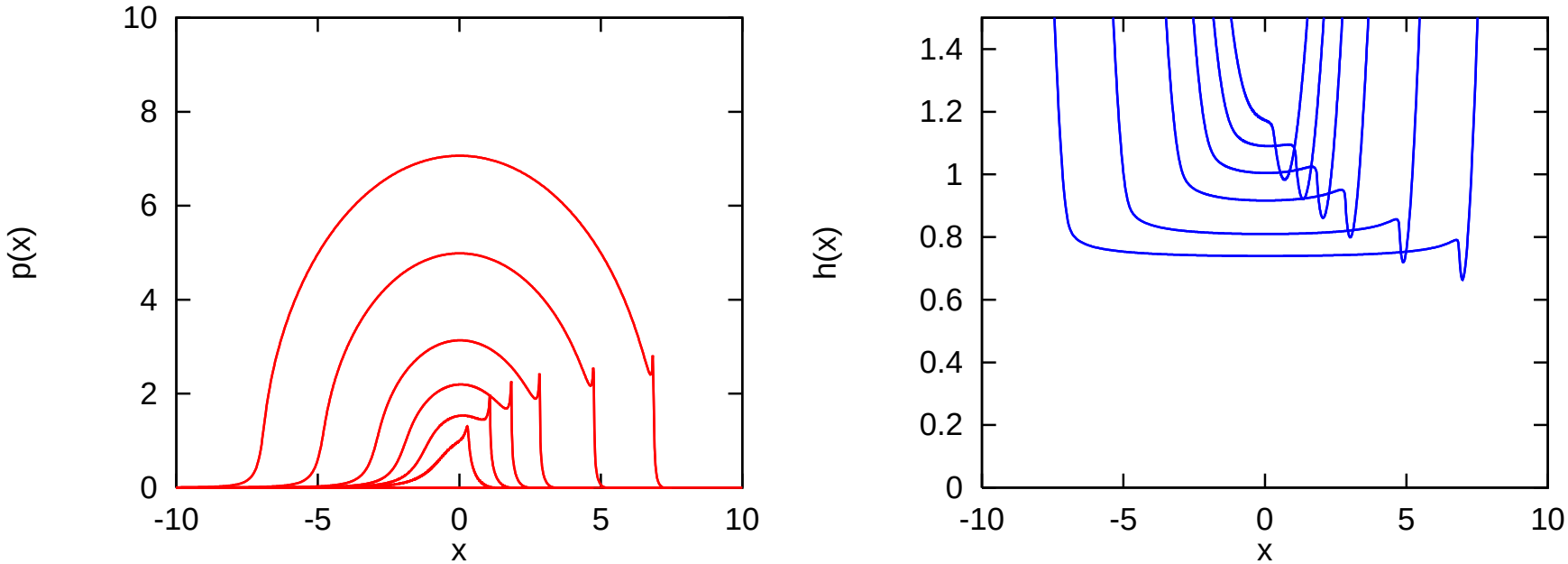


Figure *Dimensional pressure and film thickness distribution*
$W_1 = 1.53, 3.84, 7.67, 15.3, 38.4, 76.7 \cdot 10^{-5}$,
$U = 5.89 \cdot 10^{-11}$, $G = 4000$.

Dowson Higginson 1d

$$H_m = 0.97\, G^{0.6} U^{0.7} W_1^{-0.13}$$

where h_m is the minimum film thickness and $H_m = h_m/R$
a comparison of the powers found by Dowson and Higginson with those of the Ertel Grubin formula

$$H^* = 1.31\, (GU)^{3/4} (W_1)^{-1/8}$$

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- The original problem (Reynolds+geometry+force balance) is a two parameter problem: α and λ.
- The DH solution is a three parameter solution: W, U and G. Furthermore, the DH parameters are NOT $O(1)$!
- The more recent Moes-Venner solution is a two parameter solution: M, and L. Furthermore, the Moes parameters M, L and H ARE $O(1)$!

Moes Venner 1d

$$H_{min} = 1.56\, L^{0.55}\, M_1^{-0.125}$$

$$H_{min} = h_m/(R\sqrt{U})$$
$$M_1 = W_1/\sqrt{U}$$
$$L = G\sqrt[4]{U}$$

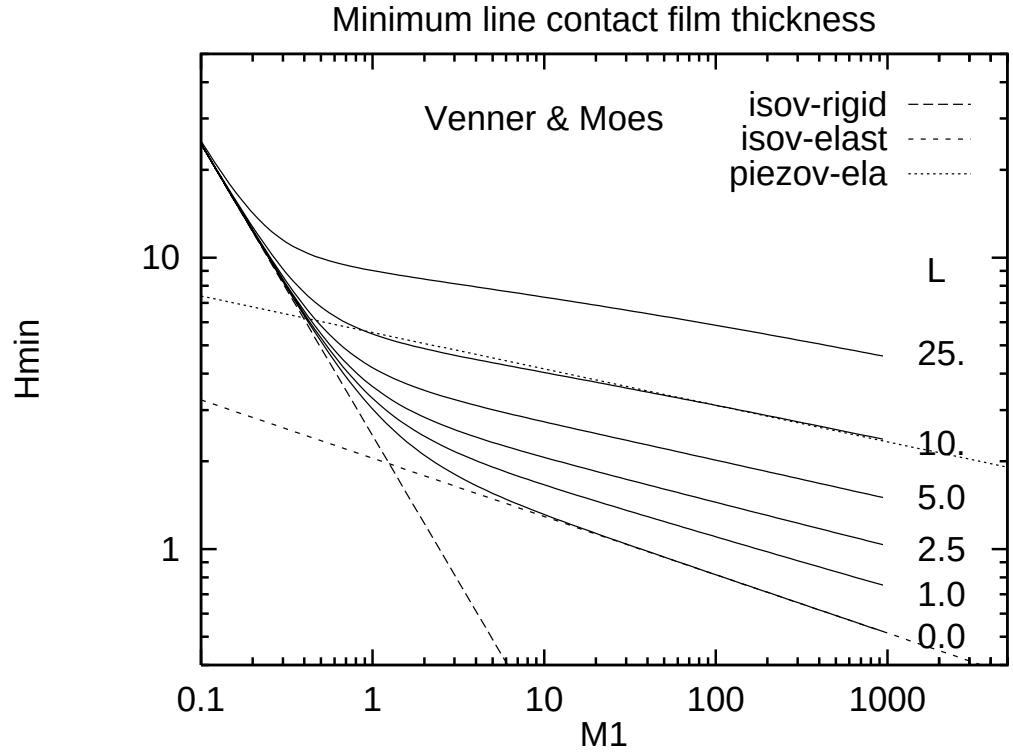

Figure *Moes-Venner 1d film thickness prediction.*

Comparison of powers

	E.G.	D.H.	M.V.
G	0.75	0.60	0.55
U	0.75	0.70	0.7
W_1	-0.125	-0.13	-0.125

Table *Comparison of the powers obtained in the Ertel Grubin, Dowson Higginson and Moes Venner film thickness equations.*

Hamrock Dowson 2d

$$H_c = 1.69\, G^{0.53} U^{0.67} W_2^{-0.067}\, (1 - 0.61 \exp(-0.73k))$$

$$H_m = 2.27\, G^{0.49} U^{0.68} W_2^{-0.073}\, (1 - \exp(-0.68k))$$

with $H_c = h_c/R_x$, $W_2 = w/(E' R_x^2)$, $U = \eta_0(u_1 + u_2)/(E' R_x)$, $G = \alpha E$.

Moes Venner 2d

using $H_{cen} = h_c/(R_x\sqrt{U})$, $M_2 = W_2/U^{3/4}$, $L = G\sqrt[4]{U}$.

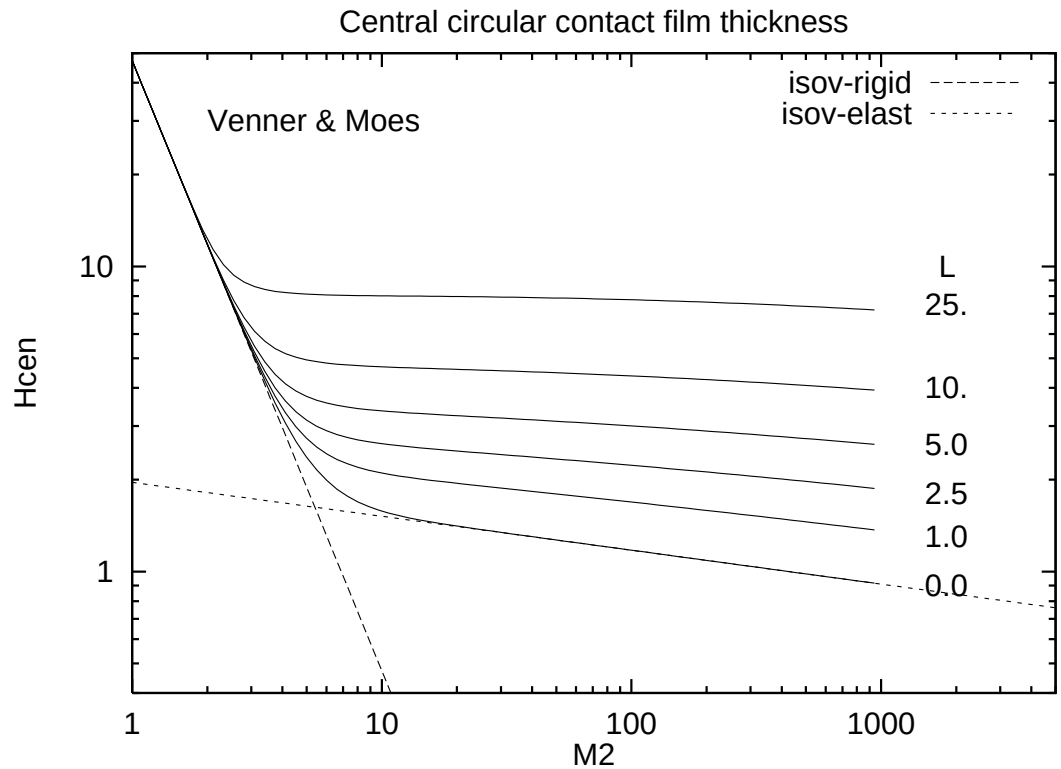

Figure *Moes-Venner 2d film thickness prediction.*

Moes Venner 1d - equations

iso-viscous rigid: $H_m = 2.45 M_1^{-1}$

iso-viscous elastic: $H_m = 2.05 M_1^{-1/5}$

piezo-viscous elastic: $H_m = 1.56 M_1^{-1/8} L^{0.55}$

Moes Venner 2d - equations

iso-viscous rigid: $H_c = 47.3 M_2^{-2}$

iso-elastic: $H_c = 1.96 M_2^{-1/9}$

Boundary layer analysis

The inlet zone can be considered as a boundary layer between a low pressure (Poiseuille) zone and a high pressure (Couette) zone. Let us study the pressure distribution in the inlet

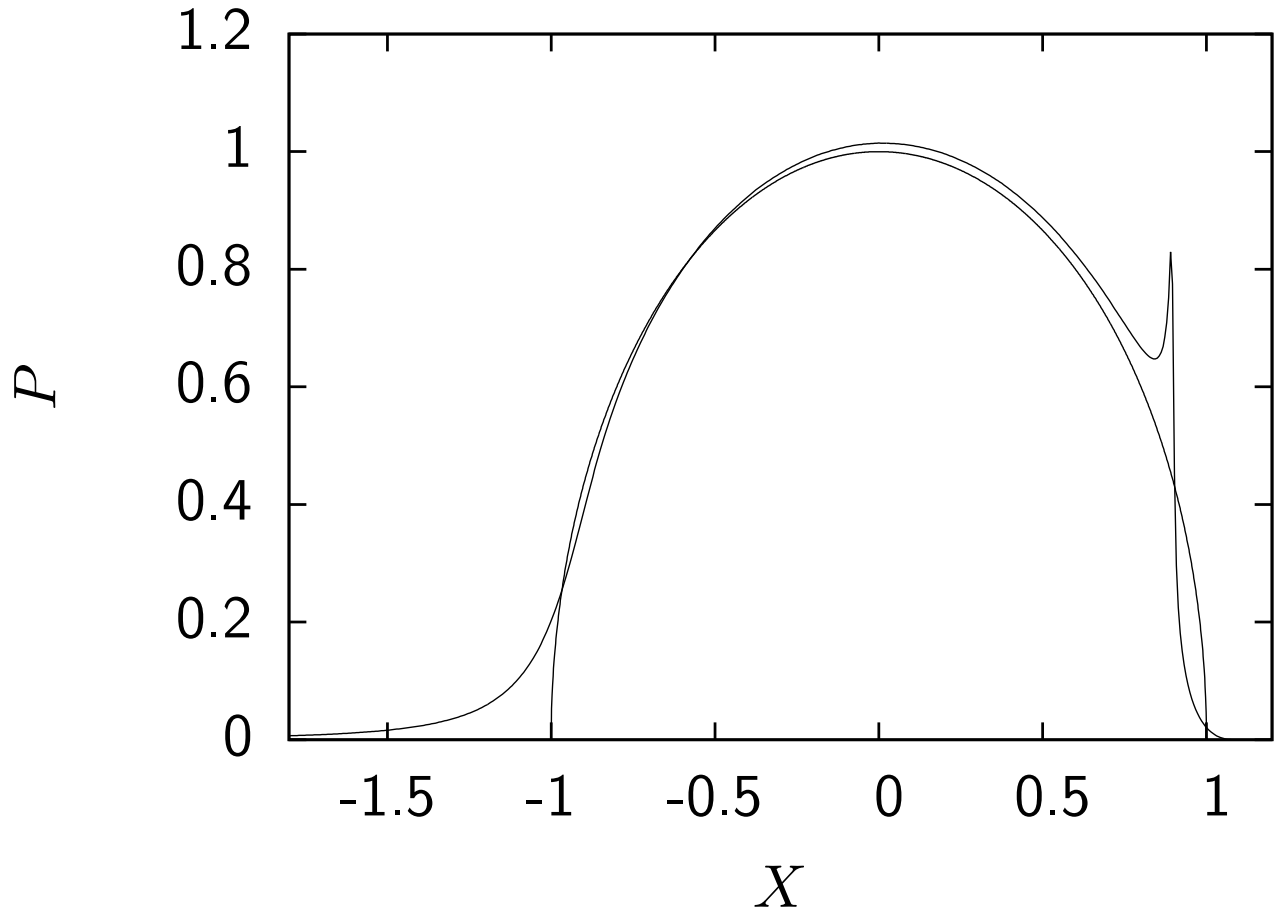

Figure *Dimensionless pressure distribution* $M = 100$, $L = 10$.

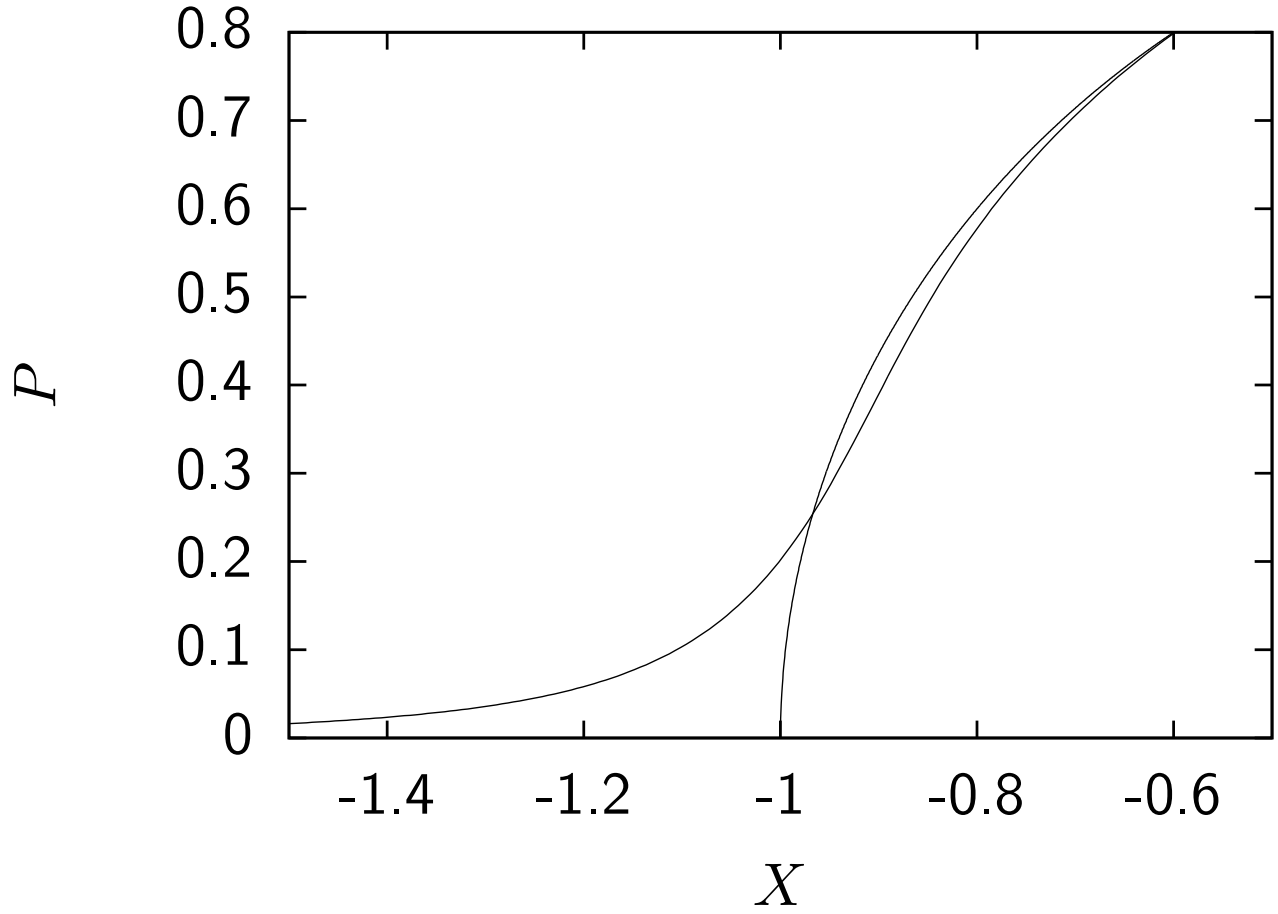

Figure *Zoom around the inlet* $X = -1$.

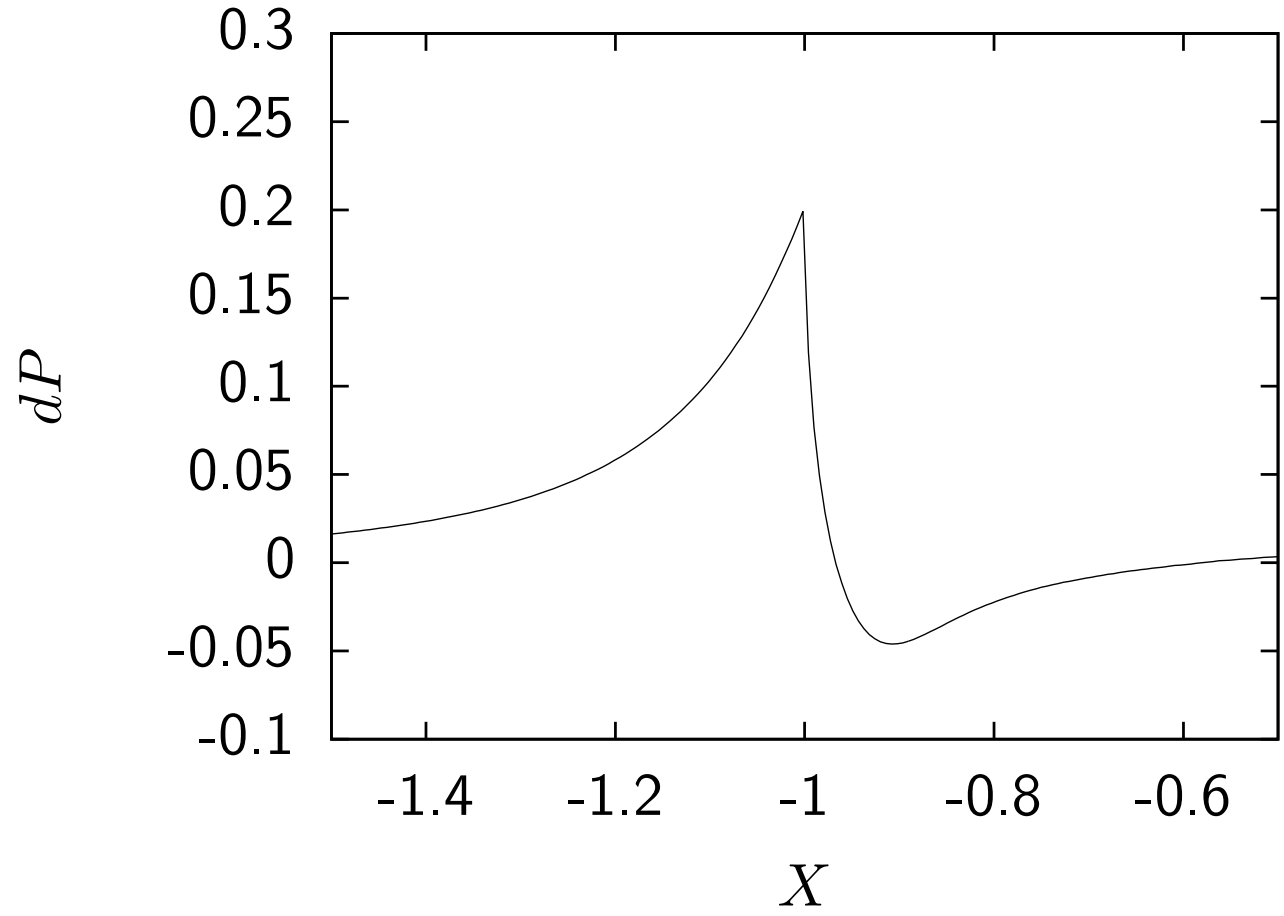

Figure *Dimensionless pressure difference distribution* $M = 100$, $L = 10$, *around* $X = -1$.

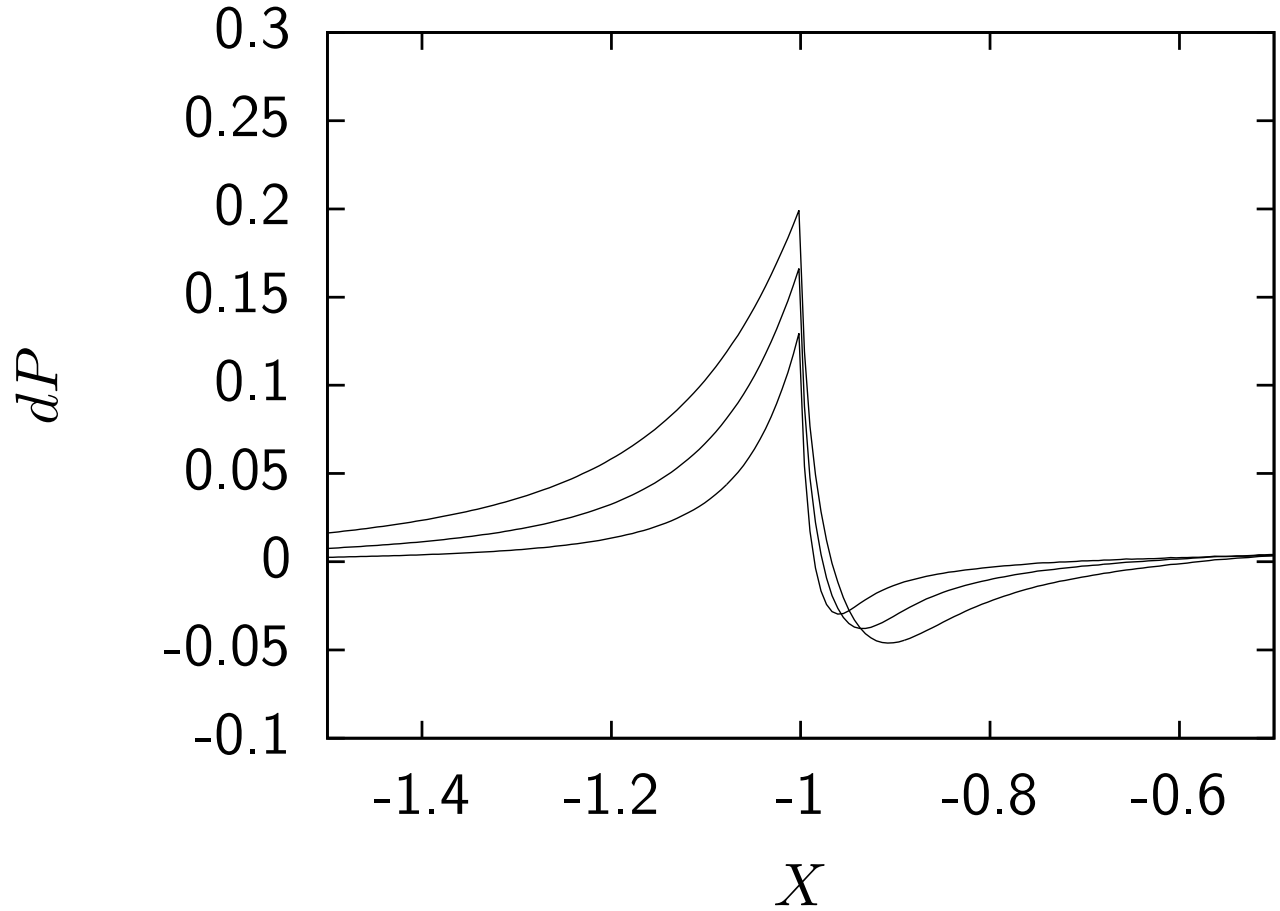

Figure *Dimensionless pressure difference distribution* $M = 100,\ 200,\ 500,\ L = 10,$ *around* $X = -1.$

It is clear that coordinate transformations in X and in P are necessary to be able to collapse the inlet pressure distribution onto a single curve:

$$\overline{dP} = dP \sqrt[3]{M} \tag{1}$$

$$\overline{X} = -1 + (X + 1) \sqrt[2]{M} \tag{2}$$

for completeness

$$\overline{H} = H \sqrt[16]{M} \tag{3}$$

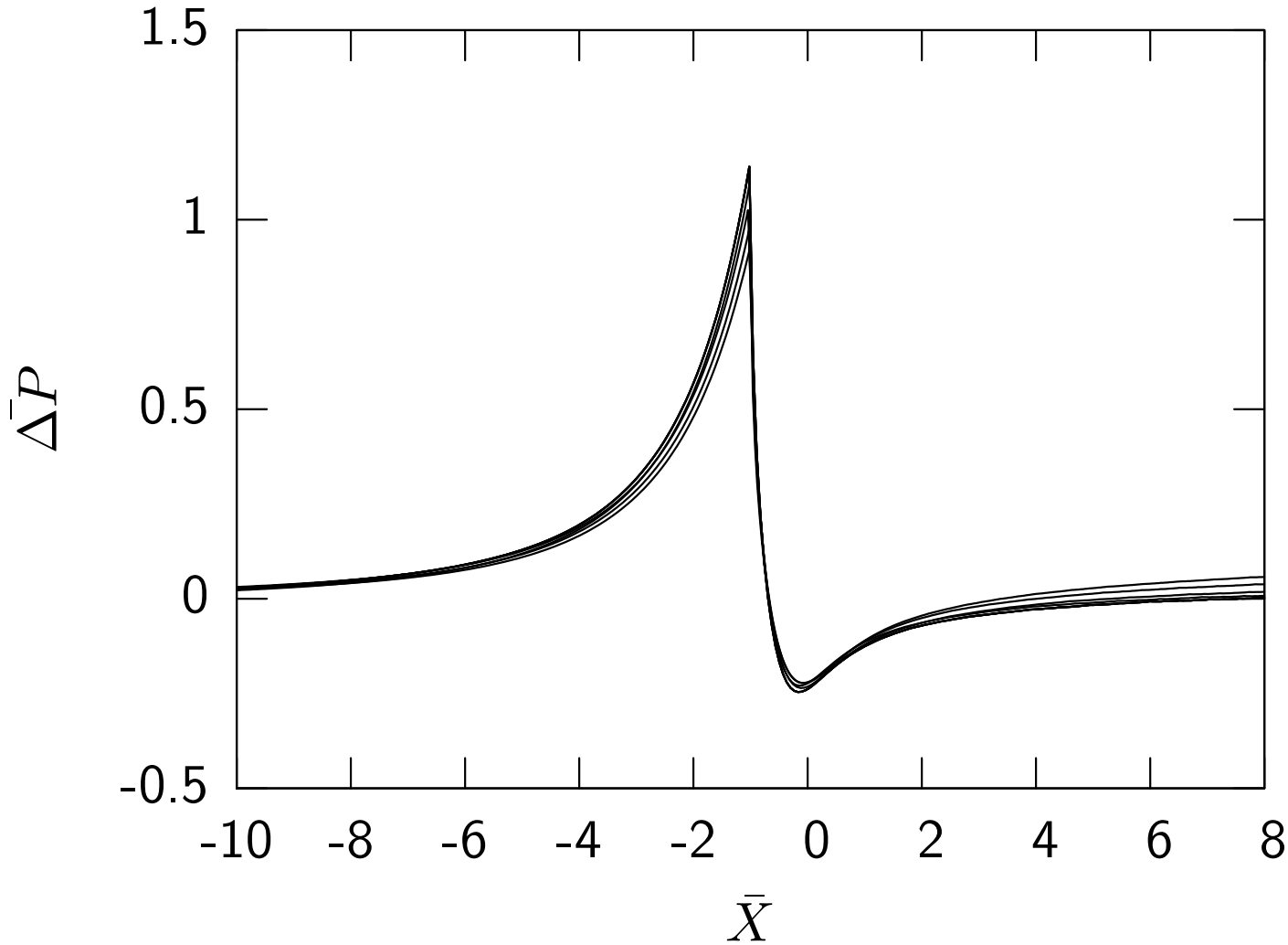

Figure $\overline{dP}$ *as a function of* $\overline{X}$ *for* $M = 100,\ 200,\ 500 \ldots 20000,$ $L = 10.$