



**PLAXIS**

**Material Models Manual**

**2015**



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## 1 INTRODUCTION

The mechanical behaviour of soils may be modelled at various degrees of accuracy. Hooke's law of linear, isotropic elasticity, for example, may be thought of as the simplest available stress-strain relationship. As it involves only two input parameters, i.e. Young's modulus,  $E$ , and Poisson's ratio,  $\nu$ , it is generally too crude to capture essential features of soil and rock behaviour. For modelling massive structural elements and bedrock layers, however, linear elasticity tends to be appropriate. An overview of the applicability of the material models is given in Appendix B.

### 1.1 ON THE USE OF DIFFERENT MODELS

#### ***Linear Elastic model (LE)***

The Linear Elastic model is based on Hooke's law of isotropic elasticity. It involves two basic elastic parameters, i.e. Young's modulus  $E$  and Poisson's ratio  $\nu$ . Although the Linear Elastic model is not suitable to model soil, it may be used to model stiff volumes in the soil, like concrete walls, or intact rock formations.

#### ***Mohr-Coulomb model (MC)***

The linear elastic perfectly-plastic Mohr-Coulomb model involves five input parameters, i.e.  $E$  and  $\nu$  for soil elasticity;  $\varphi$  and  $c$  for soil plasticity and  $\psi$  as an angle of dilatancy. This Mohr-Coulomb model represents a 'first-order' approximation of soil or rock behaviour. It is recommended to use this model for a first analysis of the problem considered. For each layer one estimates a constant average stiffness or a stiffness that increases linearly with depth. Due to this constant stiffness, computations tend to be relatively fast and one obtains a first estimate of deformations.

#### ***Hardening Soil model (HS)***

The Hardening Soil model is an advanced model for the simulation of soil behaviour. As for the Mohr-Coulomb model, limiting states of stress are described by means of the friction angle,  $\varphi$ , the cohesion,  $c$ , and the dilatancy angle,  $\psi$ . However, soil stiffness is described much more accurately by using three different input stiffnesses: the triaxial loading stiffness,  $E_{50}$ , the triaxial unloading stiffness,  $E_{ur}$ , and the oedometer loading stiffness,  $E_{oed}$ . As average values for various soil types,  $E_{ur} \approx 3E_{50}$  and  $E_{oed} \approx E_{50}$  are suggested as default settings, but both very soft and very stiff soils tend to give other ratios of  $E_{oed}/E_{50}$ , which can be entered by the user.

In contrast to the Mohr-Coulomb model, the Hardening Soil model also accounts for stress-dependency of stiffness moduli. This means that all stiffnesses increase with pressure. Hence, all three input stiffnesses relate to a reference stress, usually taken as 100 kPa (1 bar).

Besides the model parameters mentioned above, initial soil conditions, such as pre-consolidation, play an essential role in most soil deformation problems. This can be taken into account in the initial stress generation.

### ***Hardening Soil model with small-strain stiffness (HSsmall)***

The Hardening Soil model with small-strain stiffness (HSsmall) is a modification of the above Hardening Soil model that accounts for the increased stiffness of soils at small strains. At low strain levels most soils exhibit a higher stiffness than at engineering strain levels, and this stiffness varies non-linearly with strain. This behaviour is described in the HSsmall model using an additional strain-history parameter and two additional material parameters, i.e.  $G_0^{ref}$  and  $\gamma_{0.7}$ .  $G_0^{ref}$  is the small-strain shear modulus and  $\gamma_{0.7}$  is the strain level at which the shear modulus has reduced to about 70% of the small-strain shear modulus. The advanced features of the HSsmall model are most apparent in working load conditions. Here, the model gives more reliable displacements than the HS model. When used in dynamic applications, the Hardening Soil model with small-strain stiffness also introduces hysteretic material damping.

### ***Soft Soil model (SS)***

The Soft Soil model is a Cam-Clay type model especially meant for primary compression of near normally-consolidated clay-type soils. Although the modelling capabilities of this model are generally superseded by the Hardening Soil model, the Soft Soil model is better capable to model the compression behaviour of very soft soils.

### ***Soft Soil Creep model (SSC)***

The Hardening Soil model is generally suitable for all soils, but it does not account for viscous effects, i.e. creep and stress relaxation. In fact, all soils exhibit some creep and primary compression is thus followed by a certain amount of secondary compression.

The latter is most dominant in soft soils, i.e. normally consolidated clays, silts and peat, and PLAXIS thus implemented a model under the name Soft Soil Creep model. The Soft Soil Creep model has been developed primarily for application to settlement problems of foundations, embankments, etc. For unloading problems, as normally encountered in tunnelling and other excavation problems, the Soft Soil Creep model hardly supersedes the simple Mohr-Coulomb model. As for the Hardening Soil model, proper initial soil conditions are also essential when using the Soft Soil Creep model. This also includes data on the pre-consolidation stress, as the model accounts for the effect of over-consolidation. Note that the initial over-consolidation ratio also determines the initial creep rate.

### ***Jointed Rock model (JR)***

The Jointed Rock model is an anisotropic elastic-plastic model, especially meant to simulate the behaviour of rock layers involving stratification and particular fault directions. Plasticity can only occur in a maximum of three shear directions (shear planes). Each plane has its own strength parameters  $\varphi$  and  $c$ . The intact rock is considered to behave fully elastic with constant stiffness properties  $E$  and  $\nu$ . Reduced elastic properties may be defined for the stratification direction.

### ***Modified Cam-Clay model (MCC)***

The Modified Cam-Clay model is a well known model from international soil modelling literature; see for example Muir Wood (1990). It is meant primarily for the modelling of near normally-consolidated clay-type soils. This model has been added to PLAXIS to

allow for a comparison with other codes.



### **NGI-ADP model (NGI-ADP)**

The NGI-ADP model is an anisotropic undrained shear strength model. The soil shear strength is defined by means of  $s_u$  values for active, passive and direct simple shear stress states. The model may be used for onshore and offshore applications in undrained clays and silts.



### **Sekiguchi-Ohta model (Sekiguchi-Ohta)**

The Sekiguchi-Ohta model is a Cam-Clay type of model with anisotropic yield contour defined by  $K_{nc}^0$ . Two versions of the model exist: The inviscid model is a time independent model which has similarities with the Soft Soil model. The viscid model is time-dependent and has similarities with the Soft Soil Creep model. Both models have been developed in Japan. These models were previously available as user-defined models, but have become standard models in PLAXIS nowadays.



### **Hoek-Brown model (HB)**

The Hoek-Brown model is an isotropic elastic perfectly-plastic model for weathered rock based on the 2002 edition of the Hoek-Brown failure criterion. This non-linear stress-dependent criterion describes shear failure and tensile failure by a continuous function, and is familiar to most geologists and rock engineers. Besides the elastic parameters ( $E$  and  $\nu$ ), the model involves practical rock parameters such as the uni-axial compressive strength of the intact rock ( $\sigma_{ci}$ ), the Geological Strength Index ( $GSI$ ), and the disturbance factor ( $D$ ).

## **Analyses with different models**

The Mohr-Coulomb model may be used for a relatively quick and simple first analysis of the problem considered.

In many cases, even if good data on dominant soil layers is limited, it is recommended to use the Hardening Soil model or the HS small model in an additional analysis. No doubt, one seldomly has test results from both triaxial and oedometer tests, but good quality data from one type of test can be supplemented by data from correlations and/or in situ testing.

Finally, a Soft Soil Creep analysis can be performed to estimate creep, i.e. secondary compression in very soft soils.

## **1.2 LIMITATIONS**

The PLAXIS code and its soil models have been developed to perform calculations of realistic geotechnical problems. In this respect PLAXIS can be considered as a geotechnical simulation tool. The soil models can be regarded as a qualitative representation of soil behaviour whereas the model parameters are used to quantify the soil characteristics. Although much care has been taken for the development of the PLAXIS code and its soil models, the simulation of reality remains an approximation, which implicitly involves some inevitable numerical and modelling errors. Moreover, the accuracy at which reality is approximated depends highly on the expertise of the user

regarding the modelling of the problem, the understanding of the soil models and their limitations, the selection of model parameters, and the ability to judge the reliability of the computational results.

Some of the limitations in the currently available models are listed below:

### ***Linear Elastic model***

Soil behaviour is highly non-linear and irreversible. The linear elastic model is insufficient to capture the essential features of soil. The use of the linear elastic model may, however, be considered to model strong massive structures in the soil or bedrock layers. Stress states in the linear elastic model are not limited in any way, which means that the model shows infinite strength. Be careful using this model for materials that are loaded up to their material strength.

### ***Mohr-Coulomb model***

The linear elastic perfectly-plastic Mohr-Coulomb model is a first order model that includes only a limited number of features that soil behaviour shows in reality. Although the increase of stiffness with depth can be taken into account, the Mohr-Coulomb model does neither include stress-dependency nor stress-path dependency nor strain dependency of stiffness or anisotropic stiffness. In general, effective stress states at failure are quite well described using the Mohr-Coulomb failure criterion with effective strength parameters  $\varphi'$  and  $c'$ . For undrained materials, the Mohr-Coulomb model may be used with the friction angle  $\varphi$  set to  $0^\circ$  and the cohesion  $c$  set to  $c_u (s_u)$ , to enable a direct control of undrained shear strength. In that case note that the model does not automatically include the increase of shear strength with consolidation.

### ***Hardening Soil model***

Although the Hardening Soil model can be regarded as an advanced soil model, there are a number of features of real soil behaviour the model does not include. It is a hardening model that does not account for softening due to soil dilatancy and de-bonding effects. In fact, it is an isotropic hardening model so that it models neither hysteretic and cyclic loading nor cyclic mobility. Moreover, the model does not distinguish between large stiffness at small strains and reduced stiffness at engineering strain levels. The user has to select the stiffness parameters in accordance with the dominant strain levels in the application. Last but not least, the use of the Hardening Soil model generally results in longer calculation times, since the material stiffness matrix is formed and decomposed in each calculation step.

### ***Hardening Soil model with small-strain stiffness***

As the Hardening Soil model with small-strain stiffness (HSsmall) incorporates the loading history of the soil and a strain-dependent stiffness, it can, to some extent, be used to model cyclic loading. However, it does not incorporate a gradual softening during cyclic loading, so is not suitable for cyclic loading problems in which softening plays a role. In fact, just as in the Hardening Soil model, softening due to soil dilatancy and debonding effects are not taken into account. Moreover, the HSsmall does not incorporate the accumulation of irreversible volumetric straining nor liquefaction behaviour with cyclic loading. The use of the HSsmall will generally result in calculation

times that are even longer than those of the Hardening Soil model.

### ***Soft Soil model***

The same limitations (including the ones for the Soft Soil Creep model) hold in the Soft Soil model. The utilization of the Soft Soil model should be limited to the situations that are dominated by compression. It is not recommended for use in excavation problems, since the model hardly supercedes the Mohr-Coulomb model in unloading problems.

### ***Soft Soil Creep model***

All above limitations also hold true for the Soft Soil Creep model. In addition this model tends to over-predict the range of elastic soil behaviour. This is especially the case for excavation problems, including tunnelling. Care must also be taken with the generation of initial stresses for normally consolidated soils. Although it would seem logical to use  $OCR = 1.0$  for normally consolidated soils, such use would generally lead to an over-prediction of deformations in problems where the stress level is dominated by the initial self-weight stresses. Therefore, for such problems it is recommended to use a slightly increased OCR-value to generate the initial stress state. In fact, in reality most soils tend to show a slightly increased pre-consolidation stress in comparison with the initial effective stress. Before starting an analysis with external loading it is suggested to perform a single calculation phase with a short time interval and without loading to verify the surface settlement rate based on common practice.

### ***Jointed Rock model***

The Jointed Rock model is a first order anisotropic model that includes a limited number of features that rock behaviour shows in reality. Plasticity can only occur in a maximum of three shear directions (shear planes). Each plane has its own shear strength parameters  $\varphi_i$  and  $c_i$  and tensile strength  $t_i$ . Hence, the maximum shear stress is linearly dependent on the normal stress, and not curved as in reality. The intact rock is considered to behave fully elastic with constant stiffness properties  $E$  and  $\nu$ . Reduced elastic properties may be defined for the stratification direction. Note that failure is limited to the predefined shear plane directions. It is possible that realistic potential failure mechanisms are not captured by the model because of the elastic behaviour in any other direction than the three shear planes.

### ***Modified Cam-Clay model***

The same limitations (including those in the Soft Soil Creep model) hold in the Modified Cam-Clay model. Moreover, the Modified Cam-Clay model may allow for unrealistically high shear stresses. This is particularly the case for overconsolidated stress states where the stress path crosses the critical state line. Furthermore, the Modified Cam-Clay model may give softening behaviour for such stress paths. Without special regularization techniques, softening behaviour may lead to mesh dependency and convergence problems of iterative procedures. Moreover, the Modified Cam-Clay model cannot be used in combination with *Safety* analysis by means of  $\phi$ - $c$  reduction.





### **NGI-ADP model (NGI-ADP)**

The NGI-ADP model is an undrained shear strength model particularly meant to model undrained clays and silts. It can be used in a drained or effective stress analysis, but note that the shear strength is not automatically updated with changes of effective stress. Also note that the NGI-ADP model does not include a tension cut-off.



### **Hoek-Brown model (HB)**

The Hoek-Brown model is an isotropic continuum model particularly meant to model weathered rock. Hence, the model is not suitable for stratified or jointed rock sections with a significant anisotropic stiffness or with one or more dominant sliding directions. For such behaviour, the Jointed Rock model is available.

### **Interfaces**

Interface elements are generally modelled by means of the bilinear Mohr-Coulomb model. When a more advanced model is used for the corresponding cluster material data set, the interface element will only pick up the relevant data ( $c$ ,  $\varphi$ ,  $\psi$ ,  $E$ ,  $\nu$ ) for the Mohr-Coulomb model, as described in the Reference Manual. In such cases the interface stiffness is set equal to the elastic soil stiffness. Hence,  $E = E_{ur}$  where  $E_{ur}$  is stress level dependent, following a power law with  $E_{ur}$  proportional to  $\sigma_m$ . For the Soft Soil model, Soft Soil Creep model and Modified Cam-Clay model the power  $m$  is equal to 1 and  $E_{ur}$  is largely determined by the swelling constant  $\kappa^*$ .

### **Undrained behaviour**

In general, care must be taken in undrained conditions, since the effective stress path that is followed in any of the models may deviate significantly from reality. Although PLAXIS has options to deal with undrained behaviour in an effective stress analysis, the use of undrained shear strength ( $c_u$  or  $s_u$ ) may be preferred over the use of effective strength properties ( $\varphi'$  and  $c'$ ) in such cases. Please note that direct input on undrained shear strength does not automatically include the increase of shear strength with consolidation. If, for any reason, the user decides to use effective strength properties in undrained conditions, it is strongly recommended to check the resulting *mobilised shear strength* using the corresponding option in the PLAXIS Output program.

## 2 PRELIMINARIES ON MATERIAL MODELLING

A material model is described by a set of mathematical equations that give a relationship between stress and strain. Material models are often expressed in a form in which infinitesimal increments of stress (or 'stress rates') are related to infinitesimal increments of strain (or 'strain rates'). All material models implemented in PLAXIS are based on a relationship between the effective stress rates,  $\dot{\sigma}'$ , and the strain rates,  $\dot{\epsilon}$ . In the following section it is described how stresses and strains are defined in PLAXIS. In subsequent sections the basic stress-strain relationship is formulated and the influence of pore pressures in undrained materials is described. Later sections focus on initial conditions for advanced material models.

This Material Models Manual is a general manual for all PLAXIS programs and uses the coordinate system as used in most programs (Figure 2.1). Please note that the new PLAXIS 3D uses a different coordinate system where  $z$  is the vertical axis. Users should realize this when reading this manual.

### 2.1 GENERAL DEFINITIONS OF STRESS

Stress is a tensor which can be represented by a matrix in Cartesian coordinates:

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad (2.1)$$

In the standard deformation theory, the stress tensor is symmetric such that  $\sigma_{xy} = \sigma_{yx}$ ,  $\sigma_{yz} = \sigma_{zy}$  and  $\sigma_{zx} = \sigma_{xz}$ . In this situation, stresses are often written in vector notation, which involve only six different components:

$$\underline{\underline{\sigma}} = \left( \sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \sigma_{xy} \ \sigma_{yz} \ \sigma_{zx} \right)^T \quad (2.2)$$

According to Terzaghi's principle, stresses in the soil are divided into effective stresses,  $\underline{\underline{\sigma}}'$ , and pore pressures,  $\underline{\underline{\sigma}}_w$ :

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}' + \underline{\underline{\sigma}}_w \quad (2.3)$$

Pore pressures are generally provided by water in the pores. Water is considered not to

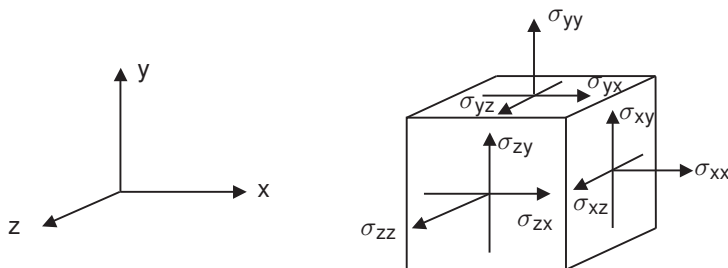


Figure 2.1 General three-dimensional coordinate system and sign convention for stresses

sustain any shear stresses. As a result, effective shear stresses are equal to total shear stresses. Positive normal stress components are considered to represent tension, whereas negative normal stress components indicate pressure (or compression). Moreover, water is considered to be fully isotropic, so all pore pressure components are equal. Hence, pore pressure can be represented by a single value,  $p_w$ :

$$\underline{\sigma}_w = \begin{pmatrix} p_w & p_w & p_w & 0 & 0 & 0 \end{pmatrix}^T \quad (2.4)$$

Material models for soil and rock are generally expressed as a relationship between infinitesimal increments of effective stress and infinitesimal increments of strain. In such a relationship, infinitesimal increments of effective stress are represented by stress rates (with a dot above the stress symbol):

$$\underline{\dot{\sigma}}' = \begin{pmatrix} \dot{\sigma}'_{xx} & \dot{\sigma}'_{yy} & \dot{\sigma}'_{zz} & \dot{\sigma}'_{xy} & \dot{\sigma}'_{yz} & \dot{\sigma}'_{zx} \end{pmatrix}^T \quad (2.5)$$

It is often useful to apply principal stresses rather than Cartesian stress components when formulating material models. Principal stresses are the stresses in such a coordinate system direction that all shear stress components are zero. Principal stresses are, in fact, the eigenvalues of the stress tensor. Principal effective stresses can be determined in the following way:

$$\det(\underline{\sigma}' - \underline{\sigma}' \underline{I}) = 0 \quad (2.6)$$

where  $\underline{I}$  is the identity matrix. This equation gives three solutions for  $\sigma'$ , i.e. the principal effective stresses ( $\sigma'_{1}, \sigma'_{2}, \sigma'_{3}$ ). In PLAXIS the principal effective stresses are arranged in algebraic order:

$$\sigma'_{1} \leq \sigma'_{2} \leq \sigma'_{3} \quad (2.7)$$

Hence,  $\sigma'_{1}$  is the largest compressive principal stress and  $\sigma'_{3}$  is the smallest compressive principal stress. In this manual, models are often presented with reference to the principal stress space, as indicated in Figure 2.2.

In addition to principal stresses it is also useful to define invariants of stress, which are stress measures that are independent of the orientation of the coordinate system. Two useful stress invariants are:

$$p' = \frac{1}{3} (\sigma'_{xx} + \sigma'_{yy} + \sigma'_{zz}) \quad (2.8a)$$

$$q = \sqrt{\frac{1}{2} ((\sigma'_{xx} - \sigma'_{yy})^2 + (\sigma'_{yy} - \sigma'_{zz})^2 + (\sigma'_{zz} - \sigma'_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2))} \quad (2.8b)$$

where  $p'$  is the isotropic effective stress, or mean effective stress, and  $q$  is the equivalent shear stress. The equivalent shear stress,  $q$ , has the important property that it reduces to  $q = |\sigma'_{1} - \sigma'_{3}|$  for triaxial stress states with  $\sigma'_{2} = \sigma'_{3}$ .

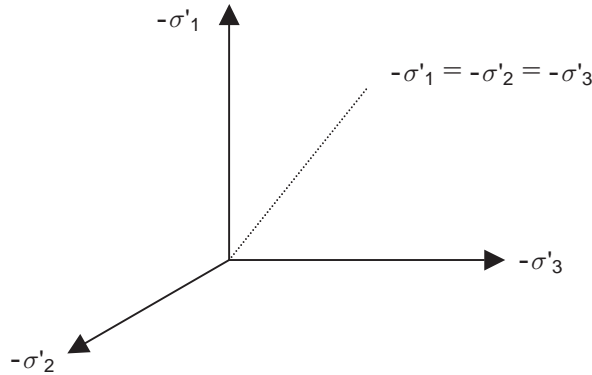


Figure 2.2 Principal stress space

Principal effective stresses can be written in terms of the invariants:

$$\sigma'_1 = p' + \frac{2}{3}q \sin\left(\theta - \frac{2}{3}\pi\right) \quad (2.9a)$$

$$\sigma'_2 = p' + \frac{2}{3}q \sin(\theta) \quad (2.9b)$$

$$\sigma'_3 = p' + \frac{2}{3}q \sin\left(\theta + \frac{2}{3}\pi\right) \quad (2.9c)$$

in which  $\theta$  is referred to as Lode's angle (a third invariant), which is defined as:

$$\theta = \frac{1}{3} \arcsin\left(\frac{27 J_3}{2 q^3}\right) \quad (2.10)$$

with

$$J_3 = (\sigma'_{xx} - p')(\sigma'_{yy} - p')(\sigma'_{zz} - p') - (\sigma'_{xx} - p')\sigma_{yz}^2 - (\sigma'_{yy} - p')\sigma_{zx}^2 - (\sigma'_{zz} - p')\sigma_{xy}^2 + 2\sigma_{xy}\sigma_{yz}\sigma_{zx} \quad (2.11)$$

## 2.2 GENERAL DEFINITIONS OF STRAIN

Strain is a tensor which can be represented by a matrix with Cartesian coordinates as:

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \quad (2.12)$$

Strains are the derivatives of the displacement components, i.e.  $\varepsilon_{ij} = \frac{1}{2}(\partial u_i / \partial j + \partial u_j / \partial i)$ , where  $i$  and  $j$  are either  $x$ ,  $y$  or  $z$ . According to the small deformation theory, only the sum of complementing Cartesian shear strain components  $\varepsilon_{ij}$  and  $\varepsilon_{ji}$  result in shear stress. This sum is denoted as the shear strain  $\gamma$ . Hence, instead of  $\varepsilon_{xy}$ ,  $\varepsilon_{yx}$ ,  $\varepsilon_{yz}$ ,  $\varepsilon_{zy}$ ,  $\varepsilon_{zx}$

and  $\varepsilon_{xz}$  the shear strain components  $\gamma_{xy}$ ,  $\gamma_{yz}$  and  $\gamma_{zx}$  are used respectively. Under the above conditions, strains are often written in vector notation, which involve only six different components:

$$\underline{\varepsilon} = \left( \varepsilon_{xx} \ \varepsilon_{yy} \ \varepsilon_{zz} \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{zx} \right)^T \quad (2.13)$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \quad (2.14a)$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y} \quad (2.14b)$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} \quad (2.14c)$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \quad (2.14d)$$

$$\gamma_{yz} = \varepsilon_{yz} + \varepsilon_{zy} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \quad (2.14e)$$

$$\gamma_{zx} = \varepsilon_{zx} + \varepsilon_{xz} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \quad (2.14f)$$

Similarly as for stresses, positive normal strain components refer to extension, whereas negative normal strain components indicate compression.

In the formulation of material models, where infinitesimal increments of strain are considered, these increments are represented by strain rates (with a dot above the strain symbol).

$$\dot{\underline{\varepsilon}} = \left( \dot{\varepsilon}_{xx} \ \dot{\varepsilon}_{yy} \ \dot{\varepsilon}_{zz} \ \dot{\gamma}_{xy} \ \dot{\gamma}_{yz} \ \dot{\gamma}_{zx} \right)^T \quad (2.15)$$

In analogy to the invariants of stress, it is also useful to define invariants of strain. A strain invariant that is often used is the volumetric strain,  $\varepsilon_v$ , which is defined as the sum of all normal strain components in a standard calculation according to small deformation theory:

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad (2.16)$$

In *Updated mesh* calculations the volumetric strain is calculated as:

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} + \varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{xx}\varepsilon_{zz} + \varepsilon_{yy}\varepsilon_{zz} + \varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz} \quad (2.17)$$

The volumetric strain is defined as negative for compaction and as positive for dilatancy.

Another invariant is the deviatoric strain ( $\varepsilon_q$ ), which is calculated as:

$$\varepsilon_q = \sqrt{\frac{2}{9} [(\varepsilon_{xx} - \varepsilon_{yy})^2 + (\varepsilon_{yy} - \varepsilon_{zz})^2 + (\varepsilon_{zz} - \varepsilon_{xx})^2] + \frac{1}{3} (\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)} \quad (2.18)$$

For triaxial conditions i.e.  $\varepsilon_2 = \varepsilon_3$  the deviatoric strain, reduces to:

$$\varepsilon_q = \frac{2}{3} |\varepsilon_1 - \varepsilon_3| \quad (2.19)$$

where  $\varepsilon_1$  and  $\varepsilon_3$  are the major and minor principal strains components respectively. Furthermore, when volumetric strains are negligible ( $\varepsilon_v = 0$ ), then  $\varepsilon_3 = -\frac{1}{2}\varepsilon_1$ , so  $\varepsilon_q = |\varepsilon_1|$

For elastoplastic models, as used in PLAXIS, strains are decomposed into elastic and plastic components:

$$\underline{\varepsilon} = \underline{\varepsilon}^e + \underline{\varepsilon}^p \quad (2.20)$$

Throughout this manual, the superscript *e* will be used to denote elastic strains and the superscript *p* will be used to denote plastic strains.

### 2.3 ELASTIC STRAINS

Material models for soil and rock are generally expressed as a relationship between infinitesimal increments of effective stress ('effective stress rates') and infinitesimal increments of strain ('strain rates'). This relationship may be expressed in the form:

$$\dot{\underline{\sigma}}' = \underline{M} \dot{\underline{\varepsilon}} \quad (2.21)$$

where  $\underline{M}$  is a material stiffness matrix. Note that in this type of approach, pore-pressures are explicitly excluded from the stress-strain relationship.

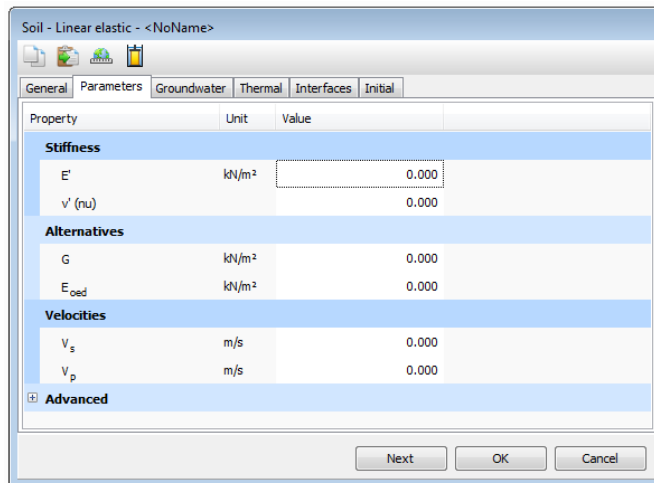


Figure 2.3 Parameters tabsheet for the Linear Elastic model

The simplest material model in PLAXIS is based on Hooke's law for isotropic linear elastic behaviour. This model is available under the name Linear Elastic model, but it is

also the basis of other models. Hooke's law can be given by the equation:

$$\begin{bmatrix} \dot{\sigma}'_{xx} \\ \dot{\sigma}'_{yy} \\ \dot{\sigma}'_{zz} \\ \dot{\sigma}'_{xy} \\ \dot{\sigma}'_{yz} \\ \dot{\sigma}'_{zx} \end{bmatrix} = \frac{E'}{(1 - 2\nu')(1 + \nu')} \begin{bmatrix} 1 - \nu' & \nu' & \nu' & 0 & 0 & 0 \\ \nu' & 1 - \nu' & \nu' & 0 & 0 & 0 \\ \nu' & \nu' & 1 - \nu' & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} - \nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} - \nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} - \nu' \end{bmatrix} \begin{bmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{zz} \\ \dot{\gamma}_{xy} \\ \dot{\gamma}_{yz} \\ \dot{\gamma}_{zx} \end{bmatrix} \quad (2.22)$$

The elastic material stiffness matrix is often denoted as  $\underline{D}^e$ . Two parameters are used in this model, the effective Young's modulus,  $E'$ , and the effective Poisson's ratio,  $\nu'$ . In the remaining part of this manual effective parameters are denoted without dash ('), unless a different meaning is explicitly stated. The symbols  $E$  and  $\nu$  are sometimes used in this manual in combination with the subscript *ur* to emphasize that the parameter is explicitly meant for unloading and reloading. A stiffness modulus may also be indicated with the subscript *ref* to emphasize that it refers to a particular reference level ( $y_{ref}$ )(see next page).

According to Hooke's law, the relationship between Young's modulus  $E$  and other stiffness moduli, such as the shear modulus  $G$ , the bulk modulus  $K$ , and the oedometer modulus  $E_{oed}$ , is given by:

$$G = \frac{E}{2(1 + \nu)} \quad (2.23a)$$

$$K = \frac{E}{3(1 - 2\nu)} \quad (2.23b)$$

$$E_{oed} = \frac{(1 - \nu)E}{(1 - 2\nu)(1 + \nu)} \quad (2.23c)$$

Using these auxiliary stiffness parameters, Hooke's law can be presented in an alternative form as:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{bmatrix} \dot{\epsilon}_v \\ \dot{\epsilon}_q \end{bmatrix} \quad (2.24)$$

During the input of material parameters for the Linear Elastic model or the Mohr-Coulomb model the values of  $G$  and  $E_{oed}$  are presented as auxiliary parameters (alternatives), calculated from Eq. (2.23). Note that the alternatives are influenced by the input values of  $E$  and  $\nu$ . Entering a particular value for one of the alternatives  $G$  or  $E_{oed}$  results in a change of the  $E$  modulus, while  $\nu$  remains the same.

It is possible for the Linear Elastic model and the Mohr-Coulomb model to specify a stiffness that varies linearly with depth. This can be done by entering a value for  $E_{inc}$  which is the increment of stiffness per unit of depth, as indicated in Figure 2.3.

Together with the input of  $E_{inc}$  the input of  $y_{ref}$  becomes relevant. Above  $y_{ref}$  the stiffness



is equal to  $E_{ref}$ . Below the stiffness is given by:

$$E(y) = E_{ref} + (y_{ref} - y) E_{inc} \quad y < y_{ref} \quad (2.25)$$

The Linear Elastic model is usually inappropriate to model the highly non-linear behaviour of soil, but it is of interest to simulate structural behaviour, such as thick concrete walls or plates, for which strength properties are usually very high compared with those of soil. For these applications, the Linear Elastic model will often be selected together with *Non-porous* drainage type in order to exclude pore pressures from these structural elements.

## 2.4 UNDRAINED EFFECTIVE STRESS ANALYSIS (EFFECTIVE STIFFNESS PARAMETERS)

In PLAXIS it is possible to specify undrained behaviour in an effective stress analysis using effective model parameters. This is achieved by identifying the type of material behaviour (*Drainage type*) of a soil layer as *Undrained (A)* or *Undrained (B)* (Section 2.5 and Section 2.6). In this section, it is explained how PLAXIS deals with this special option.

The presence of pore pressures in a soil body, usually caused by water, contributes to the total stress level. According to Terzaghi's principle, total stresses  $\underline{\sigma}$  can be divided into effective stresses  $\underline{\sigma}'$ , active pore pressure  $p_{active}$  and pore water pressures  $p_w$  (see also Eq. 2.3). However, water is supposed not to sustain any shear stress, and therefore the effective shear stresses are equal to the total shear stresses:

$$\underline{\sigma} = \underline{\sigma}' + m p_{active} \quad (2.26a)$$

where,

$$m = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad p_{active} = \alpha S_e p_w \quad (2.26b)$$

$$\sigma_{xx} = \sigma'_{xx} + \alpha S_e p_w \quad (2.26c)$$

$$\sigma_{yy} = \sigma'_{yy} + \alpha S_e p_w \quad (2.26d)$$

$$\sigma_{zz} = \sigma'_{zz} + \alpha S_e p_w \quad (2.26e)$$

$$\sigma_{xy} = \sigma'_{xy} \quad (2.26f)$$

$$\sigma_{yz} = \sigma'_{yz} \quad (2.26g)$$

$$\sigma_{zx} = \sigma'_{zx} \quad (2.26h)$$

where  $\alpha$  is Biot's pore pressure coefficient and  $S_e$  is the effective degree of saturation. Considering incompressible grains, Biot's coefficient  $\alpha$  is equal to unity ( $\alpha = 1$ ). The situation of compressible grains or compressible solid material ( $\alpha < 1$ ) is explained in more detail at the end of this section.

Note that, similar to the total and the effective stress components,  $p_w$  is considered negative for pressure.

The product  $\alpha S_e p_w$  is termed 'Active pore pressure',  $p_{active}$  in PLAXIS. A further distinction is made between steady state pore stress,  $p_{steady}$ , and excess pore stress,  $p_{excess}$ :

$$p_w = p_{steady} + p_{excess} \quad (2.27)$$

Steady state pore pressures are considered to be input data, i.e. generated on the basis of phreatic levels or by means of a groundwater flow calculation. Excess pore pressures are generated during plastic calculations for the case of undrained (A) or (B) material behaviour or during a consolidation analysis. Undrained material behaviour and the corresponding calculation of excess pore pressures are described below.

Since the time derivative of the steady state component equals zero, it follows:

$$\dot{p}_w = \dot{p}_{excess} \quad (2.28)$$

Hooke's law can be inverted to obtain:

$$\begin{bmatrix} \dot{\epsilon}_{xx}^e \\ \dot{\epsilon}_{yy}^e \\ \dot{\epsilon}_{zz}^e \\ \dot{\gamma}_{xy}^e \\ \dot{\gamma}_{yz}^e \\ \dot{\gamma}_{zx}^e \end{bmatrix} = \frac{1}{E'} \begin{bmatrix} 1 & -\nu' & -\nu' & 0 & 0 & 0 \\ -\nu' & 1 & -\nu' & 0 & 0 & 0 \\ -\nu' & -\nu' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 + 2\nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 + 2\nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 + 2\nu' \end{bmatrix} \begin{bmatrix} \dot{\sigma}'_{xx} \\ \dot{\sigma}'_{yy} \\ \dot{\sigma}'_{zz} \\ \dot{\sigma}'_{xy} \\ \dot{\sigma}'_{yz} \\ \dot{\sigma}'_{zx} \end{bmatrix} \quad (2.29)$$

Substituting Eq. (2.26) gives:

$$\begin{bmatrix} \dot{\epsilon}_{xx}^e \\ \dot{\epsilon}_{yy}^e \\ \dot{\epsilon}_{zz}^e \\ \dot{\gamma}_{xy}^e \\ \dot{\gamma}_{yz}^e \\ \dot{\gamma}_{zx}^e \end{bmatrix} = \frac{1}{E'} \begin{bmatrix} 1 & -\nu' & -\nu' & 0 & 0 & 0 \\ -\nu' & 1 & -\nu' & 0 & 0 & 0 \\ -\nu' & -\nu' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu' \end{bmatrix} \begin{bmatrix} \dot{\sigma}_{xx} - \alpha \dot{p}_w \\ \dot{\sigma}_{yy} - \alpha \dot{p}_w \\ \dot{\sigma}_{zz} - \alpha \dot{p}_w \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix} \quad (2.30)$$

Considering slightly compressible water, the rate of excess pore pressure is written as:

$$\dot{p}_{excess} = \frac{\alpha \dot{\epsilon}_v}{nC_w + (\alpha - n)C_s} \quad (2.31a)$$

$$C_w = \frac{1}{K_w} \quad (2.31b)$$

$$C_s = \frac{1}{K_s} \quad (2.31c)$$

in which  $K_w$  is the bulk modulus of the water,  $K_s$  is the bulk modulus of the solid material,  $C_w$  is the compressibility of the water,  $C_s$  is the compressibility of the solid material and  $n$  is the soil porosity.

$$n = \frac{e_0}{1 + e_0} \quad (2.32)$$

where  $e_0$  is the initial void ratio as specified in the general soil properties.

The inverted form of Hooke's law may be written in terms of the total stress rates and the undrained parameters  $E_u$  and  $\nu_u$ :

$$\begin{bmatrix} \dot{\epsilon}_{xx}^e \\ \dot{\epsilon}_{yy}^e \\ \dot{\epsilon}_{zz}^e \\ \dot{\gamma}_{xy}^e \\ \dot{\gamma}_{yz}^e \\ \dot{\gamma}_{zx}^e \end{bmatrix} = \frac{1}{E_u} \begin{bmatrix} 1 & -\nu_u & -\nu_u & 0 & 0 & 0 \\ -\nu_u & 1 & -\nu_u & 0 & 0 & 0 \\ -\nu_u & -\nu_u & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu_u & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu_u \end{bmatrix} \begin{bmatrix} \dot{\sigma}_{xx} \\ \dot{\sigma}_{yy} \\ \dot{\sigma}_{zz} \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix} \quad (2.33)$$

where:

$$E_u = 2G(1 + \nu_u); \quad \nu_u = \frac{3\nu' + \alpha B(1 - 2\nu')}{3 - \alpha B(1 - 2\nu')}; \quad B = \frac{\alpha}{\alpha + n \left( \frac{K'}{K_w} + \alpha - 1 \right)} \quad (2.34)$$

where  $B$  is Skempton's B-parameter.

Hence, the special option for undrained behaviour in PLAXIS (*Undrained (A)* or

*Undrained (B)* is such that the effective parameters  $G$  and  $\nu'$  are transformed into undrained parameters  $E_u$  and  $\nu_u$  according to Eq. (2.34). Note that the index  $u$  is used to indicate auxiliary parameters for undrained soil. Hence,  $E_u$  and  $\nu_u$  should not be confused with  $E_{ur}$  and  $\nu_{ur}$  as used to denote unloading / reloading.

Fully incompressible behaviour is obtained for  $\nu_u = 0.5$ . However, taking  $\nu_u = 0.5$  leads to singularity of the stiffness matrix. In fact, water is not fully incompressible, but a realistic bulk modulus for water is very large. In order to avoid numerical problems caused by an extremely low compressibility,  $\nu_u$  is, by default, taken as 0.495, which makes the undrained soil body slightly compressible. In order to ensure realistic computational results, the bulk modulus of the water must be high compared with the bulk modulus of the soil skeleton, i.e.  $K_w \gg nK'$ . This condition is sufficiently ensured by requiring  $\nu' \leq 0.35$ .

Consequently, for material behaviour *Undrained (A)* or *Undrained (B)*, a bulk modulus for water is automatically added to the stiffness matrix. The bulk modulus of water is obtained in two ways: either automatically from Eq. (2.35), or manually as specified by the user. For  $\alpha < 1$ , manual input of  $K_w$  is always required.

$$\frac{K_w}{n} = \frac{3(\nu_u - \nu')}{(1 - 2\nu_u)(1 + \nu')} K' = \frac{0.495 - \nu'}{1 + \nu'} 300K' \geq 30K' \quad (\text{For } \alpha = 1) \quad (2.35)$$

Hence,  $K_w/n$  is larger than  $30K'$ , at least for  $\nu' \leq 0.35$  and  $\alpha = 1$ . The bulk stiffness of water  $K_w$ , calculated in this way, is a numerical value related to the soil stiffness. It is lower than or equal to the real bulk stiffness of pure water,  $K_w^0$  ( $2 \cdot 10^6$  kN/m<sup>2</sup>). In retrospect it is worth mentioning here a review about the Skempton B-parameter.

### Skempton B-parameter

When the *Drainage type* is set to *Undrained (A)* or *Undrained (B)*, PLAXIS automatically assumes an implicit undrained bulk modulus,  $K_u$ , for the soil as a whole (soil skeleton + water) and distinguishes between total stress rates, effective stress rates and rates of excess pore pressure:

$$\text{Total stress:} \quad \dot{p} = K_u \dot{\epsilon}_v$$

$$\text{Excess pore pressure:} \quad \dot{p}_{\text{excess}} = B\dot{p} = \frac{\alpha \dot{\epsilon}_v}{nC_w + (\alpha - n)C_s}$$

$$\text{Effective stress:} \quad \dot{p}' = (1 - \alpha B)\dot{p} = K' \dot{\epsilon}_v$$

Note that for *Undrained (A)* or *Undrained (B)* effective stiffness parameters should be entered in the material data set, i.e.  $E'$  and  $\nu'$  and not  $E_u$  and  $\nu_u$ , or the respective stiffness parameters in advanced models; the latter should be done for *Undrained (C)* behaviour in a total stress analysis (Section 2.7). The undrained bulk modulus is automatically calculated by PLAXIS using Hooke's law of elasticity:

$$K_u = \frac{2G(1 + \nu_u)}{3(1 - 2\nu_u)} \quad \text{where } G = \frac{E'}{2(1 + \nu')}$$

When using the Standard setting,  $\nu_u = 0.495$  and  $\alpha = 1$ , whereas when using the Manual

setting with input of Skempton's B-parameter and Biot's pore pressure coefficient,  $\nu_u$  is calculated as:

$$\nu_u = \frac{3\nu' + \alpha B(1 - 2\nu')}{3 - \alpha B(1 - 2\nu')}$$

If the value of Skempton's B-parameter is unknown, but the degree of saturation, S is known instead, and automatic determination of  $K_w$  is selected, the bulk stiffness of the pore fluid can be estimated from:

$$K_w = \frac{K_w^0 K_{air}}{S K_{air} + (1 - S) K_w^0} \quad (2.36)$$

Where  $K_w^0$  ( $2 \cdot 10^6$  kN/m<sup>2</sup>) is the bulk modulus of pure water and  $K_{air}$  is the bulk modulus of air which is about 100 kPa under atmospheric pressure.

The value of Skempton's B-parameter is calculated from the ratio of the bulk stiffnesses of the soil skeleton and the pore fluid, as already defined in Eq. (2.34):

$$B = \frac{\alpha}{\alpha + n \left( \frac{K'}{K_w} + \alpha - 1 \right)}$$

The rate of excess pore pressure is calculated from the (small) volumetric strain rate, according to:

$$\dot{p}_{excess} = \frac{\alpha \dot{\epsilon}_v}{nC_w + (\alpha - n)C_s} \quad (2.37)$$

The types of elements used in PLAXIS are sufficiently adequate to avoid mesh locking effects for nearly incompressible materials.

This special option to model undrained material behaviour on the basis of effective model parameters is available for most material models in PLAXIS. This enables undrained calculations to be executed with effective stiffness parameters, with explicit distinction between effective stresses and (excess) pore pressures. However, shear induced (excess) pore pressure may not be sufficiently included.

Such an analysis requires effective soil parameters and is therefore highly convenient when such parameters are available. For soft soil projects, accurate data on effective parameters may not always be available. Instead, in situ tests and laboratory tests may have been performed to obtain undrained soil parameters. In such situations measured undrained Young's moduli can be easily converted into effective Young's moduli based on Hooke's law:

$$E' = \frac{2(1 + \nu')}{3} E_u \quad (2.38)$$

For advanced models there is no such direct conversion possible. In that case it is recommended to estimate the required effective stiffness parameter from the measured undrained stiffness parameter, then perform a simple undrained test to check the resulting undrained stiffness and adapt the effective stiffness if needed. The *Soil test* facility (Reference Manual) may be used as a convenient tool to perform such test.

**Biot pore pressure coefficient  $\alpha$** 

In general, for geotechnical applications, the compressibility of the soil skeleton is much higher than the compressibility of the individual grains, so deformations of the grains themselves can be ignored. However, in the case of very deep soil layers at very high pressures, the stiffness of the soil or rock matrix comes close to the stiffness of the solid material of which the soil grains or the rock is composed of, and, therefore, the compressibility of the solid material cannot be ignored. This has consequences for the division of total stress into effective stress and pore pressure. Considering compressible solid material, Terzaghi's effective stress definition changes into:

$$\underline{\sigma}' = \underline{\sigma} - \alpha S_e \underline{m} p_w \quad (2.39)$$

Where  $\alpha$  is Biot's pore pressure coefficient,  $S_e$  is the effective degree of saturation,  $\underline{m}$  is a vector with unity values (1) for the normal components and 0-values for the shear components, and  $p_w$  is the pore water pressure. The alpha coefficient is defined as:

$$\alpha = 1 - \frac{K'}{K_s} \quad (2.40)$$

Where  $K'$  is the effective bulk modulus of the soil matrix and  $K_s$  is the bulk modulus of the solid material. Indeed, for incompressible solid material ( $K_s = \infty$ ) Terzaghi's original stress definition is retained. A lower value of  $\alpha$  implies that for a given value of total stress and pore water pressure, the resulting effective stress is higher than when considering incompressible solid material ( $\alpha = 1$ ).

In the case of undrained soil behaviour (*Undrained A* or *B* in PLAXIS), Biot's pore pressure coefficient also affects the undrained Poisson's ratio  $\nu_u$  that is automatically calculated by PLAXIS based on a manual input of  $K_w$  parameter (see Eq. (2.34)).

The default value of Biot's pore pressure coefficient is 1.0 (incompressible grains), but users may change this value in material data sets for soil and interfaces in the range [0.001, 1.0].

**2.5 UNDRAINED EFFECTIVE STRESS ANALYSIS WITH EFFECTIVE STRENGTH PARAMETERS (*UNDRAINED A*)**

In principle, undrained effective stress analysis as described in Section 2.4 can be used in combination with effective strength parameters  $\varphi'$  and  $c'$  to model the material's undrained shear strength (*Undrained (A)*). In this case, the development of the pore pressure plays a crucial role in providing the right effective stress path that leads to failure at a realistic value of undrained shear strength ( $c_u$  or  $s_u$ ). However, note that most soil models are not capable of providing the right effective stress path in undrained loading. As a result, they will produce the wrong undrained shear strength if the material strength has been specified on the basis of effective strength parameters. Another problem is that for undrained materials effective strength parameters are usually not available from soil investigation data. In order to overcome these problems, some models allow for a direct input of undrained shear strength. This approach is described in Section 2.6.

If the user wants to model the material strength of undrained materials using the effective strength parameters  $\varphi'$  and  $c'$ , this can be done in PLAXIS in the same way as for

drained materials. However, in this case the *Drainage type* must be set to *Undrained (A)*. As a result, PLAXIS will automatically add the stiffness of water to the stiffness matrix (see Section 2.4) in order to distinguish between effective stresses and (excess) pore pressures (= effective stress analysis). The advantage of using effective strength parameters in undrained loading conditions is that after consolidation a qualitatively increased shear strength is obtained, although this increased shear strength could also be quantitatively wrong, for the same reason as explained before.

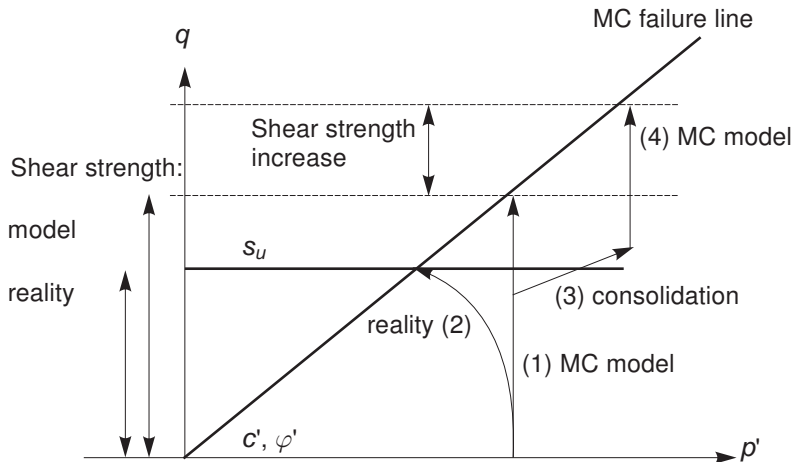


Figure 2.4 Illustration of stress paths; reality vs. Mohr-Coulomb model

Figure 2.4 illustrates an example using the Mohr-Coulomb model. When the *Drainage type* is set to *Undrained (A)*, the model will follow an effective stress path where the mean effective stress,  $p'$ , remains constant all the way up to failure (1). It is known that especially soft soils, like normally consolidated clays and peat, will follow an effective stress path in undrained loading where  $p'$  reduces significantly as a result of shear induced pore pressure (2). As a result, the maximum deviatoric stress that can be reached in the model is over-estimated in the Mohr-Coulomb model. In other words, the mobilized shear strength in the model supersedes the available undrained shear strength.

If, at some stress state, the soil is consolidated, the mean effective stress will increase (3). Upon further undrained loading with the Mohr-Coulomb model, the observed shear strength will be increased (4) compared to the previous shear strength, but this increased shear strength may again be unrealistic, especially for soft soils.

On the other hand, advanced models do include, to some extent, the reduction of mean effective stress in undrained loading, but even when using advanced models it is generally advised to check the mobilised shear strength in the Output program against the available (undrained) shear strength when this approach is followed.

Note that whenever the *Drainage type* parameter is set to *Undrained (A)*, effective values must be entered for the stiffness parameters (Young's modulus  $E'$  and Poisson ratio  $\nu'$  in case of the Mohr-Coulomb model or the respective stiffness parameters in the advanced models).

Care must be taken when using *Undrained (A)* together with a non-zero dilatancy angle



$\psi$ . The use of a positive dilatancy angle may lead to unrealistically large tensile pore stresses and, as a result, an unrealistically large shear strength. The use of a negative dilatancy angle may lead to unrealistically high pore pressure and unrealistic liquefaction type of behaviour. Hence, for *Undrained (A)* it is recommended to use  $\psi = 0$ .

## 2.6 UNDRAINED EFFECTIVE STRESS ANALYSIS WITH UNDRAINED STRENGTH PARAMETERS (*UNDRAINED B*)

For undrained soil layers with a known undrained shear strength profile, PLAXIS offers for some models the possibility of an undrained effective stress analysis, as described in Section 2.4, with direct input of the undrained shear strength, i.e. setting the friction angle to zero and the cohesion equal to the undrained shear strength ( $\varphi = \varphi_u = 0^\circ$ ;  $c = s_u$ ) (*Drainage type = Undrained (B)*). Also in this case, distinction is made between pore pressures and effective stresses. Although the pore pressures and effective stress path may not be fully correct, the resulting undrained shear strength is not affected, since it is directly specified as an input parameter.

The option to perform an undrained effective stress analysis with undrained strength properties is only available for the Mohr-Coulomb model, the Hardening Soil model, the HS small model and the NGI-ADP model. Since most soils show an increasing shear strength with depth, it is possible to specify the increase per unit of depth in PLAXIS in the *Advanced* subtree in the *Parameters* tabsheet of the *Soil* window.

Note that if the Hardening Soil model or the HS small model is used with  $\varphi = 0^\circ$ , the stiffness moduli in the model are no longer stress-dependent and the model exhibits no compression hardening, although the model retains its separate unloading-reloading modulus and shear hardening. Also note that a direct input of undrained shear strength does not automatically give the increase of shear strength with consolidation.

Also note that a direct input of undrained shear strength does not automatically give the increase of shear strength with consolidation.

Further note that whenever the *Drainage type* parameter is set to *Undrained (B)*, effective values must be entered for the stiffness parameters (Young's modulus  $E'$  and Poisson ratio  $\nu'$  in case of the Mohr-Coulomb model or the respective stiffness parameters in the advanced models).

## 2.7 UNDRAINED TOTAL STRESS ANALYSIS WITH UNDRAINED PARAMETERS (*UNDRAINED C*)

If, for any reason, it is desired not to use the *Undrained (A)* or *Undrained (B)* options in PLAXIS to perform an undrained effective stress analysis, one may simulate undrained behaviour using a conventional total stress analysis with all parameters specified as undrained. In that case, stiffness is modelled using an undrained Young's modulus  $E_u$  and an undrained Poisson ratio  $\nu_u$ , and strength is modelled using an undrained shear strength  $s_u$  and  $\varphi = \varphi_u = 0^\circ$ . Typically, for the undrained Poisson ratio a value close to 0.5 is selected (between 0.495 and 0.499). A value of 0.5 exactly is not possible, since this would lead to singularity of the stiffness matrix.

In PLAXIS it is possible to perform a total stress analysis with undrained parameters if the

Mohr-Coulomb model or the NGI-ADP model is used. In this case, one should select *Undrained (C)* as the *Drainage type*. The disadvantage of the undrained total stress analysis is that no distinction is made between effective stresses and pore pressures. Hence, all output referring to effective stresses should now be interpreted as total stresses and all pore pressures are equal to zero.

Note that a direct input of undrained shear strength does not automatically give the increase of shear strength with consolidation. In fact, it does not make sense to perform a consolidation analysis since there are no pore pressures to consolidate. Also note that the  $K_0$ -value to generate initial stresses refers to total stresses rather than effective stresses in this case. This type of approach is not possible for most advanced models.

### ***Overview of models and allowable drainage types***

Material model	Drainage type
Linear Elastic model	Drained
	Undrained (A)
	Undrained (C)
	Non-porous
Mohr-Coulomb model	Drained
	Undrained (A)
	Undrained (B)
	Undrained (C)
Hardening Soil model	Non-porous
	Drained
	Undrained (A)
HS small model	Undrained (B)
	Drained
	Undrained (A)
Soft Soil model	Undrained (B)
	Drained
Soft Soil Creep model	Undrained (A)
	Drained
Jointed Rock model	Undrained (A)
	Drained
	Non-porous

Modified Cam-Clay model	Drained Undrained (A)
NGI-ADP model	Drained Undrained (B) Undrained (C)
Hoek-Brown model	Drained Non-porous
Sekiguchi-Ohta model	Drained Undrained (A)
User-defined soil models	Drained Undrained (A) Non-porous

## 2.8 THE INITIAL PRE-CONSOLIDATION STRESS IN ADVANCED MODELS

When using advanced models in PLAXIS an initial pre-consolidation stress has to be determined. In the engineering practice it is common to use a vertical pre-consolidation stress,  $\sigma_p$ , but PLAXIS needs an equivalent isotropic pre-consolidation stress,  $p_p^{eq}$  to determine the initial position of a cap-type yield surface. If a material is over-consolidated, information is required about the Over-Consolidation Ratio (OCR), i.e. the ratio of the greatest effective vertical stress previously reached,  $\sigma_p$  (see Figure 2.5), and the in-situ effective vertical stress,  $\sigma_{yy}^0$  (note that in PLAXIS 3D the vertical (effective) stress is  $\sigma_{zz}^0$ ).

$$OCR = \frac{\sigma_p}{\sigma_{yy}^0} \tag{2.41}$$

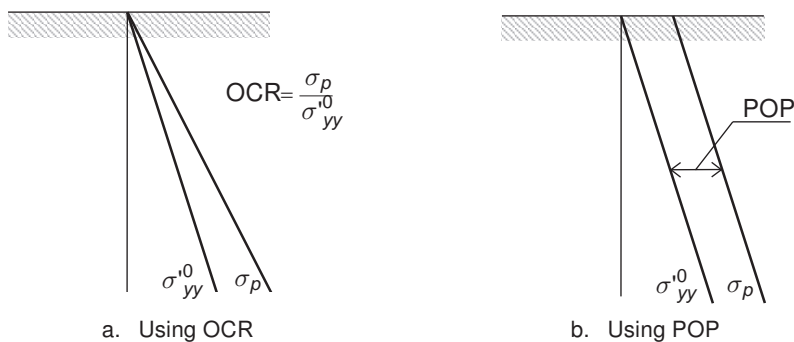


Figure 2.5 Illustration of vertical pre-consolidation stress in relation to the in-situ vertical stress

It is also possible to specify the initial stress state using the Pre-Overburden Pressure (POP) as an alternative to prescribing the over-consolidation ratio. The Pre-Overburden

Pressure is defined by:

$$POP = |\sigma_p - \sigma_{yy}^0| \quad (2.42)$$

These two ways of specifying the vertical pre-consolidation stress are illustrated in Figure 2.5.

The vertical pre-consolidation stress  $\sigma_p$  is used to compute the equivalent isotropic pre-consolidation stress,  $p_p^{eq}$ , which determines the initial position of a cap-type yield surface in the advanced soil models. The calculation of  $p_p^{eq}$  is based on the stress state:

$$\sigma'_{11} = \sigma_p \quad \text{and} \quad \sigma'_{22} = \sigma'_{33} = K_0^{nc} \sigma_p \quad (2.43)$$

Where  $K_0^{nc}$  is the  $K_0$ -value associated with normally consolidated states of stress, which is an input parameter for the advanced soil models except for the Modified Cam-Clay model.

For the Modified Cam-Clay model,  $K_0^{nc}$  is automatically determined based on the parameter  $M$  as entered by the user. The exact relationship between  $M$  and  $K_0^{nc}$  can be formulated as (Brinkgreve, 1994):

$$M = 3 \sqrt{\frac{(1 - K_0^{nc})^2}{(1 + 2K_0^{nc})^2} + \frac{(1 - K_0^{nc})(1 - 2\nu_{ur})(\lambda^*/\kappa^* - 1)}{(1 + 2K_0^{nc})(1 - 2\nu_{ur})\lambda^*/\kappa^* - (1 - K_0^{nc})(1 + \nu_{ur})}} \quad (2.44)$$

For more details about the Modified Cam-Clay model see Chapter 10.

The stress state at pre-consolidation stress level is expressed in (p, q):

$$p' = -\frac{1}{3}(1 + 2K_0^{nc})\sigma_p \quad \text{and} \quad q = (1 - K_0^{nc})\sigma_p \quad (2.45)$$

Depending on the model used, the equivalent isotropic pre-consolidation stress is calculated as:

Hardening Soil model and Hardening Soil model with small-strain stiffness:

$$p_p^{eq} = \sqrt{(p')^2 + \frac{q^2}{\alpha^2}} \quad (\text{where } \alpha \text{ is an internal model parameter}) \quad (2.46)$$

Soft Soil model, Soft Soil Creep model and Modified Cam-Clay model:

$$p_p^{eq} = p' + \frac{q^2}{M^2(p' + c \cot \varphi)} \quad (\text{where } M \text{ is an internal model parameter}) \quad (2.47)$$

Note that *OCR* and *POP* are only taken into account in the  $K_0$ -procedure (initial calculation phase). Gravity loading does not consider *OCR* or *POP*, and always gives a normally-consolidated stress state. If an advanced soil model (involving pre-consolidation stress) is activated in a later calculation phase, i.e.

- The corresponding soil cluster is activated for the first time, or
- The material data of a soil is changed from a 'simple' soil model without

pre-consolidation stress to an 'advanced' soil model with the pre-consolidation stress as a state parameter,

then the stress state at the beginning of that phase is assumed to be normally-consolidated, i.e. the pre-consolidation stress is initiated in accordance with the current stress state. If, in such a case, an over-consolidated stress state is to be modelled, the over-consolidation has to be simulated by applying and removing an overburden load.

## 2.9 ON THE INITIAL STRESSES

In overconsolidated soils the coefficient of lateral earth pressure for the initial stress state is larger than for normally consolidated soils. This effect is automatically taken into account for advanced soil models when generating the initial stresses using the  $K_0$ -procedure. The procedure that is followed here is described below. The procedure is described for the lateral stress in x-direction ( $\sigma'_{xx}$  based on  $K_{0,x}$ ), but a similar procedure is followed for the lateral stress in z-direction ( $\sigma'_{zz}$  based on  $K_{0,z}$ ).

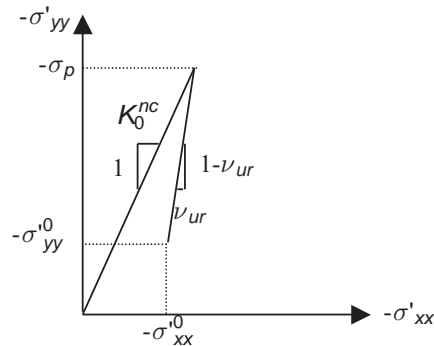


Figure 2.6 Overconsolidated stress state obtained from primary loading and subsequent unloading

Consider a one-dimensional compression test, preloaded to  $\sigma'_{yy} = \sigma_p$  and subsequently unloaded to  $\sigma'_{yy} = \sigma'_{yy}^0$ . During unloading the sample behaves elastically and the incremental stress ratio is, according to Hooke's law, given by (see Figure 2.6):

$$\frac{\Delta\sigma'_{xx}}{\Delta\sigma'_{yy}} = \frac{K_0^{nc}\sigma_p - \sigma'_{xx}^0}{\sigma_p - \sigma'_{yy}^0} = \frac{K_0^{nc}OCR\sigma'_{yy}^0 - \sigma'_{xx}^0}{(OCR - 1)\sigma'_{yy}^0} = \frac{\nu_{ur}}{1 - \nu_{ur}} \quad (2.48)$$

where  $K_0^{nc}$  is the stress ratio in the normally consolidated state. Hence, the default stress ratio of the overconsolidated soil sample is given by:

$$K_{0,x} = \frac{\sigma'_{xx}^0}{\sigma'_{yy}^0} = K_0^{nc}OCR - \frac{\nu_{ur}}{1 - \nu_{ur}}(OCR - 1) \quad (2.49)$$

When using POP as an alternative way to define overconsolidation, the default initial

horizontal stress is defined by:

$$\sigma_{xx}^0 = K_0^{nc} \sigma_p - \frac{\nu_{ur}}{1 - \nu_{ur}} POP \quad (2.50)$$

The use of a small Poisson's ratio will lead to a relatively large ratio of lateral stress and vertical stress, as generally observed in overconsolidated soils. Note that Eq. (2.49) and Eq. (2.50) are only valid in the elastic domain, because the formulas are derived from Hooke's law of elasticity. If a soil sample is unloaded by a large amount, resulting in a high degree of over-consolidation, the stress ratio will be limited by the Mohr-Coulomb failure condition.

Note that the above initial stress ratio's are only suggested (default) values, and may be overruled by the user if more precise data are available or if other values seem more appropriate.





### 3 LINEAR ELASTIC PERFECTLY PLASTIC MODEL (MOHR-COULOMB MODEL)

Soils behave rather non-linear when subjected to changes of stress or strain. In reality, the stiffness of soil depends at least on the stress level, the stress path and the strain level. Some such features are included in the advanced soil models in PLAXIS. The Mohr-Coulomb model however, is a simple and well-known linear elastic perfectly plastic model, which can be used as a first approximation of soil behaviour. The linear elastic part of the Mohr-Coulomb model is based on Hooke's law of isotropic elasticity (Section 3.1). The perfectly plastic part is based on the Mohr-Coulomb failure criterion, formulated in a non-associated plasticity framework.

Plasticity involves the development of irreversible strains. In order to evaluate whether or not plasticity occurs in a calculation, a yield function,  $f$ , is introduced as a function of stress and strain. Plastic yielding is related with the condition  $f = 0$ . This condition can often be presented as a surface in principal stress space. A perfectly-plastic model is a constitutive model with a fixed yield surface, i.e. a yield surface that is fully defined by model parameters and not affected by (plastic) straining. For stress states represented by points within the yield surface, the behaviour is purely elastic and all strains are reversible.

#### 3.1 LINEAR ELASTIC PERFECTLY-PLASTIC BEHAVIOUR

The basic principle of elastoplasticity is that strains and strain rates are decomposed into an elastic part and a plastic part:

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^p \qquad \underline{\underline{\dot{\varepsilon}}} = \underline{\underline{\dot{\varepsilon}}}^e + \underline{\underline{\dot{\varepsilon}}}^p \qquad (3.1)$$

Hooke's law is used to relate the stress rates to the elastic strain rates. Substitution of Eq. (3.1) into Hooke's law Eq. (2.22) leads to:

$$\underline{\underline{\dot{\sigma}'}} = \underline{\underline{D}}^e \underline{\underline{\dot{\varepsilon}}}^e = \underline{\underline{D}}^e (\underline{\underline{\dot{\varepsilon}}} - \underline{\underline{\dot{\varepsilon}}}^p) \qquad (3.2)$$

According to the classical theory of plasticity (Hill, 1950), plastic strain rates are proportional to the derivative of the yield function with respect to the stresses. This means that the plastic strain rates can be represented as vectors perpendicular to the yield surface. This classical form of the theory is referred to as associated plasticity. However, for Mohr-Coulomb type yield functions, the theory of associated plasticity overestimates dilatancy. Therefore, in addition to the yield function, a plastic potential function  $g$  is introduced. The case  $g \neq f$  is denoted as non-associated plasticity. In general, the plastic strain rates are written as:

$$\underline{\underline{\dot{\varepsilon}}}^p = \lambda \frac{\partial g}{\partial \underline{\underline{\sigma}'}} \qquad (3.3)$$

in which  $\lambda$  is the plastic multiplier. For purely elastic behaviour  $\lambda$  is zero, whereas in the

case of plastic behaviour  $\lambda$  is positive:

$$\lambda = 0 \quad \text{for:} \quad f < 0 \quad \text{or:} \quad \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{D}^e \dot{\underline{\epsilon}} \leq 0 \quad (\text{Elasticity}) \quad (3.4a)$$

$$\lambda > 0 \quad \text{for:} \quad f = 0 \quad \text{and:} \quad \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{D}^e \dot{\underline{\epsilon}} > 0 \quad (\text{Plasticity}) \quad (3.4b)$$

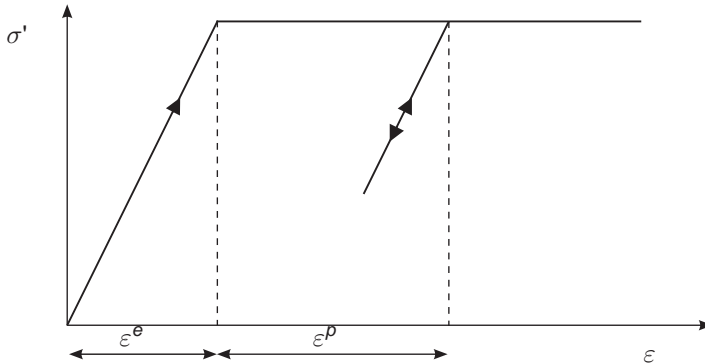


Figure 3.1 Basic idea of an elastic perfectly plastic model

These equations may be used to obtain the following relationship between the effective stress rates and strain rates for elastic perfectly-plastic behaviour (Smith & Griffiths, 1982; Vermeer & Borst, 1984):

$$\dot{\underline{\sigma}}' = \left( \underline{D}^e - \frac{\alpha}{d} \underline{D}^e \frac{\partial g}{\partial \underline{\sigma}'} \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{D}^e \right) \dot{\underline{\epsilon}} \quad (3.5a)$$

where:

$$d = \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{D}^e \frac{\partial g}{\partial \underline{\sigma}'} \quad (3.5b)$$

The parameter  $\alpha$  is used as a switch. If the material behaviour is elastic, as defined by Eq. (3.4a), the value of  $\alpha$  is equal to zero, whilst for plasticity, as defined by Eq. (3.4b), the value of  $\alpha$  is equal to unity.

The above theory of plasticity is restricted to smooth yield surfaces and does not cover a multi surface yield contour as present in the full Mohr-Coulomb model. For such a yield surface the theory of plasticity has been extended by Koiter (1960) and others to account for flow vertices involving two or more plastic potential functions:

$$\dot{\underline{\epsilon}}^p = \lambda_1 \frac{\partial g_1}{\partial \underline{\sigma}'} + \lambda_2 \frac{\partial g_2}{\partial \underline{\sigma}'} + \dots \quad (3.6)$$

Similarly, several quasi independent yield functions ( $f_1, f_2, \dots$ ) are used to determine the magnitude of the multipliers ( $\lambda_1, \lambda_2, \dots$ ).

### 3.2 FORMULATION OF THE MOHR-COULOMB MODEL

The Mohr-Coulomb yield condition is an extension of Coulomb's friction law to general states of stress. In fact, this condition ensures that Coulomb's friction law is obeyed in any plane within a material element.

The full Mohr-Coulomb yield condition consists of six yield functions when formulated in terms of principal stresses (see for instance Smith & Griffiths, 1982):

$$f_{1a} = \frac{1}{2} (\sigma'_{2} - \sigma'_{3}) + \frac{1}{2} (\sigma'_{2} + \sigma'_{3}) \sin \varphi - c \cos \varphi \leq 0 \quad (3.7a)$$

$$f_{1b} = \frac{1}{2} (\sigma'_{3} - \sigma'_{2}) + \frac{1}{2} (\sigma'_{3} + \sigma'_{2}) \sin \varphi - c \cos \varphi \leq 0 \quad (3.7b)$$

$$f_{2a} = \frac{1}{2} (\sigma'_{3} - \sigma'_{1}) + \frac{1}{2} (\sigma'_{3} + \sigma'_{1}) \sin \varphi - c \cos \varphi \leq 0 \quad (3.7c)$$

$$f_{2b} = \frac{1}{2} (\sigma'_{1} - \sigma'_{3}) + \frac{1}{2} (\sigma'_{1} + \sigma'_{3}) \sin \varphi - c \cos \varphi \leq 0 \quad (3.7d)$$

$$f_{3a} = \frac{1}{2} (\sigma'_{1} - \sigma'_{2}) + \frac{1}{2} (\sigma'_{1} + \sigma'_{2}) \sin \varphi - c \cos \varphi \leq 0 \quad (3.7e)$$

$$f_{3b} = \frac{1}{2} (\sigma'_{2} - \sigma'_{1}) + \frac{1}{2} (\sigma'_{2} + \sigma'_{1}) \sin \varphi - c \cos \varphi \leq 0 \quad (3.7f)$$

The two plastic model parameters appearing in the yield functions are the well-known friction angle  $\varphi$  and the cohesion  $c$ . The condition  $f_i = 0$  for all yield functions together (where  $f_i$  is used to denote each individual yield function) represents a fixed hexagonal cone in principal stress space as shown in Figure 3.2.

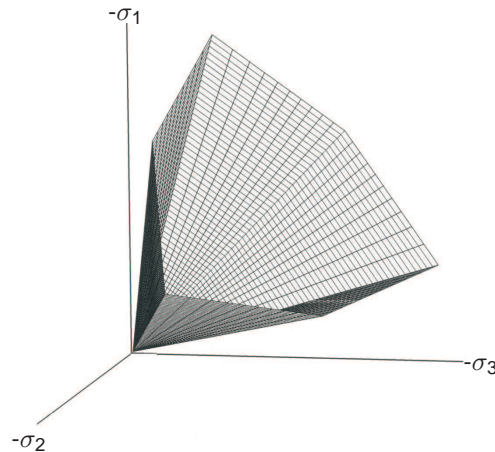


Figure 3.2 The Mohr-Coulomb yield surface in principal stress space ( $c = 0$ )

In addition to the yield functions, six plastic potential functions are defined for the Mohr-Coulomb model:

$$g_{1a} = \frac{1}{2} (\sigma'_{2} - \sigma'_{3}) + \frac{1}{2} (\sigma'_{2} + \sigma'_{3}) \sin \psi \quad (3.8a)$$

$$g_{1b} = \frac{1}{2} (\sigma'_3 - \sigma'_2) + \frac{1}{2} (\sigma'_3 + \sigma'_2) \sin \psi \quad (3.8b)$$

$$g_{2a} = \frac{1}{2} (\sigma'_3 - \sigma'_1) + \frac{1}{2} (\sigma'_3 + \sigma'_1) \sin \psi \quad (3.8c)$$

$$g_{2b} = \frac{1}{2} (\sigma'_1 - \sigma'_3) + \frac{1}{2} (\sigma'_1 + \sigma'_3) \sin \psi \quad (3.8d)$$

$$g_{3a} = \frac{1}{2} (\sigma'_1 - \sigma'_2) + \frac{1}{2} (\sigma'_1 + \sigma'_2) \sin \psi \quad (3.8e)$$

$$g_{3b} = \frac{1}{2} (\sigma'_2 - \sigma'_1) + \frac{1}{2} (\sigma'_2 + \sigma'_1) \sin \psi \quad (3.8f)$$

The plastic potential functions contain a third plasticity parameter, the dilatancy angle  $\psi$ . This parameter is required to model positive plastic volumetric strain increments (dilatancy) as actually observed for dense soils. A discussion of all of the model parameters used in the Mohr-Coulomb model is given in the next section.

When implementing the Mohr-Coulomb model for general stress states, special treatment is required for the intersection of two yield surfaces. Some programs use a smooth transition from one yield surface to another, i.e. the rounding-off of the corners (see for example Smith & Griffiths, 1982). In PLAXIS, however, the exact form of the full Mohr-Coulomb model is implemented, using a sharp transition from one yield surface to another. For a detailed description of the corner treatment the reader is referred to the literature (Koiter, 1960; van Langen & Vermeer, 1990).

For  $c > 0$ , the standard Mohr-Coulomb criterion allows for tension. In fact, allowable tensile stresses increase with cohesion. In reality, soil can sustain none or only very small tensile stresses. This behaviour can be included in a PLAXIS analysis by specifying a tension cut-off. In this case, Mohr circles with positive principal stresses are not allowed. The tension cut-off introduces three additional yield functions, defined as:

$$f_4 = \sigma'_1 - \sigma_t \leq 0 \quad (3.9a)$$

$$f_5 = \sigma'_2 - \sigma_t \leq 0 \quad (3.9b)$$

$$f_6 = \sigma'_3 - \sigma_t \leq 0 \quad (3.9c)$$

When this tension cut-off procedure is used, the allowable tensile stress,  $\sigma_t$ , is, by default, taken equal to zero, but this value can be changed by the user. For these three yield functions an associated flow rule is adopted.

For stress states within the yield surface, the behaviour is elastic and obeys Hooke's law for isotropic linear elasticity, as discussed in Section 2.3. Hence, besides the plasticity parameters  $c$ ,  $\varphi$ , and  $\psi$ , input is required on the elastic Young's modulus  $E$  and Poisson's ratio  $\nu$ . The model described here is officially called the linear elastic perfectly plastic model with Mohr-Coulomb failure criterion. For simplicity, this model is called the Mohr-Coulomb model in PLAXIS.

### 3.3 BASIC PARAMETERS OF THE MOHR-COULOMB MODEL

The linear elastic perfectly-plastic Mohr-Coulomb model requires a total of five parameters, which are generally familiar to most geotechnical engineers and which can be obtained from basic tests on soil samples. These parameters with their standard units are listed below:

$E$	: Young's modulus	[kN/m <sup>2</sup> ]
$\nu$	: Poisson's ratio	[-]
$c$	: Cohesion	[kN/m <sup>2</sup> ]
$\varphi$	: Friction angle	[°]
$\psi$	: Dilatancy angle	[°]
$\sigma_t$	: Tension cut-off and tensile strength	[kN/m <sup>2</sup> ]

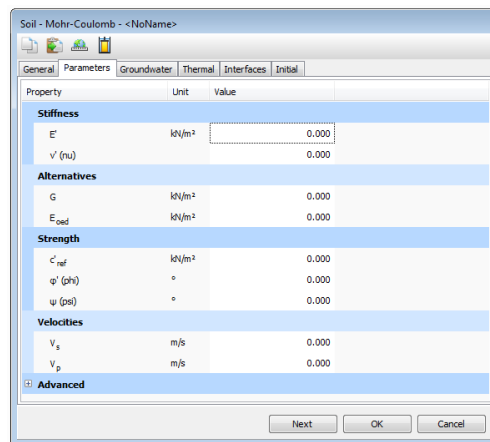


Figure 3.3 Parameter tabsheet for Mohr-Coulomb model

Instead of using the Young's modulus as a stiffness parameter, alternative stiffness parameters can be entered (Section 2.3). These parameters with their standard units are listed below:

$G$	: Shear modulus	[kN/m <sup>2</sup> ]
$E_{oed}$	: Oedometer modulus	[kN/m <sup>2</sup> ]

Parameters can either be effective parameters (indicated by a prime sign (')) or undrained parameters (indicated by a subscript u), depending on the selected drainage type.



In the case of dynamic applications, alternative and/or additional parameters may be used to define stiffness based on wave velocities. These parameters are listed below:

$V_p$	: Compression wave velocity	[m/s]
$V_s$	: Shear wave velocity	[m/s]

### **Young's modulus ( $E$ )**

PLAXIS uses the Young's modulus as the basic stiffness modulus in the elastic model and the Mohr-Coulomb model, but some alternative stiffness moduli are displayed as well. A stiffness modulus has the dimension of stress. The values of the stiffness parameter adopted in a calculation require special attention as many geomaterials show a non-linear behaviour from the very beginning of loading. In triaxial testing of soil samples the initial slope of the stress-strain curve (tangent modulus) is usually indicated as  $E_0$  and the secant modulus at 50% strength is denoted as  $E_{50}$  (see Figure 3.4). For materials with a large linear elastic range it is realistic to use  $E_0$ , but for loading of soils one generally uses  $E_{50}$ . Considering unloading problems, as in the case of tunnelling and excavations, one needs an unload-reload modulus ( $E_{ur}$ ) instead of  $E_{50}$ .

For soils, both the unloading modulus,  $E_{ur}$ , and the first loading modulus,  $E_{50}$ , tend to increase with the confining pressure. Hence, deep soil layers tend to have greater stiffness than shallow layers. Moreover, the observed stiffness depends on the stress path that is followed. The stiffness is much higher for unloading and reloading than for primary loading. Also, the observed soil stiffness in terms of a Young's modulus may be lower for (drained) compression than for shearing. Hence, when using a constant stiffness modulus to represent soil behaviour one should choose a value that is consistent with the stress level and the stress path development. Note that some stress-dependency of soil behaviour is taken into account in the advanced models in PLAXIS which are described in subsequent chapters. For the Mohr-Coulomb model, PLAXIS offers a special option for the input of a stiffness increasing with depth (see Section 3.4). Note that for material data sets where the drainage type is set to *Undrained (A)* or *Undrained (B)*, Young's modulus has the meaning of an effective Young's modulus, whilst PLAXIS automatically takes care of the incompressibility (Section 2.4).

### **Poisson's ratio ( $\nu$ )**

Standard drained triaxial tests may yield a significant rate of volume decrease at the very beginning of axial loading and, consequently, a low initial value of Poisson's ratio ( $\nu_0$ ). For some cases, such as particular unloading problems, it may be realistic to use such a low initial value, but in general when using the Mohr-Coulomb model the use of a higher value is recommended.

The selection of a Poisson's ratio is particularly simple when the elastic model or Mohr-Coulomb model is used for gravity loading under conditions of one-dimensional compression. For this type of loading PLAXIS should give realistic ratios of  $K_0 = \sigma_h' / \sigma_v'$ .

As both models will give the well-known ratio of  $\sigma_h' / \sigma_v' = \nu / (1 - \nu)$  for one-dimensional compression it is easy to select a Poisson's ratio that gives a realistic value of  $K_0$ . Hence,  $\nu$  is evaluated by matching  $K_0$ . In many cases one will obtain  $\nu$  values in the range between 0.3 and 0.4. In general, such values can also be used for loading conditions other than one-dimensional compression. Please note that in this way it is not possible to create  $K_0$  values larger than 1, as may be observed in highly overconsolidated stress

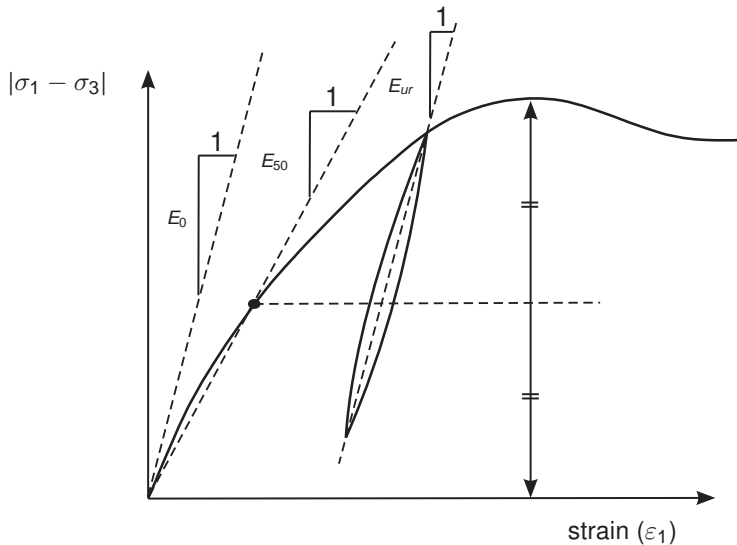


Figure 3.4 Definition of  $E_0$ ,  $E_{50}$  and  $E_{ur}$  for drained triaxial test results

states. For unloading conditions, however, it is more appropriate to use values in the range between 0.15 and 0.25.

Further note that for material data sets where the drainage type is set to *Undrained (A)* or *Undrained (B)*, Poisson's ratio has the meaning of an effective Poisson's ratio, whilst PLAXIS automatically takes care of the incompressibility (Section 2.4). To ensure that the soil skeleton is much more compressible than the pore water, the effective Poisson's ratio should be smaller than 0.35 for *Undrained (A)* or *Undrained (B)* materials.

### **Cohesion ( $c$ ) or undrained shear strength ( $s_u$ )**

The cohesive strength has the dimension of stress. In the Mohr-Coulomb model, the cohesion parameter may be used to model the effective cohesion  $c'$  of the soil (cohesion intercept), in combination with a realistic effective friction angle  $\varphi'$  (see Figure 3.5a). This may not only be done for drained soil behaviour, but also if the type of material behaviour is set to *Undrained (A)*, as in both cases PLAXIS will perform an effective stress analysis. Alternatively, the cohesion parameter may be used to model the undrained shear strength  $s_u$  of the soil, in combination with  $\varphi = \varphi_u = 0$  when the *Drainage type* is set to *Undrained (B)* or *Undrained (C)*. In that case the Mohr-Coulomb failure criterion reduces to the well-known Tresca criterion. PLAXIS allows for an increase of shear strength with depth using the  $s_{u,inc}$  parameter (Section 3.4).

The disadvantage of using effective strength parameters  $c'$  and  $\varphi'$  in combination with the drainage type being set to *Undrained (A)* is that the undrained shear strength as obtained from the model may deviate from the undrained shear strength in reality because of differences in the actual stress path being followed. In this respect, advanced soil models generally perform better than the Mohr-Coulomb model, but in all cases it is recommended to compare the resulting stress state in all calculation phases with the present shear strength in reality ( $|\sigma_1 - \sigma_3| \leq 2s_u$ ).

On the other hand, the advantage of using effective strength parameters is that the

change in shear strength with consolidation is obtained automatically, although it is still recommended to check the resulting stress state after consolidation.

The advantage of using the cohesion parameter to model undrained shear strength in combination with  $\varphi = 0$  (*Undrained (B)* or *Undrained (C)*) is that the user has direct control over the shear strength, independent of the actual stress state and stress path followed. Please note that this option may not be appropriate when using advanced soil models.

PLAXIS can handle cohesionless sands ( $c = 0$ ), but some options may not perform well. To avoid complications, non-experienced users are advised to enter at least a small value in soil layers near the ground surface (use  $c > 0.2$  kPa). Please note that a positive value for the cohesion may lead to a tensile strength, which may be unrealistic for soils. By default, the *Tension cut-off* option is used to reduce the tensile strength.

PLAXIS offers a special option for the input of layers in which the cohesion increases with depth (see Section 3.4).

### Friction angle ( $\varphi$ )

The friction angle  $\varphi$  (phi) is entered in degrees. In general the friction angle is used to model the effective friction of the soil, in combination with an effective cohesion  $c'$  (Figure 3.5a). This may not only be done for drained soil behaviour, but also if the type of material behaviour is set to *Undrained (A)*, since in both cases PLAXIS will perform an effective stress analysis. Alternatively, the soil strength is modelled by setting the cohesion parameter equal to the undrained shear strength of the soil, in combination with  $\varphi = 0$  (*Undrained (B)* or *Undrained (C)*) (Figure 3.5b). In that case the Mohr-Coulomb failure criterion reduces to the well-known Tresca criterion.

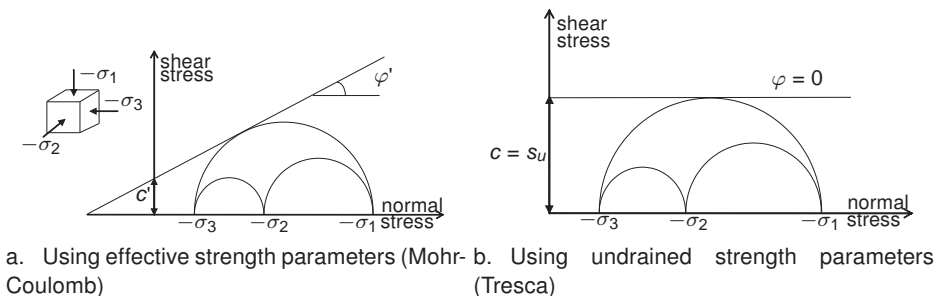


Figure 3.5 Stress circles at yield; one touches Coulomb's envelope.

High friction angles, as sometimes obtained for dense sands, will substantially increase plastic computational effort. Moreover, high friction may be subjected to strain-softening behaviour, which means that such high friction angles are not sustainable under (large) deformation. Hence, high friction angles should be avoided when performing preliminary computations for a particular project \*. The friction angle largely determines the shear strength as shown in Figure 3.5 by means of Mohr's stress circles. A more general representation of the yield criterion is shown in Figure 3.6. The Mohr-Coulomb failure criterion proves to be better for describing soil strength for general stress states than the

\* Typical effective friction angles are in the order of 20 - 30 degrees for clay and silt (the more plastic the clay, the lower the friction), and 30 - 40 degrees for sand and gravel (the denser the sand, the higher the friction).



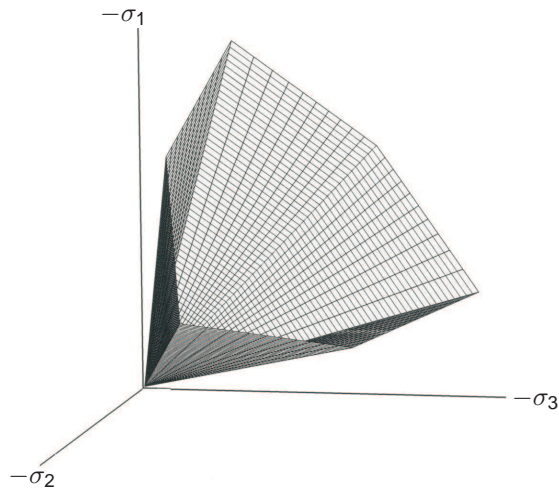


Figure 3.6 Failure surface in principal stress space for cohesionless soil

Drucker-Prager approximation.

### ***Dilatancy angle ( $\psi$ )***

The dilatancy angle,  $\psi$  (psi), is specified in degrees. Apart from heavily over-consolidated layers, clay soils tend to show little dilatancy ( $\psi \approx 0$ ). The dilatancy of sand depends on both the density and the friction angle. In general the dilatancy angle of soils is much smaller than the friction angle. For quartz sands the order of magnitude is  $\psi \approx \varphi - 30^\circ$ . For  $\varphi$ -values of less than  $30^\circ$ , however, the angle of dilatancy is mostly zero. A small negative value for  $\psi$  is only realistic for extremely loose sands. In the Hardening Soil model or Hardening Soil model with small-strain stiffness the end of dilatancy, as generally observed when the soil reaches the critical state, can be modelled using the *Dilatancy cut-off*. However, this option is not available for the Mohr-Coulomb model. For further information about the link between the friction angle and dilatancy, see Bolton (1986).

A positive dilatancy angle implies that in drained conditions the soil will continue to dilate as long as shear deformation occurs. This is clearly unrealistic, as most soils will reach a critical state at some point and further shear deformation will occur without volume changes. In undrained conditions a positive dilatancy angle, combined with the restriction on volume changes, leads to a generation of tensile pore stresses. In an undrained effective stress analysis therefore the strength of the soil may be overestimated.

When the soil strength is modelled as undrained shear strength,  $s_u$ , and  $\varphi = 0$ , (*Undrained (B)* or *Undrained (C)*) the dilatancy angle is automatically set to zero. Great care must be taken when using a positive value of dilatancy in combination with drainage type set to *Undrained (A)*. In that case the model will show unlimited soil strength due to tensile pore stresses. These tensile pore stresses can be limited by setting the cavitation cut-off.

### Shear modulus ( $G$ )

The shear modulus,  $G$ , has the dimension of stress. According to Hooke's law, the relationship between Young's modulus  $E$  and the shear modulus is given by (see Eq. 2.23a):

$$G = \frac{E}{2(1 + \nu)} \quad (3.10)$$

Entering a particular value for one of the alternatives  $G$  or  $E_{oed}$  results in a change of the  $E$  modulus whilst  $\nu$  remains the same.

### Oedometer modulus ( $E_{oed}$ )

The oedometer modulus,  $E_{oed}$ , or constrained modulus, has the dimension of stress. According to Hooke's law, the relationship between Young's modulus  $E$  and the oedometer modulus is given by (see Eq. 2.23c).

$$E_{oed} = \frac{(1 - \nu)E}{(1 - 2\nu)(1 + \nu)} \quad (3.11)$$

Entering a particular value for one of the alternatives  $G$  or  $E_{oed}$  results in a change of the  $E$  modulus whilst  $\nu$  remains the same.



### Compression wave velocity $V_p$

The compression wave velocity,  $V_p$ , in a confined one-dimensional soil is a function of stiffness,  $E_{oed}$ , and the mass density,  $\rho$ , as:

$$V_p = \sqrt{\frac{E_{oed}}{\rho}} \quad \text{where} \quad E_{oed} = \frac{(1 - \nu)E}{(1 + \nu)(1 - 2\nu)} \quad \text{and} \quad \rho = \frac{\gamma_{unsat}}{g} \quad (3.12)$$

in which  $\gamma_{unsat}$  is the total unsaturated unit weight and  $g$  is the gravity acceleration (9.8 m/s<sup>2</sup>).



### Shear wave velocity $V_s$

The shear wave velocity,  $V_s$ , in a confined one-dimensional soil is a function of shear stiffness,  $G$ , and the mass density,  $\rho$ , as:

$$V_s = \sqrt{\frac{G}{\rho}} \quad \text{where} \quad G = \frac{E}{2(1 + \nu)} \quad \text{and} \quad \rho = \frac{\gamma_{unsat}}{g} \quad (3.13)$$

in which  $\gamma_{unsat}$  is the total unsaturated unit weight and  $g$  is the gravity acceleration (9.8 m/s<sup>2</sup>).

## 3.4 ADVANCED PARAMETERS OF THE MOHR-COULOMB MODEL

The advanced features comprise the increase of stiffness and cohesive strength with depth and the use of a tension cut-off. In fact, the latter option is used by default, but it may be deactivated here, if desired. These parameters are defined in the *Advanced* subtree in the *Parameters* tabsheet of the *Soil* window.

### ***Increase of stiffness ( $E_{inc}$ )***

In real soils, the stiffness depends significantly on the stress level, which means that the stiffness generally increases with depth. When using the Mohr-Coulomb model, the stiffness is a constant value. In order to account for the increase of the stiffness with depth the  $E_{inc}$ -value may be used, which is the increase of the Young's modulus per unit of depth (expressed in the unit of stress per unit depth). At the level given by the  $y_{ref}$  parameter, and above, the stiffness is equal to the reference Young's modulus,  $E_{ref}$ , as entered in the *Parameters* tabsheet. Below, the stiffness is given by:

$$E(y) = E_{ref} + (y_{ref} - y)E_{inc} \quad (y < y_{ref}) \quad (3.14)$$

where  $y$  represents the vertical direction. The actual value of Young's modulus in the stress points is obtained from the reference value and  $E_{inc}$ . Note that during calculations a stiffness increasing with depth does not change as a function of the stress state.

**Hint:** Note that in PLAXIS 3D the vertical coordinate is  $z$  instead of  $y$ .

### ***Increase of cohesion or shear strength with depth ( $C_{inc}$ or $S_{u,inc}$ )***

PLAXIS offers an advanced option for the input of clay layers in which the cohesion,  $c$ , (or undrained shear strength,  $s_u$ ) increases with depth. In order to account for the increase of the cohesion with depth the  $C_{inc}$ -value may be used, which is the increase of cohesion per unit of depth (expressed in the unit of stress per unit depth). At the level given by the  $y_{ref}$  parameter, and above, the cohesion is equal to the (reference) cohesion,  $C_{ref}$ , as entered in the *Parameters* tabsheet. Below, the cohesive strength is given by:

$$c(y) = C_{ref} + (y_{ref} - y)C_{inc} \quad (y < y_{ref}) \quad (3.15a)$$

$$s_u(y) = S_{u,ref} + (y_{ref} - y)S_{u,inc} \quad (y < y_{ref}) \quad (3.15b)$$

where  $y$  represents the vertical direction. Note that when using effective strength properties ( $\varphi' > 0$ ) it is generally not necessary to use an increase of cohesion with depth, since the friction together with the initial effective stress will result in an increasing shear strength with depth.

### ***Tension cut-off***

In some practical problems an area with tensile stresses may develop. According to the Coulomb envelope shown in Figure 3.5 this is allowed when the shear stress (radius of Mohr circle) is sufficiently small. However, the soil surface near a trench in clay sometimes shows tensile cracks. This indicates that soil may also fail in tension instead of in shear. Such behaviour can be included in a PLAXIS analysis by selecting the tension cut-off. In this case Mohr circles with positive principal stresses are not allowed. When selecting the tension cut-off the allowable tensile strength may be entered. For the Mohr-Coulomb model the tension cut-off is, by default, selected with a tensile strength of zero.

### 3.5 ON THE USE OF THE MOHR-COULOMB MODEL IN DYNAMIC CALCULATIONS

When using the Mohr-Coulomb model in dynamic calculations, the stiffness parameters need to be selected such that the model correctly predicts wave velocities in the soil (Equations (3.12) and (3.13)). This generally requires a much larger small strain stiffness rather than a stiffness at engineering strain levels. When subjected to dynamic or cyclic loading, the Mohr-Coulomb model may generate plastic strains if stress points reach the Mohr-Coulomb failure criterion, which will lead to damping in dynamic calculations. However, it should be noted that the stress cycles within the Mohr-Coulomb failure contour will only generate elastic strains and no (hysteretic) damping, nor accumulation of strains or pore pressure or liquefaction. In order to simulate the soil's damping characteristics in cyclic loading, Rayleigh damping may be defined.



## 4 THE HOEK-BROWN MODEL (ROCK BEHAVIOUR)

The material behaviour of rock differs from the behaviour of soils in the sense that it is generally stiffer and stronger. The dependency of the stiffness on the stress level is almost negligible, so stiffness of rocks can be considered constant. On the other hand, the dependency of the (shear) strength on the stress level is significant. In this respect, heavily jointed or weathered rock can be regarded a frictional material. A first approach is to model the shear strength of rock by means of the Mohr-Coulomb failure criterion. However, considering the large range of stress levels where rock may be subjected to, a linear stress-dependency, as obtained from the Mohr-Coulomb model, is generally not sufficient. Furthermore, rock may also show a significant tensile strength. The Hoek-Brown failure criterion is a better non-linear approximation of the strength of rocks. It involves shear strength as well as tensile strength in a continuous formulation. Together with Hooke's law of isotropic linear elastic behaviour it forms the Hoek-Brown model for rock behaviour. The 2002 edition of this model (Hoek, Carranza-Torres & Corkum, 2002) has been implemented in PLAXIS to simulate the isotropic behaviour of rock-type materials. The implementation of the model, including the material strength factorization, is based on Benz, Schwab, Vermeer & Kauter (2007). More background information on the Hoek-Brown model and the selection of model parameters can be found in Hoek (2006). For anisotropic behaviour of stratified rock reference is made to Chapter 5.

### 4.1 FORMULATION OF THE HOEK-BROWN MODEL

The generalized Hoek-Brown failure criterion can be formulated as a non-linear relationship between the major and minor effective principal stresses (considering tension positive and pressure negative):

$$\sigma_1' = \sigma_3' - \sigma_{ci} \left( m_b \frac{-\sigma_3'}{\sigma_{ci}} + s \right)^a \quad (4.1)$$

where  $m_b$  is a reduced value of the intact rock parameter  $m_i$ , which also depends on the Geological Strength Index (GSI) and the Disturbance Factor (D):

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (4.2)$$

$s$  and  $a$  are auxiliary material constants for the rock mass, that can be expressed as:

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (4.3)$$

$$a = \frac{1}{2} + \frac{1}{6} \left[ \exp\left(\frac{-GSI}{15}\right) - \exp\left(\frac{-20}{3}\right) \right] \quad (4.4)$$

$\sigma_{ci}$  is the uni-axial compressive strength of the intact rock material (defined as a positive value). From this value, the uni-axial compressive strength of the specific rock under consideration,  $\sigma_c$ , can be obtained by:

$$\sigma_c = \sigma_{ci} S^a \quad (4.5)$$

The tensile strength of the specific rock under consideration,  $\sigma_t$ , can be obtained by:

$$\sigma_t = \frac{S\sigma_{ci}}{m_b} \quad (4.6)$$

The Hoek-Brown failure criterion is illustrated in Figure 4.1.

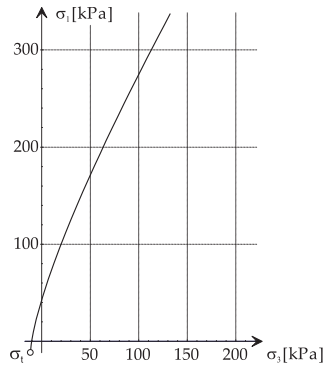


Figure 4.1 Hoek-Brown failure criterion in principal stresses

In the framework of plasticity theory, the Hoek-Brown failure criterion is reformulated into the following yield function:

$$f_{HB} = \sigma'_1 - \sigma'_3 + \bar{f}(\sigma'_3) \quad \text{where} \quad \bar{f}(\sigma'_3) = \sigma_{ci} \left( m_b \frac{-\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (4.7)$$

For general three-dimensional stress states, more than one yield function is required to deal with the corners of the yield contour, similar as for the full Mohr-Coulomb criterion. Defining pressure as negative and considering ordering of principal stresses such that  $\sigma'_1 \leq \sigma'_2 \leq \sigma'_3$ , the full criterion can be captured by two yield functions:

$$f_{HB,13} = \sigma'_1 - \sigma'_3 + \bar{f}(\sigma'_3) \quad \text{where} \quad \bar{f}(\sigma'_3) = \sigma_{ci} \left( m_b \frac{-\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (4.8a)$$

$$f_{HB,12} = \sigma'_1 - \sigma'_2 + \bar{f}(\sigma'_2) \quad \text{where} \quad \bar{f}(\sigma'_2) = \sigma_{ci} \left( m_b \frac{-\sigma'_2}{\sigma_{ci}} + s \right)^a \quad (4.8b)$$

The full Hoek-Brown failure contour ( $f_i = 0$ ) in principal stress space is illustrated in Figure 4.2.

In addition to the two yield functions, two corresponding plastic potential functions are defined for the Hoek-Brown model:

$$g_{HB,13} = S_1 - \left( \frac{1 + \sin \psi_{mob}}{1 - \sin \psi_{mob}} \right) S_3 \quad (4.9a)$$

$$g_{HB,12} = S_1 - \left( \frac{1 + \sin \psi_{mob}}{1 - \sin \psi_{mob}} \right) S_2 \quad (4.9b)$$

where  $S_i$  are the transformed stresses, defined as:

$$S_i = \frac{-\sigma_1}{m_b \sigma_{ci}} + \frac{s}{m_b^2} \quad \text{for} \quad i = 1, 2, 3 \quad (4.10)$$

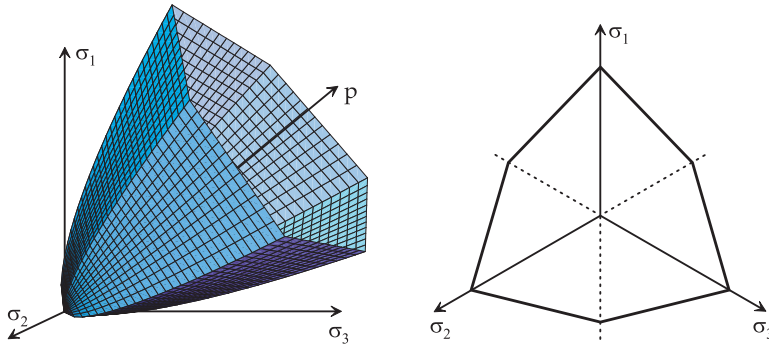


Figure 4.2 The Hoek-Brown failure contour in principal stress space

$\psi_{mob}$  is the mobilized dilatancy angle, varying with  $\sigma'_3$  from its input value at ( $\sigma'_3 = 0$ ) down to zero at  $-\sigma'_3 = \sigma_{\psi}$  and beyond:

$$\psi_{mob} = \frac{\sigma_{\psi} + \sigma'_3}{\sigma_{\psi}} \psi \geq 0 \quad (0 \geq -\sigma'_3 \geq \sigma_{\psi}) \quad (4.11)$$

Moreover, in order to allow for plastic expansion in the tensile zone, an increased artificial value of the mobilised dilatancy is used:

$$\psi_{mob} = \psi + \frac{\sigma'_3}{\sigma_t} (90^\circ - \psi) \quad (\sigma_t \geq \sigma'_3 \geq 0) \quad (4.12)$$

The evolution of the mobilized dilatancy angle as a function of  $\sigma'_3$  is visualized in Figure 4.3.

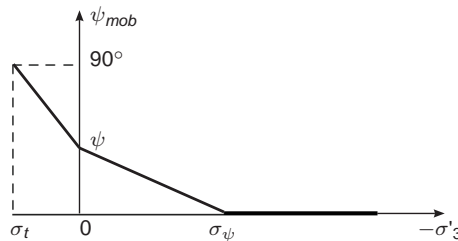


Figure 4.3 Evolution of mobilized dilatancy angle

Regarding the elastic behaviour of the Hoek-Brown model, Hooke's law of isotropic linear elastic behaviour, as described in Section 2.3, is adopted. This part of the model involves Young's modulus,  $E$ , representing the in-situ stiffness of the jointed rock mass before failure, and Poisson's ratio,  $\nu$ , describing transverse straining.

## 4.2 CONVERSION OF HOEK-BROWN TO MOHR-COULOMB

In order to compare the Hoek-Brown failure criterion with the well-known Mohr-Coulomb failure criterion for practical applications involving a particular stress range, a balanced fit can be made for confining stresses in the range (considering tension positive and

compression negative):

$$-\sigma_t < \sigma'_3 < -\sigma_{3,max}$$

This gives the following expressions for the Mohr-Coulomb effective strength parameters  $\varphi'$  and  $c'$  (Carranza-Torres, 2004):

$$\sin \varphi' = \frac{6 a m_b (s + m_b \sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6 a m_b (s + m_b \sigma'_{3n})^{a-1}} \quad (4.13)$$

$$c' = \frac{\sigma_{ci} [(1+2a)s + (1-a)m_b \sigma'_{3n}] (s + m_b \sigma'_{3n})^{a-1}}{(1+a)(2+a) \sqrt{1 + \frac{6 a m_b (s + m_b \sigma'_{3n})^{a-1}}{(1+a)(2+a)}}} \quad (4.14)$$

where  $\sigma'_{3n} = -\sigma'_{3,max} / \sigma_{ci}$ . The upper limit of the confining stress,  $\sigma'_{3,max}$ , depends on the application.

### 4.3 PARAMETERS OF THE HOEK-BROWN MODEL

The Hoek-Brown model involves a total of 8 parameters, which are generally familiar to geologists and mining engineers. These parameters with their standard units are listed below:

$E$	: Young's modulus	[kN/m <sup>2</sup> ]
$\nu$	: Poisson's ratio	[-]
$\sigma_{ci}$	: Uni-axial compressive strength of the intact rock (>0)	[kN/m <sup>2</sup> ]
$m_i$	: Intact rock parameter	[-]
$GSI$	: Geological Strength Index	[-]
$D$	: Disturbance factor	[-]
$\psi_{max}$	: Dilatancy angle (at $\sigma'_3 = 0$ )	[°]
$\sigma_\psi$	: Absolute value of confining pressure $\sigma'_3$ at which $\psi = 0^\circ$	[kN/m <sup>2</sup> ]

**Hint:** Note that it is common in rock mechanics to express  $E$ ,  $\sigma_{ci}$  and  $\sigma_\psi$  in the unit MPa (megaPascal = MN/m<sup>2</sup>), whereas the input values in PLAXIS are given in standard units as defined in the project properties.

#### **Young's modulus ( $E$ ):**

Young's modulus  $E$  is assumed to be a constant value for the considered rock layer. This modulus can be estimated on the basis of the rock quality parameters in the Hoek-Brown model (Hoek, Carranza-Torres & Corkum, 2002):

$$E \text{ (GPa)} = \left(1 - \frac{D}{2}\right) \sqrt{\frac{\sigma_{ci}}{p^{ref}}} \cdot 10^{\left(\frac{GSI - 10}{40}\right)} \quad (4.15)$$



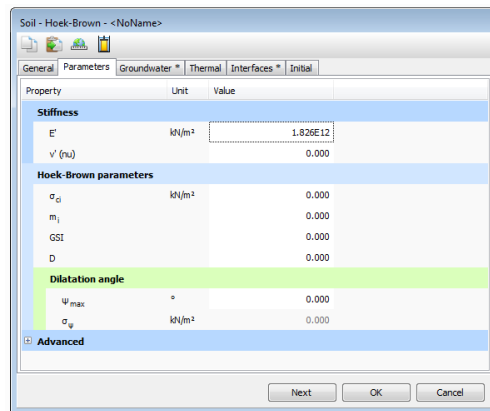


Figure 4.4 Parameter tabsheet for Hoek-Brown model

with  $p_{ref} = 10^5 \text{ kPa}$  and considering a maximum result of the square root of 1.

Note that the input of Young's modulus in PLAXIS is generally in  $\text{kN/m}^2$  ( $= \text{kPa} = 10^{-6} \text{ GPa}$ ), which means that the value obtained from the above formula must be multiplied by  $10^6$ .

In principle, Young's modulus can be measured from axial compression tests or direct shear tests on rock samples. However, this modulus is more applicable to the intact rock material, and should be reduced to obtain a representative stiffness of the in-situ rock mass. Note that, in contrast to most soils, the initial stiffness,  $E_i$ , is a good representation of the rock behaviour in the elastic range.

#### **Poisson's ratio ( $\nu$ ):**

Poisson's ratio,  $\nu$ , is generally in the range [0.1, 0.4]. Typical values for particular rock types are listed in Figure 4.5.

#### **Uni-axial compressive strength of intact rock ( $\sigma_{ci}$ ):**

The uni-axial strength of intact rock  $\sigma_{ci}$  can be determined in laboratory testing, e.g. axial compression. Laboratory testing is often conducted on intact rock so that ( $GSI = 100$  and  $D = 0$  hold). Typical values are shown in Table 4.1 (Hoek, 1999).

#### **Intact rock parameter ( $m_i$ ):**

The intact rock parameter is an empirical model parameter that depends on the rock type. Typical values are given in Figure 4.6.

#### **Geological Strength Index (GSI):**

The GSI parameter can be selected on the basis of the chart depicted in Figure 4.7.

#### **Disturbance factor ( $D$ ):**

The *Disturbance factor*,  $D$ , is a parameter that depends on the amount of disturbance of the rock as a result of mechanical processes in open excavations, tunnels or mines, such as blasting, tunnel boring, machine driven or manual excavation. No disturbance is

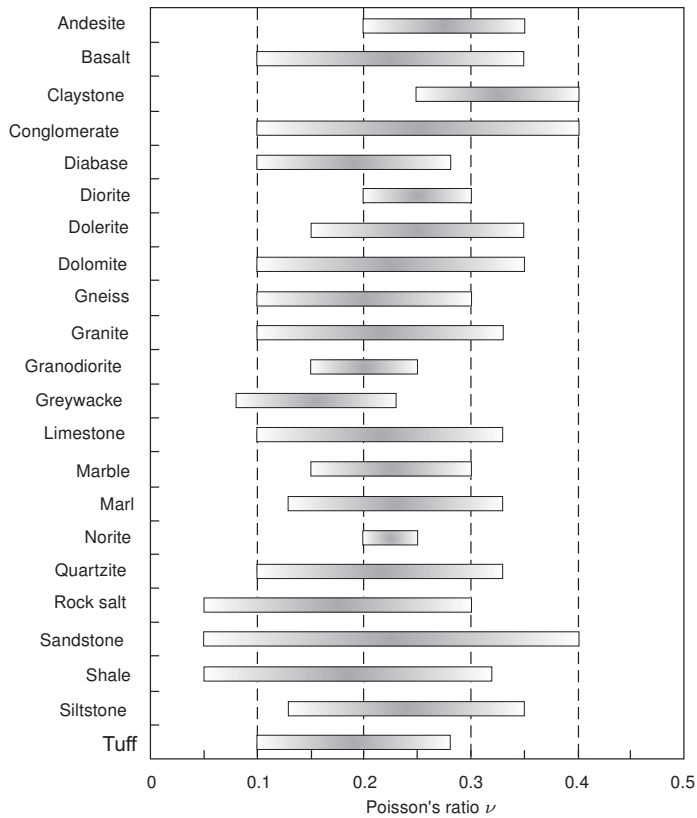


Figure 4.5 Typical Poisson's ratio values

Table 4.1 Intact uniaxial compressive strength

Grade	Term	Uniaxial Comp. Strength (MPa)	Field estimate of strength	Examples
R6	Extremely strong	> 250	Specimen can be chipped with a geological hammer	Fresh basalt, chert, diabase, gneiss, granite, quartzite
R5	Very strong	100 – 250	Specimen requires many blows of a geological hammer to fracture it	Amphibolite, sandstone, basalt, gabbro, gneiss, granodiorite, limestone, marble, rhyolite, tuff
R4	Strong	50 – 100	Specimen requires more than one blow of a geological hammer to fracture it	Limestone, marble, phyllite, sandstone, schist, shale
R3	Medium strong	25 – 50	Cannot be scraped or peeled with a pocket knife, specimen can be fractured with a single blow from a geological hammer.	Claystone, coal, concrete, schist, shale, siltstone
R2	Weak	5 – 25	Can be peeled with a pocket knife with difficulty, shallow indentation made by firm blow with point of a geological hammer.	Chalk, rocksalt, potash.
R1	Very weak	1 – 5	Crumbles under firm blows with point of a geological hammer, can be peeled by a pocket knife.	Highly weathered or altered rock.
R0	Extremely weak	0.25 – 1	Indented by thumbnail	Stiff fault gouge

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerate (20)	Sandstone 19 Greywacke (18)	Siltstone 9 Chalk 7	Claystone 4
		Organic		Coal (8-21)		
	Non - Clastic	Carbonate	Breccia (22)	Sparitic Limestone (10)	Micritic Limestone 8	
		Chemical		Gypstone 16	Anhydrite 13	
METAMORPHIC	Non Foliated		Marble 9	Hornfels (19)	Quartzite 24	
	Slightly foliated		Migmatite (30)	Amphibolite (25 - 31)	Mylonites (6)	
	Foliated*		Gneiss 33	Schists 4 - 8	Phyllites (10)	Slate 9
IGNEOUS	Light		Granite 33		Rhyolite (16)	
			Granodiorite (30)		Dacite (17)	
	Dark		Diorite (28)		Andesite 19	
		Gabbro 27 Norite 22	Dolerite (19)	Basalt (17)		
	Extrusive pyroclastic type	Agglomerate (20)	Breccia (18)	Tuff (15)		

Figure 4.6 Values for the constant  $m_i$  for intact rock after Hoek (1999)

GEOLOGICAL STRENGTH INDEX		SURFACE CONDITIONS				
<p>From the description of structure and surface conditions of the rock mass, pick an appropriate box in this chart. Estimate the average value of the Geological Strength Index (GSI) from the contours. Do not attempt to be too precise. Quoting a range of GSI from 36 to 42 is more realistic than stating that GSI = 38. It is also important to recognize that the Hoek-Brown criterion should only be applied to rock masses where the size of the individual blocks or pieces is small compared with the size of the excavation under consideration. When individual block sizes are more than approximately one quarter of the excavation dimension, failure will generally be structurally controlled and the Hoek-Brown criterion should not be used.</p>		VERY GOOD Very rough, fresh unweathered surfaces	GOOD Slightly weathered, iron stained surfaces	FAIR Smooth, moderately weathered and altered surfaces	POOR Stickensided, highly weathered surfaces with compact coatings or fillings of angular fragments	VERY POOR Stickensided, highly weathered surfaces with soft clay coatings or fillings
STRUCTURE		DECREASING SURFACE QUALITY				
	INTACT OR MASSIVE – intact rock specimens or massive in situ rock with very few widely spaced discontinuities	90		N/A	N/A	N/A
	BLOCKY - very well interlocked undisturbed rock mass consisting of cubical blocks formed by three orthogonal discontinuity sets	80	70			
	VERY BLOCKY - interlocked, partially disturbed rock mass with multifaceted angular blocks formed by four or more discontinuity sets		60	50		
	BLOCKY/DISTURBED - folded and/or faulted with angular blocks formed by many intersecting discontinuity sets			40	30	
	DISINTEGRATED - poorly interlocked, heavily broken rock mass with a mixture of angular and rounded rock pieces				20	
	FOLIATED/LAMINATED – Folded and tectonically sheared foliated rocks. Schistosity prevails over any other discontinuity set, resulting in complete lack of blockiness	N/A	N/A		10	5

Figure 4.7 Selection of Geological Strength Index, Hoek (1999)

equivalent to  $D = 0$ , whereas severe disturbance is equivalent to  $D = 1$ . For more information see Hoek (2006).

***Dilatancy ( $\psi$  and  $\sigma_{\psi}$ ):***

Rocks may show dilatant material behaviour when subjected to shear under relatively low confining stress. At larger confining stress, dilatancy is suppressed. This behaviour is modelled by means of a specified value of  $\psi$  for  $\sigma_3 = 0$ , with a linear decrease down to zero for  $\sigma'_3 = \sigma_{\psi}$ , where  $\sigma_{\psi}$  is an additional input parameter (Figure 4.3).

#### **4.4 ON THE USE OF THE HOEK-BROWN MODEL IN DYNAMIC CALCULATIONS**

When using the Hoek-Brown model in dynamic calculations, the stiffness need to be selected such that the model correctly predicts wave velocities in the soil (Equation (3.13)). When subjected to dynamic or cyclic loading, the Hoek-Brown model may generate plastic strains if stress points reach the Hoek-Brown failure criterion, which will lead to damping in dynamic calculations. However, it should be noted that the stress cycles within the Hoek-Brown failure contour will only generate elastic strains and no (hysteretic) damping, nor accumulation of strains or pore pressure or liquefaction. In order to simulate the rock's damping characteristics in cyclic loading, Rayleigh damping may be defined.

## 5 THE JOINTED ROCK MODEL (ANISOTROPY)

Materials may have different properties in different directions. As a result, they may respond differently when subjected to particular conditions in one direction or another. This aspect of material behaviour is called anisotropy. When modelling anisotropy, distinction can be made between elastic anisotropy and plastic anisotropy. Elastic anisotropy refers to the use of different elastic stiffness properties in different directions. Plastic anisotropy may involve the use of different strength properties in different directions, as considered in the Jointed Rock model. Another form of plastic anisotropy is kinematic hardening. The latter is not considered in PLAXIS.

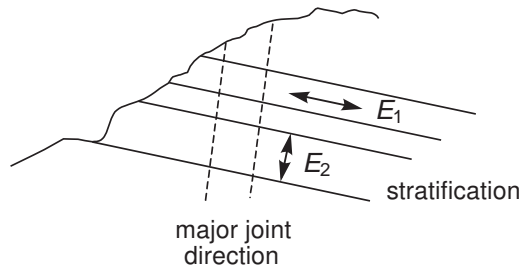


Figure 5.1 Visualization of concept behind the Jointed Rock model

The Jointed Rock model is an anisotropic elastic perfectly-plastic model, especially meant to simulate the behaviour of stratified and jointed rock layers. In this model it is assumed that there is intact rock with an optional stratification direction and major joint directions. The intact rock is considered to behave as a transversely anisotropic elastic material, quantified by five parameters and a direction. The anisotropy may result from stratification or from other phenomena. In the major joint directions it is assumed that shear stresses are limited according to Coulomb's criterion. Upon reaching the maximum shear stress in such a direction, plastic sliding will occur. A maximum of three sliding directions ('planes') can be defined, of which the first plane is assumed to coincide with the direction of elastic anisotropy. Each plane may have different shear strength properties. In addition to plastic shearing, the tensile stresses perpendicular to the three planes are limited according to a predefined tensile strength (tension cut-off).

The application of the Jointed Rock model is justified when families of joints or joint sets are present. These joint sets have to be parallel, not filled with fault gouge, and their spacing has to be small compared to the characteristic dimension of the structure.

Some basic characteristics of the Jointed Rock model are:

- Anisotropic elastic behaviour for intact rock                      Parameters  $E_1, E_2, \nu_1, \nu_2, G_2$
- Shear failure according to Coulomb in 3 directions,  $i$               Parameters  $c_i, \varphi_i$  and  $\psi_i$
- Limited tensile strength in three directions,  $i$                       Parameters  $\sigma_{t,i}$

## 5.1 ANISOTROPIC ELASTIC MATERIAL STIFFNESS MATRIX

The elastic material behaviour in the Jointed Rock model is described by an elastic material stiffness matrix,  $\underline{D}^*$ . In contrast to Hooke's law, the  $\underline{D}^*$ -matrix as used in the Jointed Rock model is transversely anisotropic. Different stiffnesses can be used normal to and in a predefined direction ('plane 1'). This direction may correspond to the stratification direction or to any other direction with significantly different elastic stiffness properties.

Consider, for example, a horizontal stratification, where the stiffness in z- direction,  $E_2$ , is different from the stiffness in the rock as a continuum,  $E_1$ . In this case the 'plane 1' direction is parallel to the x-yplane and the following constitutive relations exist (See: Zienkiewicz & Taylor: The Finite Element Method, 4th Ed.):

$$\dot{\epsilon}_{xx} = \frac{\dot{\sigma}_{xx}}{E_1} - \frac{\nu_1 \dot{\sigma}_{yy}}{E_1} - \frac{\nu_2 \dot{\sigma}_{zz}}{E_2} \quad (5.1a)$$

$$\dot{\epsilon}_{yy} = -\frac{\nu_1 \dot{\sigma}_{xx}}{E_1} + \frac{\dot{\sigma}_{yy}}{E_1} - \frac{\nu_2 \dot{\sigma}_{zz}}{E_2} \quad (5.1b)$$

$$\dot{\epsilon}_{zz} = -\frac{\nu_2 \dot{\sigma}_{xx}}{E_2} - \frac{\nu_2 \dot{\sigma}_{yy}}{E_2} + \frac{\dot{\sigma}_{zz}}{E_2} \quad (5.1c)$$

$$\dot{\gamma}_{xy} = \frac{2(1 + \nu_1)}{E_1} \sigma_{xy} \quad (5.1d)$$

$$\dot{\gamma}_{yz} = \frac{\dot{\sigma}_{yz}}{G_2} \quad (5.1e)$$

$$\dot{\gamma}_{zx} = \frac{\dot{\sigma}_{zx}}{G_2} \quad (5.1f)$$

The inverse of the anisotropic elastic material stiffness matrix,  $(\underline{D}^*)^{-1}$ , follows from the above relations. This matrix is symmetric. The regular material stiffness matrix  $\underline{D}^*$  can only be obtained by numerical inversion.

In general, the stratification plane will not be parallel to the global x-yplane, but the above relations will generally hold for a local ( $n,s,t$ ) coordinate system where the stratification plane is parallel to the  $s$ - $t$ -plane. The orientation of this plane is defined by the *dip angle* (or shortly *dip*) and *strike* (see Section 5.3). As a consequence, the local material stiffness matrix has to be transformed from the local to the global coordinate system. Therefore we consider first a transformation of stresses and strains:

$$\underline{\sigma}_{nst} = \underline{R}_\sigma \underline{\sigma}_{xyz} \quad \underline{\sigma}_{xyz} = \underline{R}_\sigma^{-1} \underline{\sigma}_{nst} \quad (5.2a)$$

$$\underline{\epsilon}_{nst} = \underline{R}_\epsilon \underline{\epsilon}_{xyz} \quad \underline{\epsilon}_{xyz} = \underline{R}_\epsilon^{-1} \underline{\epsilon}_{nst} \quad (5.2b)$$

where

$$\underline{\underline{R}}_{\sigma} = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & 2n_x n_y & 2n_y n_z & 2n_x n_z \\ s_x^2 & s_y^2 & s_z^2 & 2s_x s_y & 2s_y s_z & 2s_x s_z \\ t_x^2 & t_y^2 & t_z^2 & 2t_x t_y & 2t_y t_z & 2t_x t_z \\ n_x s_x & n_y s_y & n_z s_z & n_x s_y + n_y s_x & n_y s_z + n_z s_y & n_z s_x + n_x s_z \\ s_x t_x & s_y t_y & s_z t_z & s_x t_y + s_y t_x & s_y t_z + s_z t_y & s_x t_z + s_z t_x \\ n_x t_x & n_y t_y & n_z t_z & n_x t_y + n_y t_x & n_y t_z + n_z t_y & n_z t_x + n_x t_z \end{bmatrix} \quad (5.3)$$

and

$$\underline{\underline{R}}_{\varepsilon} = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & n_x n_y & n_y n_z & n_x n_z \\ s_x^2 & s_y^2 & s_z^2 & s_x s_y & s_y s_z & s_x s_z \\ t_x^2 & t_y^2 & t_z^2 & t_x t_y & t_y t_z & t_x t_z \\ 2n_x s_x & 2n_y s_y & 2n_z s_z & n_x s_y + n_y s_x & n_y s_z + n_z s_y & n_z s_x + n_x s_z \\ 2s_x t_x & 2s_y t_y & 2s_z t_z & s_x t_y + s_y t_x & s_y t_z + s_z t_y & s_x t_z + s_z t_x \\ 2n_x t_x & 2n_y t_y & 2n_z t_z & n_x t_y + n_y t_x & n_y t_z + n_z t_y & n_z t_x + n_x t_z \end{bmatrix} \quad (5.4)$$

$n_x, n_y, n_z, s_x, s_y, s_z, t_x, t_y$  and  $t_z$  are the components of the normalized  $n, s$  and  $t$ -vectors in global  $(x,y,z)$ -coordinates (i.e. 'sines' and 'cosines'; see Section 5.3). For plane strain condition  $n_z = s_z = t_x = t_y = 0$  and  $t_z = 1$ .

It further holds that:

$$\underline{\underline{R}}_{\varepsilon}^T = \underline{\underline{R}}_{\sigma}^{-1} \quad \underline{\underline{R}}_{\sigma}^T = \underline{\underline{R}}_{\varepsilon}^{-1} \quad (5.5)$$

A local stress-strain relationship in  $(n,s,t)$ -coordinates can be transformed to a global relationship in  $(x,y,z)$ -coordinates in the following way:

$$\left. \begin{aligned} \sigma_{nst} &= \underline{\underline{D}}_{nst}^* \varepsilon_{nst} \\ \sigma_{nst} &= \underline{\underline{R}}_{\sigma} \sigma_{xyz} \\ \varepsilon_{nst} &= \underline{\underline{R}}_{\varepsilon} \varepsilon_{xyz} \end{aligned} \right\} \Rightarrow \underline{\underline{R}}_{\sigma} \sigma_{xyz} = \underline{\underline{D}}_{nst}^* \underline{\underline{R}}_{\varepsilon} \varepsilon_{xyz} \quad (5.6)$$

Hence,

$$\sigma_{xyz} = \underline{\underline{R}}_{\sigma}^{-1} \underline{\underline{D}}_{nst}^* \underline{\underline{R}}_{\varepsilon} \varepsilon_{xyz} \quad (5.7)$$

Using to above condition (Eq. 5.5):

$$\sigma_{xyz} = \underline{\underline{R}}_{\varepsilon}^T \underline{\underline{D}}_{nst}^* \underline{\underline{R}}_{\varepsilon} \varepsilon_{xyz} = \underline{\underline{D}}_{xyz}^* \varepsilon_{xyz} \quad \text{or} \quad \underline{\underline{D}}_{xyz}^* = \underline{\underline{R}}_{\varepsilon}^T \underline{\underline{D}}_{nst}^* \underline{\underline{R}}_{\varepsilon} \quad (5.8)$$

Actually, not the  $D^*$ -matrix is given in local coordinates but the inverse matrix ( $\underline{D}^{*-1}$ ).

$$\left. \begin{aligned} \underline{\varepsilon}_{nst} &= \underline{D}_{nst}^{*-1} \underline{\sigma}_{nst} \\ \underline{\sigma}_{nst} &= \underline{R}_{\sigma} \underline{\sigma}_{xyz} \\ \underline{\varepsilon}_{nst} &= \underline{R}_{\varepsilon} \underline{\varepsilon}_{xyz} \end{aligned} \right\} \Rightarrow \underline{\varepsilon}_{xyz} = \underline{R}_{\varepsilon}^{-1} \underline{D}_{nst}^{*-1} \underline{R}_{\sigma} \underline{\sigma}_{xyz} = \underline{R}_{\sigma}^T \underline{D}_{nst}^{*-1} \underline{R}_{\sigma} \underline{\sigma}_{xyz} \quad (5.9)$$

Hence,

$$\underline{D}_{xyz}^{*-1} = \underline{R}_{\sigma}^T \underline{D}_{nst}^{*-1} \underline{R}_{\sigma} \quad \text{or} \quad \underline{D}_{xyz}^* = \left[ \underline{R}_{\sigma}^T \underline{D}_{nst}^{*-1} \underline{R}_{\sigma} \right]^{-1} \quad (5.10)$$

Instead of inverting the ( $\underline{D}_{nst}^*$ )<sup>-1</sup>-matrix in the first place, the transformation is considered first, after which the total is numerically inverted to obtain the global material stiffness matrix  $\underline{D}_{xyz}^*$ .

## 5.2 PLASTIC BEHAVIOUR IN THREE DIRECTIONS

A maximum of 3 sliding directions (sliding planes) can be defined in the Jointed Rock model. The first sliding plane corresponds to the direction of elastic anisotropy. In addition, a maximum of two other sliding directions may be defined. However, the formulation of plasticity on all planes is similar. On each plane a local Coulomb condition applies to limit the shear stress,  $|\tau|$ . Moreover, a tension cut-off criterion is used to limit the tensile stress on a plane. Each plane,  $i$ , has its own strength parameters  $c_i$ ,  $\varphi_i$ ,  $\psi_i$  and  $\sigma_{t,i}$ .

In order to check the plasticity conditions for a plane with local ( $n$ ,  $s$ ,  $t$ )-coordinates it is necessary to calculate the local stresses from the Cartesian stresses. The local stresses involve three components, i.e. a normal stress component,  $\sigma_n$ , and two independent shear stress components,  $\tau_s$  and  $\tau_t$ .

$$\underline{\sigma}_i = \underline{T}_i^T \underline{\sigma} \quad (5.11)$$

where

$$\underline{\sigma}_i = \left( \sigma_n \ \tau_s \ \tau_t \right)^T \quad (5.12a)$$

$$\underline{\sigma} = \left( \sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \sigma_{xy} \ \sigma_{yz} \ \sigma_{zx} \right)^T \quad (5.12b)$$

$$\underline{T}_i^T = \text{transformation matrix (3x6), for plane } i$$

As usual in PLAXIS, tensile (normal) stresses are defined as positive whereas compression is defined as negative.

Consider a plane strain situation as visualized in Figure 5.2. Here a sliding plane is considered under an angle  $\alpha_1$  (= *dip*) with respect to the  $x$ -axis. In this case the



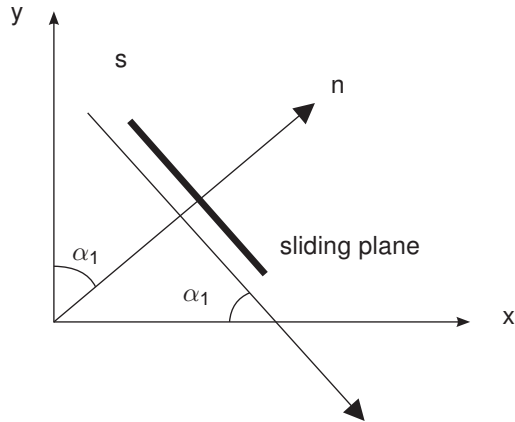


Figure 5.2 Plane strain situation with a single sliding plane and vectors  $n$ ,  $s$

transformation matrix  $\underline{\underline{T}}^T$  becomes:

$$\underline{\underline{T}}^T = \begin{bmatrix} s^2 & c^2 & 0 & -2sc & 0 & 0 \\ sc & -sc & 0 & -s^2 + c^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c & -s \end{bmatrix} \quad (5.13)$$

where

$$s = \sin \alpha_1$$

$$c = \cos \alpha_1$$

In the general three-dimensional case the transformation matrix is more complex, since it involves both *dip* and *strike* (see Section 5.3):

$$\underline{\underline{T}}^T = \begin{bmatrix} n_x^2 & n_y^2 & n_z^2 & 2n_x n_y & 2n_y n_z & 2n_z n_x \\ n_x s_x & n_y s_y & n_z s_z & n_x s_y + n_y s_x & n_z s_y + n_y s_z & n_z s_x + n_x s_z \\ n_x t_x & n_y t_y & n_z t_z & n_y t_x + n_x t_y & n_y t_z + n_z t_y & n_z t_x + n_x t_z \end{bmatrix} \quad (5.14)$$

Note that the general transformation matrix,  $\underline{\underline{T}}^T$ , for the calculation of local stresses corresponds to rows 1, 4 and 6 of  $\underline{\underline{R}}_{\sigma}$  (see Eq. 5.3).

After having determined the local stress components, the plasticity conditions can be checked on the basis of yield functions. The yield functions for plane  $i$  are defined as:

$$f_i^c = |\tau_s| + \sigma_n \tan \varphi_i - c_i \quad (\text{Coulomb}) \quad (5.15a)$$

$$f_i^t = \sigma_n - \sigma_{t,i} (\sigma_{t,i} \leq c_i \cot \varphi_i) \quad (\text{Tension cut-off}) \quad (5.15b)$$

Figure 5.3 visualizes the full yield criterion on a single plane.

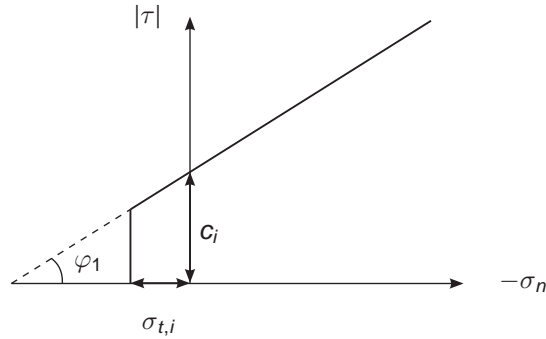


Figure 5.3 Yield criterion for individual plane

The local plastic strains are defined by:

$$\Delta \underline{\varepsilon}_j^p = \lambda_j \frac{\partial g_j^c}{\partial \underline{\sigma}_j} \quad (5.16)$$

where  $g_j$  is the local plastic potential function for plane  $j$ :

$$g_j^c = |\tau_j| + \sigma_n \tan \varphi_j - c_j \quad (\text{Coulomb}) \quad (5.17a)$$

$$g_j^t = \sigma_n - \sigma_{t,j} \quad (\text{Tension cut-off}) \quad (5.17b)$$

The transformation matrix,  $\underline{T}_j$ , is also used to transform the local plastic strain increments of plane  $j$ ,  $\Delta \underline{\varepsilon}_j^p$ , into global plastic strain increments,  $\Delta \underline{\varepsilon}^p$ :

$$\Delta \underline{\varepsilon}^p = \underline{T}_j \Delta \underline{\varepsilon}_j^p \quad (5.18)$$

The consistency condition requires that at yielding the value of the yield function must remain zero for all active yield functions. For all planes together, a maximum of 6 yield functions exist, so up to 6 plastic multipliers must be found such that all yield functions are at most zero and the plastic multipliers are non-negative.

$$f_i^c = f_i^{c(e)} - \sum_{j=1}^{np} \langle \lambda_j^c \rangle \frac{\partial f_i^{cT}}{\partial \underline{\sigma}} T_i^T D T_j \frac{\partial g_j^c}{\partial \underline{\sigma}} - \sum_{j=1}^{np} \langle \lambda_j^t \rangle \frac{\partial f_i^{tT}}{\partial \underline{\sigma}} T_i^T D T_j \frac{\partial g_j^t}{\partial \underline{\sigma}} \quad (5.19a)$$

$$f_i^t = f_i^{t(e)} - \sum_{j=1}^{np} \langle \lambda_j^c \rangle \frac{\partial f_i^{cT}}{\partial \underline{\sigma}} T_i^T D T_j \frac{\partial g_j^c}{\partial \underline{\sigma}} - \sum_{j=1}^{np} \langle \lambda_j^t \rangle \frac{\partial f_i^{tT}}{\partial \underline{\sigma}} T_i^T D T_j \frac{\partial g_j^t}{\partial \underline{\sigma}} \quad (5.19b)$$

This means finding up to 6 values of  $\lambda_j \geq 0$  such that all  $f_i \leq 0$  and  $\lambda_j f_i = 0$ .

When the maximum of 3 planes are used, there are  $2^6 = 64$  possibilities of (combined) yielding. In the calculation process, all these possibilities are taken into account in order to provide an exact calculation of stresses.

### 5.3 PARAMETERS OF THE JOINTED ROCK MODEL

Most parameters of the Jointed Rock model coincide with those of the isotropic Mohr-Coulomb model. These are the basic elastic parameters and the basic strength

parameters.

*Elastic parameters as in Mohr-Coulomb model (see Section 3.3):*

$E_1$	: Young's modulus for rock as a continuum	[kN/m <sup>2</sup> ]
$\nu_1$	: Poisson's ratio for rock as a continuum	[-]

*Anisotropic elastic parameters 'Plane 1' direction (e.g. stratification direction):*

$E_2$	: Young's modulus perpendicular on 'Plane 1' direction	[kN/m <sup>2</sup> ]
$G_2$	: Shear modulus perpendicular on 'Plane 1' direction	[kN/m <sup>2</sup> ]
$\nu_2$	: Poisson's ratio perpendicular on 'Plane 1' direction	[-]

*Strength parameters in joint directions (Plane  $i=1, 2, 3$ ):*

$c_i$	: Cohesion	[kN/m <sup>2</sup> ]
$\varphi_i$	: Friction angle	[°]
$\psi_i$	: Dilatancy angle	[°]
$\sigma_{t,i}$	: Tensile strength	[kN/m <sup>2</sup> ]

*Definition of joint directions (Plane  $i=1, 2, 3$ ):*

$n$	: Number of joint directions ( $1 \leq n \leq 3$ )	[-]
$\alpha_{1,i}$	: Dip ( $-180 \leq \alpha_{1,i} \leq 180$ )	[°]
$\alpha_{2,i}$	: Strike ( $-180 \leq \alpha_{1,i} \leq 180$ ) ( $\alpha_{2,i} = 90$ in PLAXIS 2D)	[°]

### **Elastic parameters**

The elastic parameters  $E_1$  and  $\nu_1$  are the (constant) stiffness (Young's modulus) and Poisson's ratio of the rock as a continuum according to Hooke's law, i.e. as if it would not be anisotropic.

Elastic anisotropy in a rock formation may be introduced by stratification. The stiffness perpendicular to the stratification direction is usually reduced compared with the general stiffness. This reduced stiffness can be represented by the parameter  $E_2$ , together with a second Poisson's ratio,  $\nu_2$ . In general, the elastic stiffness normal to the direction of elastic anisotropy is defined by the parameters  $E_2$  and  $\nu_2$ .

Elastic shearing in the stratification direction is also considered to be 'weaker' than elastic shearing in other directions. In general, the shear stiffness in the anisotropic direction can explicitly be defined by means of the elastic shear modulus  $G_2$ . In contrast to Hooke's law of isotropic elasticity,  $G_2$  is a separate parameter and is not simply related to Young's modulus by means of Poisson's ratio (see Eq. 5.1d and e).

If the elastic behaviour of the rock is fully isotropic, then the parameters  $E_2$  and  $\nu_2$  can be simply set equal to  $E_1$  and  $\nu_1$  respectively, whereas  $G_2$  should be set to  $\frac{1}{2}E_1/(1 + \nu_1)$ .

Property	Unit	Value
<b>Stiffness</b>		
$E'$	kN/m <sup>2</sup>	0.000
$\nu_1$ (nu)		0.000
$E_2$	kN/m <sup>2</sup>	0.000
$\nu_2$ (nu)		0.000
$G_2$	kN/m <sup>2</sup>	0.000
<b>Strength</b>		
Number of planes		1 plane
<b>Plane 1</b>		
$c_{ref}$	kN/m <sup>2</sup>	0.000
$\phi$ (phi)	°	0.000
$\psi$ (psi)	°	0.000
$\alpha_1$ (alpha 1)	°	0.000
<b>Advanced</b>		
Tension cut-off		<input checked="" type="checkbox"/>
Tensile strength	kN/m <sup>2</sup>	0.000

Figure 5.4 Parameters for the Jointed Rock model (PLAXIS 3D)

### Strength parameters

Each sliding direction (plane) has its own strength properties  $c_i$ ,  $\varphi_i$  and  $\sigma_{t,i}$  and dilatancy angle  $\psi_i$ . The strength properties  $c_i$  and  $\varphi_i$  determine the allowable shear strength according to Coulomb's criterion and  $\sigma_t$  determines the tensile strength according to the tension cut-off criterion. The latter is displayed after pressing *Advanced* button. By default, the tension cut-off is active and the tensile strength is set to zero. The dilatancy angle,  $\psi_i$ , is used in the plastic potential function  $g$ , and determines the plastic volume expansion due to shearing.

### Definition of joint directions

It is assumed that the direction of elastic anisotropy corresponds with the first direction where plastic shearing may occur ('plane 1'). This direction must always be specified. In the case the rock formation is stratified without major joints, the *number of sliding planes* (= sliding directions) is still 1, and strength parameters must be specified for this direction anyway. A maximum of three sliding directions can be defined. These directions may correspond to the most critical directions of joints in the rock formation.

The sliding directions are defined by means of two parameters: The *Dip angle* ( $\alpha_1$ ) (or shortly *Dip*) and the *Strike* ( $\alpha_2$ ). The definition of both parameters is visualized in Figure 5.5.

Consider a sliding plane, as indicated in Figure 5.5. The sliding plane can be defined by

the vectors  $(s, t)$ , which are both normal to the vector  $n$ . The vector  $n$  is the 'normal' to the sliding plane, whereas the vector  $s$  is the 'fall line' of the sliding plane (*dip*) and the vector  $t$  is the 'horizontal line' of the sliding plane (*strike*). The sliding plane makes an angle  $\alpha_1$  with respect to the horizontal plane, where the horizontal plane can be defined by the vectors  $(s^*, t)$ , which are both normal to the vertical axis. The angle  $\alpha_1$  is the *dip*, which is defined as the positive 'downward' inclination angle between the horizontal plane and the sliding plane. Hence,  $\alpha_1$  is the angle between the vectors  $s^*$  and  $s$ , measured clockwise from  $s^*$  to  $s$  when looking in the positive  $t$ -direction. The dip ought to be entered in the range  $[0^\circ, 90^\circ]$ , but negative values as well as values larger than  $90^\circ$  can also be entered.

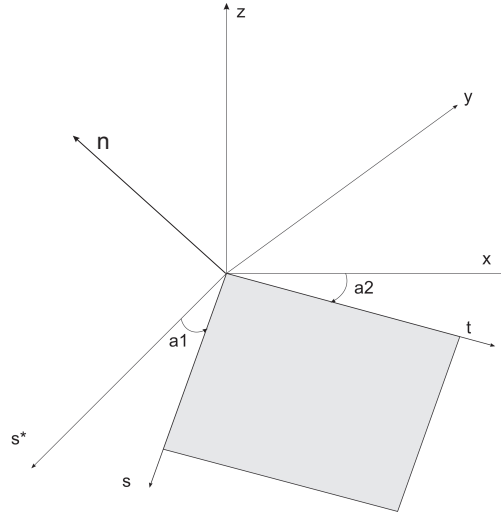


Figure 5.5 Definition of dip angle and strike

The orientation of the sliding plane is further defined by the *strike*,  $\alpha_2$ , which is defined in PLAXIS as the orientation of the vector  $t$  with respect to the  $x$ -direction. The strike is defined as the positive angle from the  $x$ -direction, measured clockwise to the  $t$ -axis. The dip direction is entered in the range  $[-180^\circ, 180^\circ]$ .

From the definitions as given above, it follows for PLAXIS 3D that:

$$\underline{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} -\sin \alpha_1 \sin \alpha_2 \\ -\sin \alpha_1 \cos \alpha_2 \\ \cos \alpha_1 \end{bmatrix} \quad (5.20a)$$

$$\underline{s} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix} = \begin{bmatrix} -\cos \alpha_1 \sin \alpha_2 \\ -\cos \alpha_1 \cos \alpha_2 \\ -\sin \alpha_1 \end{bmatrix} \quad (5.20b)$$

$$\underline{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \cos \alpha_2 \\ -\sin \alpha_2 \\ 0 \end{bmatrix} \quad (5.20c)$$

whereas for PLAXIS 2D  $\alpha_2$  is taken by definition as  $\alpha_2 = 90^\circ$ , such that:

$$\underline{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} -\sin \alpha_1 \\ \cos \alpha_1 \\ 0 \end{bmatrix} \quad (5.21a)$$

$$\underline{s} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix} = \begin{bmatrix} -\cos \alpha_1 \\ -\sin \alpha_1 \\ 0 \end{bmatrix} \quad (5.21b)$$

$$\underline{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (5.21c)$$

Figure 5.6 shows some examples of how sliding planes occur in a 3D models for different values of  $\alpha_1$  and  $\alpha_2$ . As it can be seen, for plane strain conditions (the cases considered in PLAXIS 2D) only  $\alpha_1$  is required. By default,  $\alpha_2$  is fixed at  $90^\circ$ .

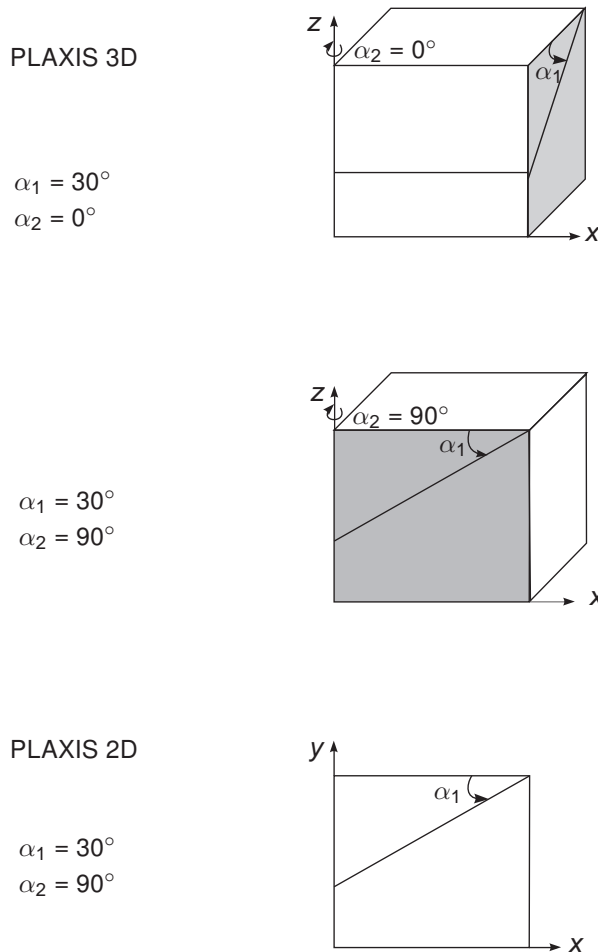


Figure 5.6 Examples of failure directions defined by  $\alpha_1$  and  $\alpha_2$

#### 5.4 ON THE USE OF THE JOINTED ROCK MODEL IN DYNAMIC CALCULATIONS

When using the Jointed Rock model in dynamic calculations, the stiffness need to be selected such that the model correctly predicts wave velocities in the soil (Equation (3.13)). When subjected to dynamic or cyclic loading, the Jointed Rock model may generate plastic strains if stress points reach the Coulomb failure criterion, which will lead to damping in dynamic calculations. However, it should be noted that stress cycles within the Coulomb failure contour will only generate elastic strains and no (hysteretic) damping, nor accumulation of strains or pore pressure or liquefaction. In order to simulate the rock's damping characteristics in cyclic loading, Rayleigh damping may be defined.

**Hint:** A slightly modified version of the Jointed Rock model with generalised Mohr-Coulomb failure criterion in addition to the individual failure directions is available as user-defined soil model. Contact Plaxis for more information.

**Hint:** In a geological context, *strike* is defined as the angle from the North to the *strike* of the *dipping plane* (positive towards East direction), whereas in PLAXIS definition *strike* is defined from the global *x – direction* to the *strike* of the dipping plane (with the same positive rotation direction). If we define the angle from the North direction to the *x – direction* of the PLAXIS model as *declination*, then *strike* in PLAXIS is the geological *strike* minus *declination*.

» Moreover, there might be confusion between true *strike* and its opposite direction ( $180^\circ$  direction). Hence, care must be taken when translating *strike* from geological data into PLAXIS input.



## 6 THE HARDENING SOIL MODEL (ISOTROPIC HARDENING)

In contrast to an elastic perfectly-plastic model, the yield surface of a hardening plasticity model is not fixed in principal stress space, but it can expand due to plastic straining. Distinction can be made between two main types of hardening, namely shear hardening and compression hardening. Shear hardening is used to model irreversible strains due to primary deviatoric loading. Compression hardening is used to model irreversible plastic strains due to primary compression in oedometer loading and isotropic loading. Both types of hardening are contained in the present model.

The Hardening Soil model is an advanced model for simulating the behaviour of different types of soil, both soft soils and stiff soils, Schanz (1998). When subjected to primary deviatoric loading, soil shows a decreasing stiffness and simultaneously irreversible plastic strains develop. In the special case of a drained triaxial test, the observed relationship between the axial strain and the deviatoric stress can be well approximated by a hyperbola. Such a relationship was first formulated by Kondner (1963) and later used in the well-known hyperbolic model (Duncan & Chang, 1970). The Hardening Soil model, however, supersedes the hyperbolic model by far: Firstly by using the theory of plasticity rather than the theory of elasticity, secondly by including soil dilatancy and thirdly by introducing a yield cap. Some basic characteristics of the model are:

- Stress dependent stiffness according to a power law    Input parameter  $m$
- Plastic straining due to primary deviatoric loading    Input parameter  $E_{50}^{ref}$
- Plastic straining due to primary compression    Input parameter  $E_{oed}^{ref}$
- Elastic unloading / reloading    Input parameters  $E_{ur}^{ref}$ ,  $\nu_{ur}$
- Failure according to the Mohr-Coulomb failure criterion    Parameters  $c$ ,  $\varphi$  and  $\psi$

A basic feature of the present Hardening Soil model is the stress dependency of soil stiffness. For oedometer conditions of stress and strain, the model implies for example the relationship  $E_{oed} = E_{oed}^{ref} (\sigma / p^{ref})^m$ . In the special case of soft soils it is realistic to use  $m = 1$ . In such situations there is also a simple relationship between the modified compression index  $\lambda^*$ , as used in models for soft soil and the oedometer loading modulus (see also Section 9.8).

$$E_{oed}^{ref} = \frac{p^{ref}}{\lambda^*} \qquad \lambda^* = \frac{\lambda}{(1 + e_0)}$$

where  $p^{ref}$  is a reference pressure. Here we consider a tangent oedometer modulus at a particular reference pressure  $p^{ref}$ . Hence, the primary loading stiffness relates to the modified compression index  $\lambda^*$  or to the standard Cam-Clay compression index  $\lambda$ .

Similarly, the unloading-reloading modulus relates to the modified swelling index  $\kappa^*$  or to the standard Cam-Clay swelling index  $\kappa$ . There is the approximate relationship:

$$E_{ur}^{ref} \approx \frac{2p^{ref}}{\kappa^*} \qquad \kappa^* = \frac{\kappa}{(1 + e_0)}$$

This relationship applies in combination with the input value  $m = 1$ .

## 6.1 HYPERBOLIC RELATIONSHIP FOR STANDARD DRAINED TRIAXIAL TEST

A basic idea for the formulation of the Hardening Soil model is the hyperbolic relationship between the vertical strain,  $\varepsilon_1$ , and the deviatoric stress,  $q$ , in primary triaxial loading. Here standard drained triaxial tests tend to yield curves that can be described by:

$$-\varepsilon_1 = \frac{1}{E_i} \frac{q}{1 - q/q_a} \quad \text{for: } q < q_f \quad (6.1)$$

Where  $q_a$  is the asymptotic value of the shear strength and  $E_i$  the initial stiffness.  $E_i$  is related to  $E_{50}$  by:

$$E_i = \frac{2E_{50}}{2 - R_f} \quad (6.2)$$

This relationship is plotted in Figure 6.1. The parameter  $E_{50}$  is the confining stress dependent stiffness modulus for primary loading and is given by the equation:

$$E_{50} = E_{50}^{ref} \left( \frac{c \cos \varphi - \sigma'_3 \sin \varphi}{c \cos \varphi + p^{ref} \sin \varphi} \right)^m \quad (6.3)$$

where  $E_{50}^{ref}$  is a reference stiffness modulus corresponding to the reference confining pressure  $p^{ref}$ . In PLAXIS, a default setting  $p^{ref} = 100$  stress units is used. The actual stiffness depends on the minor principal stress,  $\sigma'_3$ , which is the confining pressure in a triaxial test. Please note that  $\sigma'_3$  is negative for compression. The amount of stress dependency is given by the power  $m$ . In order to simulate a logarithmic compression behaviour, as observed for soft clays, the power should be taken equal to 1.0. Janbu (1963) reports values of  $m$  around 0.5 for Norwegian sands and silts, whilst von Soos (1990) reports various different values in the range  $0.5 < m < 1.0$ .

The ultimate deviatoric stress,  $q_f$ , and the quantity  $q_a$  in Eq. (6.1) are defined as:

$$q_f = (c \cot \varphi - \sigma'_3) \frac{2 \sin \varphi}{1 - \sin \varphi} \quad \text{and: } q_a = \frac{q_f}{R_f} \quad (6.4)$$

Again it is remarked that  $\sigma'_3$  is usually negative. The above relationship for  $q_f$  is derived from the Mohr-Coulomb failure criterion, which involves the strength parameters  $c$  and  $\varphi$ . As soon as  $q = q_f$ , the failure criterion is satisfied and perfectly plastic yielding occurs as described by the Mohr-Coulomb model.

The ratio between  $q_f$  and  $q_a$  is given by the failure ratio  $R_f$ , which should obviously be smaller than or equal to 1. In PLAXIS,  $R_f = 0.9$  is chosen as a suitable default setting.

For unloading and reloading stress paths, another stress-dependent stiffness modulus is used:

$$E_{ur} = E_{ur}^{ref} \left( \frac{c \cos \varphi - \sigma'_3 \sin \varphi}{c \cos \varphi + p^{ref} \sin \varphi} \right)^m \quad (6.5)$$

where  $E_{ur}^{ref}$  is the reference Young's modulus for unloading and reloading, corresponding to the reference pressure  $p^{ref}$ . In many practical cases it is appropriate to set  $E_{ur}^{ref}$  equal to  $3E_{50}^{ref}$ ; this is the default setting used in PLAXIS.

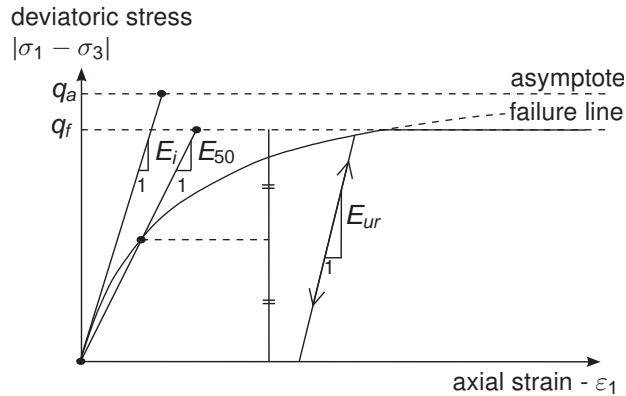


Figure 6.1 Hyperbolic stress-strain relation in primary loading for a standard drained triaxial test

## 6.2 APPROXIMATION OF HYPERBOLA BY THE HARDENING SOIL MODEL

For the sake of convenience, restriction is made here to triaxial loading conditions with  $\sigma'_2 = \sigma'_3$  and  $\sigma'_1$  being the major compressive stress. In fact, for general states of stress,  $q$  can be replaced by  $\tilde{q}$  where:

$$\tilde{q} = \sigma_1 + (\alpha - 1)\sigma_2' - \alpha\sigma_3' \quad \text{with} \quad \alpha = \frac{3 + \sin\varphi}{3 - \sin\varphi}$$

(refer Section 6.5 for more details). Moreover, it is assumed that  $q < q_f$ , as also indicated in Figure 6.1. It should also be realised that compressive stress and strain are considered negative. For a more general presentation of the Hardening Soil model the reader is referred to Schanz, Vermeer & Bonnier (1999). In this section it will be shown that this model gives virtually the hyperbolic stress strain curve of Eq. (6.1) when considering stress paths of standard drained triaxial tests. Let us first consider the corresponding plastic strains. This stems from a shear hardening yield function of the form:

$$f = \bar{f} - \gamma^p \quad (6.6)$$

where  $\bar{f}$  is a function of stress and  $\gamma^p$  is a function of plastic strains:

$$\bar{f} = \frac{2}{E_i} \frac{q}{1 - q/q_a} - \frac{2q}{E_{ur}} \quad \gamma^p = -(2\varepsilon_1^p - \varepsilon_v^p) \approx -2\varepsilon_1^p \quad (6.7)$$

with  $q$ ,  $q_a$ ,  $E_i$  and  $E_{ur}$  as defined by Eq. (6.1) to Eq. (6.5), whilst the superscript  $p$  is used to denote plastic strains. For hard soils, plastic volume changes ( $\varepsilon_v^p$ ) tend to be relatively small and this leads to the approximation  $\gamma^p \approx -2\varepsilon_1^p$ . The above definition of the strain-hardening parameter  $\gamma^p$  will be referred to later.

An essential feature of the above definitions for  $\bar{f}$  is that it matches the well-known hyperbolic law Eq. (6.1). For checking this statement, one has to consider primary loading, as this implies the yield condition  $f = 0$ . For primary loading, it thus yields  $\gamma^p = \bar{f}$  and it follows from Eq. (6.6) that:

$$\varepsilon_1^p \approx \frac{1}{2}\bar{f} = \frac{1}{E_i} \frac{q}{1 - q/q_a} - \frac{q}{E_{ur}} \quad (6.8)$$

In addition to the plastic strains, the model accounts for elastic strains. Plastic strains develop in primary loading alone, but elastic strains develop both in primary loading and unloading / reloading. For drained triaxial test stress paths with  $\sigma'_2 = \sigma'_3 = \text{constant}$ , the elastic Young's modulus  $E_{ur}$  remains constant and the elastic strains are given by the equations:

$$-\varepsilon_1^e = \frac{q}{E_{ur}} \qquad -\varepsilon_2^e = -\varepsilon_3^e = -\nu_{ur} \frac{q}{E_{ur}} \qquad (6.9)$$

where  $\nu_{ur}$  is the unloading / reloading Poisson's ratio. Here it should be realised that restriction is made to strains that develop during deviatoric loading, whilst the strains that develop during the very first stage of the test (isotropic compression with consolidation) are not considered.

For the deviatoric loading stage of the triaxial test, the axial strain is the sum of an elastic component given by Eq. (6.9) and a plastic component according to Eq. (6.8). Hence, it follows that:

$$-\varepsilon_1 = -\varepsilon_1^e - \varepsilon_1^p \approx \frac{1}{E_i} \frac{q}{1 - q/q_a} \qquad (6.10)$$

This relationship holds exactly in absence of plastic volume strains, i.e. when  $\varepsilon_V^p = 0$ .

In reality, plastic volumetric strains will never be precisely equal to zero, but for hard soils plastic volume changes tend to be small when compared with the axial strain so that this formulation yields a hyperbolic stress-strain curve under triaxial testing conditions.

For a given constant value of the hardening parameter,  $\gamma^p$ , the yield condition  $f = 0$ , can be visualised in  $p'$ - $q$ -plane by means of a yield locus. Hence,  $\gamma^p$  is associated with mobilised friction. When plotting such yield loci, one has to use Eq. (6.7) as well as Eqs. (6.3) and (6.5) for  $E_{50}$  and  $E_{ur}$  respectively. Because of the latter expressions, the shape of the yield loci depends on the exponent  $m$ . For  $m = 1$ , straight lines are obtained, but slightly curved yield loci correspond to lower values of the exponent. Figure 6.2 shows the shape of yield loci for increasing values of  $\gamma^p$  considering  $m = 0.5$ , being typical for hard soils. Hence,  $\gamma^p$  can be regarded as the plastic shear strain related to the mobilised shear resistance.

**Hint:** Note that there is approximately a factor 2 difference between the plastic shear strain parameter  $\gamma^p$  and the deviatoric plastic strain  $\varepsilon_q^p$  ( $\gamma^p \approx 2\varepsilon_q^p$ )

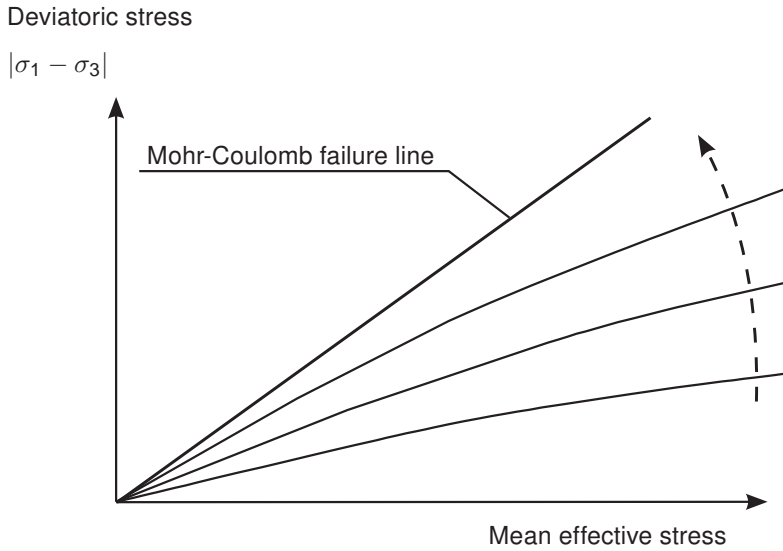


Figure 6.2 Successive yield loci for various constant values of the hardening parameter  $\gamma^P$

### 6.3 PLASTIC VOLUMETRIC STRAIN FOR TRIAXIAL STATES OF STRESS

Having presented a relationship for the plastic shear strain,  $\gamma^P$ , attention is now focused on the plastic volumetric strain,  $\varepsilon_v^P$ . As for all plasticity models, the Hardening Soil model involves a relationship between rates of plastic strain, i.e. a relationship between  $\dot{\varepsilon}_v^P$  and  $\dot{\gamma}^P$ . This shear hardening flow rule has the linear form:

$$\dot{\varepsilon}_v^P = \sin \psi_m \dot{\gamma}^P \quad (6.11)$$

Clearly, further detail is needed by specifying the mobilised dilatancy angle  $\psi_m$ . For the present model, the following is considered (see also Figure 6.3):

$$\begin{aligned} \text{For } \sin \varphi_m < 3/4 \sin \varphi : & \quad \psi_m = 0 \\ \text{For } \sin \varphi_m \geq 3/4 \sin \varphi \text{ and } \psi > 0 & \quad \sin \psi_m = \max \left( \frac{\sin \varphi_m - \sin \varphi_{cv}}{1 - \sin \varphi_m \sin \varphi_{cv}}, 0 \right) \\ \text{For } \sin \varphi_m \geq 3/4 \sin \varphi \text{ and } \psi \leq 0 & \quad \psi_m = \psi \\ \text{If } \varphi = 0 & \quad \psi_m = 0 \end{aligned} \quad (6.12)$$

where  $\varphi_{cv}$  is the critical state friction angle, being a material constant independent of density, and  $\varphi_m$  is the mobilised friction angle:

$$\sin \varphi_m = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3 - 2c \cot \varphi} \quad (6.13)$$

The above equations are a small adaptation from the well-known stress-dilatancy theory by Rowe (1962), as explained by Schanz & Vermeer (1996). The mobilised dilatancy angle,  $\psi_m$ , follows Rowe's theory for larger values of the mobilised friction angle, as long as this results in a positive value of  $\psi_m$ . For small mobilised friction angles and for negative values of  $\psi_m$ , as computed by Rowe's formula (as long as the dilatancy angle  $\psi$

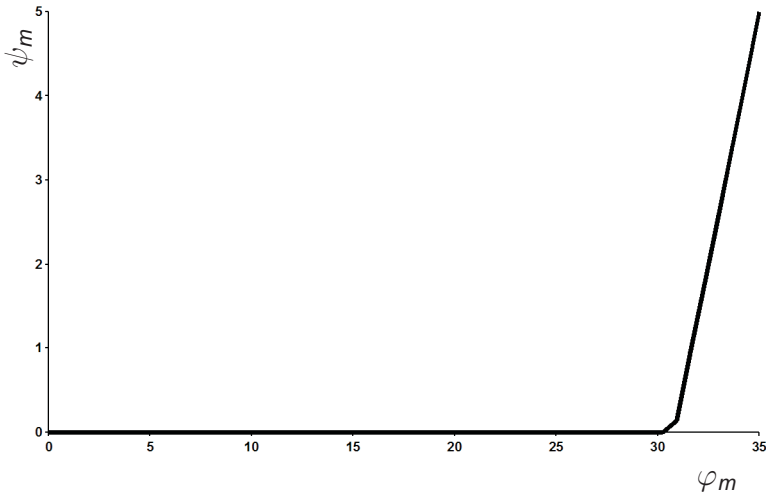


Figure 6.3 Plot of mobilized dilatancy angle  $\psi_m$  and mobilized friction angle  $\varphi_m$  for Hardening Soil model

is positive),  $\psi_m$  is taken zero. Furthermore, in all cases when  $\varphi = 0$ ,  $\psi_m$  is set equal to zero.

The essential property of the stress-dilatancy theory is that the material contracts for small stress ratios  $\varphi_m < \varphi_{cv}$ , whilst dilatancy occurs for high stress ratios  $\varphi_m > \varphi_{cv}$ . At failure, when the mobilised friction angle equals the failure angle,  $\varphi$ , it is found from Eq. (6.12) that:

$$\sin \psi = \frac{\sin \varphi - \sin \varphi_{cv}}{1 - \sin \varphi \sin \varphi_{cv}} \quad (6.14a)$$

or equivalently:

$$\sin \varphi_{cv} = \frac{\sin \varphi - \sin \psi}{1 - \sin \varphi \sin \psi} \quad (6.14b)$$

Hence, the critical state angle can be computed from the failure angles  $\varphi$  and  $\psi$ . PLAXIS performs this computation automatically and therefore users do not need to specify a value for  $\varphi_{cv}$ . Instead, one has to provide input data on the peak friction angle,  $\varphi$ , and the peak dilatancy angle,  $\psi$ .

The shear hardening process will continue with the mobilization of the shear strength, until the maximum shear strength according to the Mohr-Coulomb model failure criterion is reached.

## 6.4 PARAMETERS OF THE HARDENING SOIL MODEL

Some parameters of the present hardening model coincide with those of the non-hardening Mohr-Coulomb model. These are the failure parameters  $c$ ,  $\varphi$  and  $\psi$ .

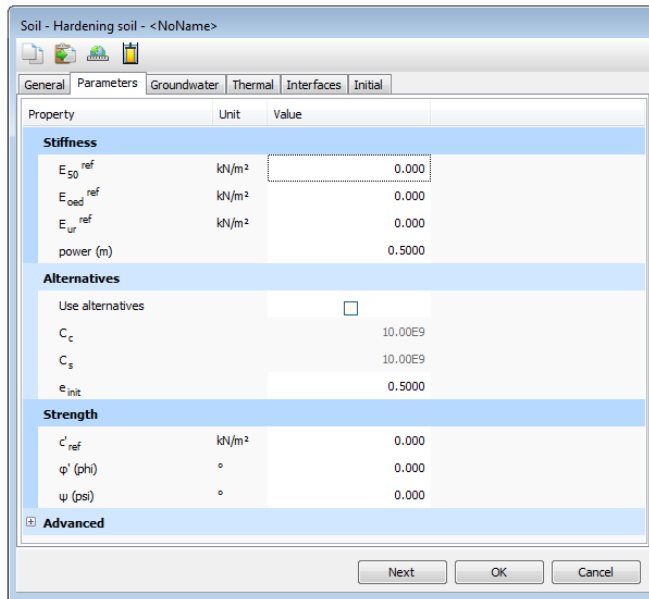


Figure 6.4 Parameters for the Hardening Soil model

Failure parameters as in Mohr-Coulomb model (see Section 3.3):

$c$	: (Effective) cohesion	[kN/m <sup>2</sup> ]
$\varphi$	: (Effective) angle of internal friction	[°]
$\psi$	: Angle of dilatancy	[°]
$\sigma_t$	: Tension cut-off and tensile strength	[kN/m <sup>2</sup> ]

Basic parameters for soil stiffness:

$E_{50}^{ref}$	: Secant stiffness in standard drained triaxial test	[kN/m <sup>2</sup> ]
$E_{oed}^{ref}$	: Tangent stiffness for primary oedometer loading	[kN/m <sup>2</sup> ]
$E_{ur}^{ref}$	: Unloading / reloading stiffness (default $E_{ur}^{ref} = 3E_{50}^{ref}$ )	[kN/m <sup>2</sup> ]
$m$	: Power for stress-level dependency of stiffness	[-]

Advanced parameters (it is advised to use the default setting):

$\nu_{ur}$	: Poisson's ratio for unloading-reloading (default $\nu_{ur} = 0.2$ )	[-]
$p^{ref}$	: Reference stress for stiffnesses (default $p^{ref} = 100$ kN/m <sup>2</sup> )	[kN/m <sup>2</sup> ]
$K_0^{nc}$	: $K_0$ -value for normal consolidation (default $K_0^{nc} = 1 - \sin \varphi$ )	[-]
$R_f$	: Failure ratio $q_f / q_a$ (default $R_f = 0.9$ ) (see Figure 6.1)	[-]
$\sigma_{tension}$	: Tensile strength (default $\sigma_{tension} = 0$ stress units)	[kN/m <sup>2</sup> ]

$C_{inc}$  : As in Mohr-Coulomb model (default  $C_{inc} = 0$ ) [kN/m<sup>3</sup>]

Instead of entering the basic parameters for soil stiffness, alternative parameters can be entered. These parameters are listed below:

$C_c$  : Compression index [-]

$C_s$  : Swelling index or reloading index [-]

$e_{init}$  : Initial void ratio [-]

**Stiffness moduli  $E_{50}^{ref}$ ,  $E_{oed}^{ref}$  &  $E_{ur}^{ref}$  and power  $m$**

The advantage of the Hardening Soil model over the Mohr-Coulomb model is not only the use of a hyperbolic stress-strain curve instead of a bi-linear curve, but also the control of stress level dependency. When using the Mohr-Coulomb model, the user has to select a fixed value of Young's modulus whereas for real soils this stiffness depends on the stress level. It is therefore necessary to estimate the stress levels within the soil and use these to obtain suitable values of stiffness. With the Hardening Soil model, however, this cumbersome selection of input parameters is not required.

Instead, a stiffness modulus  $E_{50}^{ref}$  is defined for a reference minor principal effective stress of  $-\sigma'_3 = p^{ref}$ . This is the secant stiffness at 50 % of the maximum deviatoric stress, at a cell pressure equal to the reference stress  $p^{ref}$  (Figure 6.5). As a default value, the program uses  $p^{ref} = 100$  kN/m<sup>2</sup>.

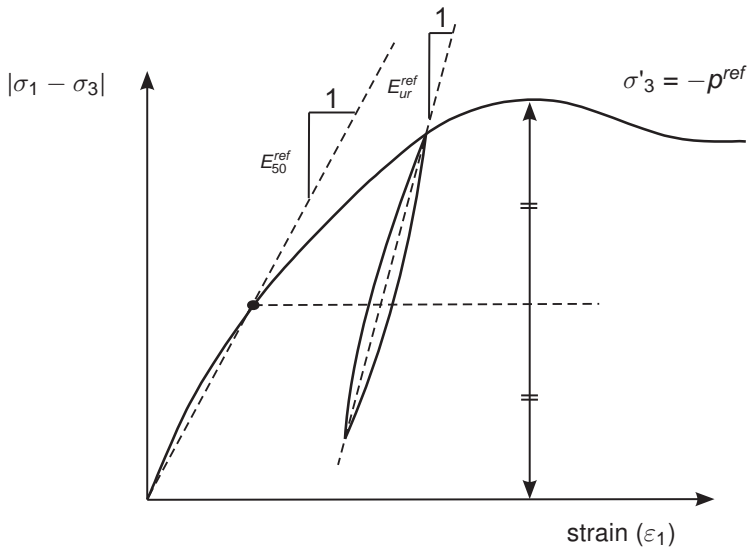


Figure 6.5 Definition of  $E_{50}^{ref}$  and  $E_{ur}^{ref}$  for drained triaxial test results

As some PLAXIS users are familiar with the input of shear moduli rather than the above stiffness moduli, shear moduli will now be discussed. Within Hooke's law of isotropic elasticity conversion between  $E$  and  $G$  goes by the equation  $E = 2(1 + \nu)G$ . As  $E_{ur}$  is a real elastic stiffness, one may thus write  $E_{ur} = 2(1 + \nu)G_{ur}$ , where  $G_{ur}$  is an elastic shear modulus. Please note that PLAXIS allows for the input of  $E_{ur}$  and  $\nu_{ur}$  but not for a direct input of  $G_{ur}$ . In contrast to  $E_{ur}$ , the secant modulus  $E_{50}$  is not used within a



concept of elasticity. As a consequence, there is no simple conversion from  $E_{50}$  to  $G_{50}$ .

In contrast to elasticity based models, the elastoplastic Hardening Soil model does not involve a fixed relationship between the (drained) triaxial stiffness  $E_{50}$  and the oedometer stiffness  $E_{oed}$  for one-dimensional compression. Instead, these stiffnesses can be inputted independently. Having defined  $E_{50}$  by Eq. (6.3), it is now important to define the oedometer stiffness. Here we use the equation:

$$E_{oed} = E_{oed}^{ref} \left( \frac{c \cos \varphi - \frac{\sigma'_3}{K_0^{nc}} \sin \varphi}{c \cos \varphi + p^{ref} \sin \varphi} \right)^m \quad (6.15)$$

where  $E_{oed}$  is a tangent stiffness modulus obtained from an oedometer test, as indicated in Figure 6.6.

Hence,  $E_{oed}^{ref}$  is a tangent stiffness at a vertical stress of  $-\sigma'_1 = \frac{-\sigma'_3}{K_0^{nc}} = p^{ref}$ . Note that we basically use  $\sigma'_1$  rather than  $\sigma'_3$  and that we consider primary loading.

When undrained behaviour is considered in the Hardening Soil model the *Drainage type* should preferably be set to *Undrained (A)*. Alternatively, *Undrained (B)* can be used in case the effective strength properties are not known or the undrained shear strength is not properly captured using *Undrained (A)*. However, it should be noted that the material loses its stress-dependency of stiffness in that case. *Undrained (C)* is not possible since the model is essentially formulated as an effective stress model.

### Alternative stiffness parameters

When soft soils are considered, the stiffness parameters can be calculated from the compression index, swelling index and the initial void ratio<sup>†</sup>. The relationship between these parameters and the compression index,  $C_c$ , is given by:

$$C_c = \frac{2.3(1 + e_{init})p_{ref}}{E_{oed}^{ref}} \quad (6.16)$$

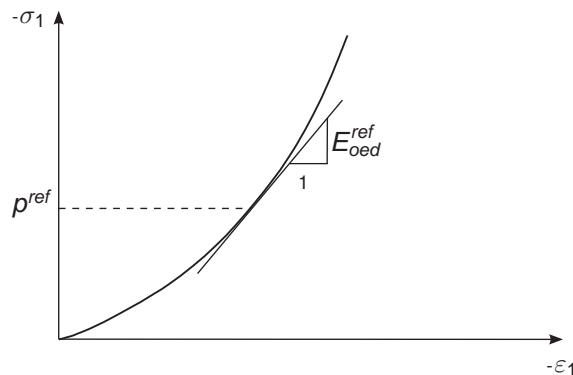


Figure 6.6 Definition of  $E_{oed}^{ref}$  in oedometer test results

<sup>†</sup> In the PLAXIS material database, these alternative parameters depend on the initial void ratio. In reality, these parameters depend on the actual void ratio, which is not a constant.

The relationship between the  $E_{ur}^{ref}$  and the swelling index,  $C_s$ , is given by:

$$C_s \approx \frac{2.3(1 + e_{init})(1 + \nu)(1 - 2\nu)\rho_{ref}}{(1 - \nu)E_{ur}^{ref}K_0} \quad (6.17)$$

**Hint:** During unloading the  $K_0$ -value will increase, where values above 1.0 may be observed for significant unloading. By default, PLAXIS assumes that on average the horizontal stress is equal to the vertical stress ( $K_0=1$ ).

Regardless the previous value of  $E_{50}$ , a new value will be automatically assigned according to:

$$E_{50}^{ref} = 1.25E_{oed}^{ref} \quad (6.18)$$

Although for *Soft soils*,  $E_{50}^{ref}$  could be as high as  $2E_{oed}^{ref}$ , this high value could lead to a limitation in the modeling; therefore a lower value is used. Changing the value of  $C_s$  will change the stiffness parameter  $E_{ur}$ .

Note that the value of the power for stress-level dependency of stiffness ( $m$ ) is automatically set to 1.

### **Advanced parameters**

Realistic values of  $\nu_{ur}$  are about 0.2 and this value is thus used as a default setting, as indicated in Figure 6.4. Note that in the Hardening Soil model,  $\nu_{ur}$  is a pure elastic parameter.

In contrast to the Mohr-Coulomb model,  $K_0^{nc}$  is not simply a function of Poisson's ratio, but an independent input parameter. As a default setting PLAXIS uses the correlation  $K_0^{nc} = 1 - \sin \varphi$ . It is suggested to maintain this value as the correlation is quite realistic. However, users do have the possibility to select different values. Not all possible input values for  $K_0^{nc}$  can be accommodated for. Depending on other parameters, such as  $E_{50}^{ref}$ ,  $E_{oed}^{ref}$ ,  $E_{ur}^{ref}$  and  $\nu_{ur}$ , there happens to be a certain range of valid  $K_0^{nc}$ -values.  $K_0^{nc}$  values outside this range are rejected by PLAXIS. On inputting values, the program shows the nearest possible value that will be used in the computations.

### **Dilatancy cut-off**

After extensive shearing, dilating materials arrive in a state of critical density where dilatancy has come to an end, as indicated in Figure 6.7. This phenomenon of soil behaviour can be included in the Hardening Soil model by means of a dilatancy cut-off. In order to specify this behaviour, the initial void ratio,  $e_{init}$ , and the maximum void ratio,  $e_{max}$ , of the material must be entered as general parameters. As soon as the volume change results in a state of maximum void, the mobilised dilatancy angle,  $\psi_m$ , is

automatically set back to zero, as indicated in Figure 6.7.

$$\text{for } e < e_{max} : \quad \sin \psi_m = \frac{\sin \varphi_m - \sin \varphi_{cv}}{1 - \sin \varphi_m \sin \varphi_{cv}}$$

$$\text{where:} \quad \sin \varphi_{cv} = \frac{\sin \varphi - \sin \psi}{1 - \sin \varphi \sin \psi} \quad (6.19a)$$

$$\text{for } e \geq e_{max} : \quad \psi_m = 0 \quad (6.19b)$$

The void ratio is related to the volumetric strain,  $\varepsilon_v$  by the relationship:

$$-(\varepsilon_v - \varepsilon_v^{init}) = \ln \left( \frac{1 + e}{1 - e_{init}} \right) \quad (6.20)$$

where an increment of  $\varepsilon_v$  is positive for dilatancy.

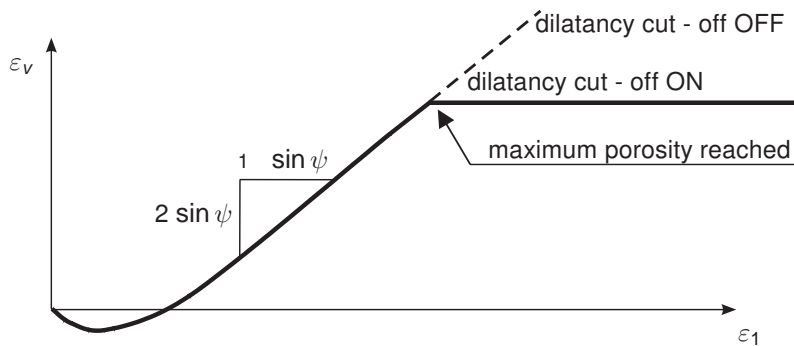


Figure 6.7 Resulting strain curve for a standard drained triaxial test when including dilatancy cut-off

The initial void ratio,  $e_{init}$ , is the in-situ void ratio of the soil body. The maximum void ratio is the void ratio of the material in a state of critical void (critical state). As soon as the maximum void ratio is reached, the dilatancy angle is set to zero. The minimum void ratio,  $e_{min}$ , of a soil can also be input, but this general soil parameter is not used within the context of the Hardening Soil model.

Please note that the selection of the dilatancy cut-off and the input of void ratios are done in the *General* tabsheet (Figure 6.8) of the *Soil* window and not in the *Parameters* tabsheet. The selection of the *Dilatancy cut-off* is only available when the Hardening Soil model or the Hardening Soil model with small-strain stiffness has been selected. By default, the *Dilatancy cut-off* is not active. Note that the dilatancy cut-off does not help in limiting the shear strength when using the *Undrained (A)* drainage type with a positive dilatancy angle. This is because the void ratio remains constant for undrained materials. Therefore, it is strongly recommended to set  $\psi = 0$  for undrained material behaviour (*Undrained A*).

Property	Unit	Value
<b>Material set</b>		
Identification		
Material model		Hardening soil
Drainage type		Drained
Colour		RGB 161, 226, 232
Comments		
<b>General properties</b>		
$\gamma_{\text{unsat}}$	kN/m <sup>3</sup>	0.000
$\gamma_{\text{sat}}$	kN/m <sup>3</sup>	0.000
<b>Advanced</b>		
<b>Void ratio</b>		
Dilatancy cut-off		<input type="checkbox"/>
$e_{\text{init}}$		0.5000
$e_{\text{min}}$		0.000
$e_{\text{max}}$		999.0
<b>Damping</b>		
Rayleigh $\alpha$		0.000
Rayleigh $\beta$		0.000

Figure 6.8 General tabsheet

## 6.5 ON THE CAP YIELD SURFACE IN THE HARDENING SOIL MODEL

Shear hardening yield surfaces as indicated in Figure 6.2 do not explain the plastic volume strain that is measured in isotropic compression which is mostly observed in softer types of soil. A second type of yield surface must therefore be introduced to close the elastic region for compressive (compaction hardening) stress paths. Without such a cap type yield surface it would not be possible to formulate a model with independent input of both  $E_{50}^{\text{ref}}$  and  $E_{\text{oad}}^{\text{ref}}$ . The triaxial modulus largely controls the shear yield surface and the oedometer modulus controls the cap yield surface. In fact,  $E_{50}^{\text{ref}}$  largely controls the magnitude of the plastic strains that are associated with the shear yield surface. Similarly,  $E_{\text{oad}}^{\text{ref}}$  is used to control the magnitude of plastic strains that originate from the yield cap. In this section the yield cap will be described in detail. To this end we consider the definition of the cap yield surface:

$$f_c = \frac{\tilde{q}^2}{M^2} + (p')^2 - p_p^2 \quad (6.21)$$

where  $M$  is an auxiliary model parameter that relates to  $K_0^{\text{nc}}$  as will be discussed later. Further more we have  $p' = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$  and  $\tilde{q} = \sigma'_1 + (\alpha - 1)\sigma'_2 - \alpha\sigma'_3$  with  $\alpha = (3 + \sin \varphi)/(3 - \sin \varphi)$ .  $\tilde{q}$  is a special stress measure for deviatoric stresses. In the special case of triaxial compression ( $-\sigma'_1 > -\sigma'_2 = -\sigma'_3$ ) it yields  $\tilde{q} = -(\sigma'_1 - \sigma'_3)$  and for triaxial extension ( $-\sigma'_1 = -\sigma'_2 > -\sigma'_3$ )  $\tilde{q}$  reduces to  $\tilde{q} = -\alpha(\sigma'_1 - \sigma'_3)$ . The magnitude of the yield cap is determined by the isotropic pre-consolidation stress  $p_p$ . In a  $(p', \tilde{q})$  plane, the yield cap ( $f_c = 0$ ) is a part of an ellipse with its centre point in the origin

(Figure 6.9). The hardening law relating  $\dot{p}_p$  to volumetric cap strain  $\dot{\varepsilon}_V^{pc}$  is:

$$\dot{\varepsilon}_V^{pc} = \frac{K_s/K_c - 1}{K_s^{ref}} \left[ \left( \frac{p_p + c \cot \varphi}{p^{ref} + c \cot \varphi} \right)^{-m} \right] \dot{p}_p \quad (6.22)$$

in which  $K_s^{ref}$  is the reference bulk modulus in unloading / reloading:

$$K_s^{ref} = \frac{E_{ur}^{ref}}{3(1 - 2\nu_{ur})} \quad (6.23)$$

and  $K_s/K_c$  is the ratio of bulk moduli in isotropic swelling and primary isotropic compression.

The volumetric cap strain is the plastic volumetric strain in isotropic compression. In addition to the well known constants  $m$  and  $p^{ref}$  there is another model constant  $K_s/K_c$ . Both  $M$  (Eq. (6.21)) and  $K_s/K_c$  (Eq. (6.22)) are cap parameters, but these are not used as direct input parameters. Instead, there is a relationship of the form:

$$M \leftrightarrow K_0^{nc} \quad (\text{default : } K_0^{nc} = 1 - \sin \varphi)$$

and the ratio  $K_s/K_c$  can be approximated as:

$$K_s/K_c \approx \frac{E_{ur}^{ref}}{E_{oed}^{ref}} \frac{K_0^{nc}}{(1 + 2K_0^{nc})(1 - 2\nu_{ur})}$$

such that  $K_0^{nc}$ ,  $E_{ur}^{ref}$  and  $E_{oed}^{ref}$  can be used as input parameters that determine the magnitude of  $M$  and  $K_s/K_c$  respectively.

The ellipse on which the yield cap is located has length  $p_p$  on the  $p$ -axis and  $Mp_p$  on the  $\tilde{q}$ -axis. Hence,  $p_p$  determines its magnitude and  $M$  its aspect ratio. High values of  $M$  lead to steep caps underneath the Mohr-Coulomb line and correspondingly small  $K_0^{nc}$ -values, whereas small  $M$ -values define caps that are much more pointed around the  $p$ -axis leading to large  $K_0^{nc}$ -values. The ellipse is used both as a yield surface and as a plastic potential (associated plasticity). Input data on initial  $p_p$ -values is provided by means of the PLAXIS procedure for initial stresses. Here,  $p_p$  is either computed from the input over-consolidation ratio (OCR) or the pre-overburden pressure (POP) (see Section 2.9).

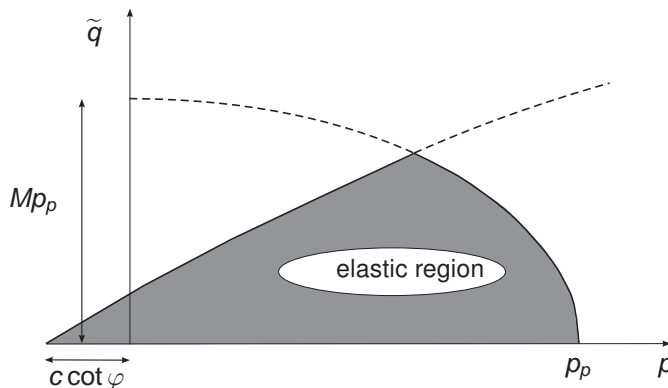


Figure 6.9 Yield surfaces of Hardening Soil model in  $p - \tilde{q}$ -plane. The elastic region can be further reduced by means of a tension cut-off

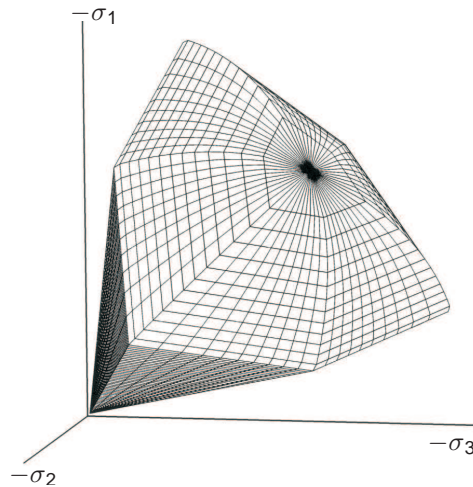


Figure 6.10 Representation of total yield contour of the Hardening Soil model in principal stress space for cohesionless soil

For understanding the yield surfaces in full detail, one should consider both Figure 6.9 and Figure 6.10. The first figure shows simple yield lines, whereas the second one depicts yield surfaces in principal stress space. Both the shear locus and the yield cap have the hexagonal shape of the classical Mohr-Coulomb failure criterion. In fact, the shear yield locus can expand up to the ultimate Mohr-Coulomb failure surface. The cap yield surface expands as a function of the pre-consolidation stress  $p_p$ .

## 6.6 STATE PARAMETERS IN THE HARDENING SOIL MODEL

In addition to the output of standard stress and strain quantities, the Hardening Soil model provides output (when being used) on state variables such as the hardening parameter  $\gamma^p$  and the isotropic pre-consolidation stress  $p_p$ . These parameters can be visualised by selecting the *State parameters* option from the *stresses* menu. An overview of available state parameters is given below:

$p^{eq}$  : Equivalent isotropic stress [kN/m<sup>2</sup>]

$$p^{eq} = \sqrt{\frac{\tilde{q}^2}{M^2} + (p')^2}$$

$p^p$  : Isotropic preconsolidation stress [kN/m<sup>2</sup>]

$OCR$  : Isotropic over-consolidation ratio ( $OCR = p^p / p^{eq}$ ) [-]

$\gamma^p$  : Hardening parameter (equivalent mobilised plastic shear strain) [-]

$E_{ur}$  : Current stress-dependent elastic Young's modulus [-]

$c$  : Current depth-dependent cohesion [-]

## 6.7 ON THE USE OF THE HARDENING SOIL MODEL IN DYNAMIC CALCULATIONS

When using the Hardening Soil model in dynamic calculations, the elastic stiffness parameter  $E_{ur}^{ref}$  needs to be selected such that the model correctly predicts wave velocities in the soil. This generally requires an even larger small strain stiffness rather than just an unloading-reloading stiffness to be entered for  $E_{ur}^{ref}$ . When subjected to dynamic or cyclic loading, the Hardening Soil model will generate plastic strains when mobilizing the soil's material strength (shear hardening) or increasing the soil's preconsolidation stress (compaction hardening). However, it should be noted that stress cycles within the current hardening contours will only generate elastic strains and no (hysteretic) damping, nor accumulation of strains or pore pressure nor liquefaction. In order to simulate the soil's damping characteristics in cyclic loading, Rayleigh damping may be defined. Note that some of the limitations of the Hardening Soil model in dynamic applications can be overcome by using the HS small model (Chapter 7).





## 7 THE HARDENING SOIL MODEL WITH SMALL-STRAIN STIFFNESS (HSSMALL)

The original Hardening Soil model assumes elastic material behaviour during unloading and reloading. However, the strain range in which soils can be considered truly elastic, i.e. where they recover from applied straining almost completely, is very small. With increasing strain amplitude, soil stiffness decays nonlinearly. Plotting soil stiffness against  $\log(\text{strain})$  yields characteristic S-shaped stiffness reduction curves. Figure 7.1 gives an example of such a stiffness reduction curve. It outlines also the characteristic shear strains that can be measured near geotechnical structures and the applicable strain ranges of laboratory tests. It turns out that at the minimum strain which can be reliably measured in classical laboratory tests, i.e. triaxial tests and oedometer tests without special instrumentation, soil stiffness is often decreased to less than half its initial value.

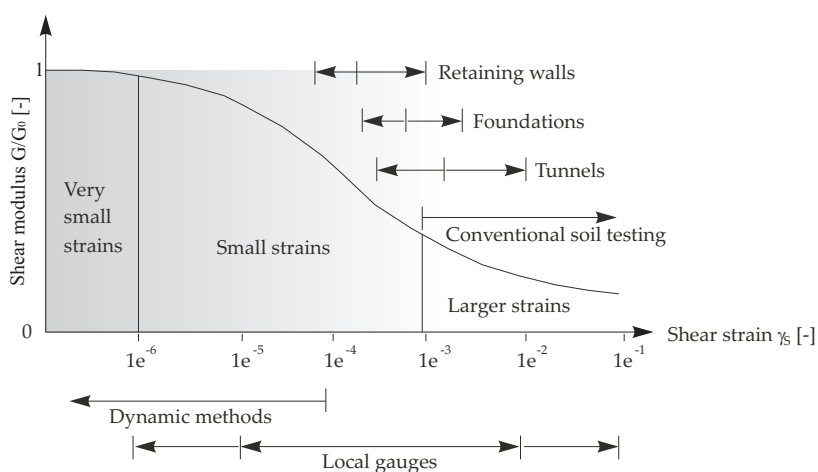


Figure 7.1 Characteristic stiffness-strain behaviour of soil with typical strain ranges for laboratory tests and structures (after Atkinson & Salfors (1991))

The soil stiffness that should be used in the analysis of geotechnical structures is not the one that relates to the strain range at the end of construction according to Figure 7.1. Instead, very small-strain soil stiffness and its non-linear dependency on strain amplitude should be properly taken into account. In addition to all features of the Hardening Soil model, the Hardening Soil model with small-strain stiffness offers the possibility to do so.

The Hardening Soil model with small-strain stiffness implemented in PLAXIS is based on the Hardening Soil model and uses almost entirely the same parameters (see Section 6.4). In fact, only two additional parameters are needed to describe the variation of stiffness with strain:

- the initial or very small-strain shear modulus  $G_0$
- the shear strain level  $\gamma_{0.7}$  at which the secant shear modulus  $G_s$  is reduced to about 70% of  $G_0$

### 7.1 DESCRIBING SMALL-STRAIN STIFFNESS WITH A SIMPLE HYPERBOLIC LAW

In soil dynamics, small-strain stiffness has been a well known phenomenon for a long time. In static analysis, the findings from soil dynamics have long been considered not to be applicable.

Seeming differences between static and dynamic soil stiffness have been attributed to the nature of loading (e.g. inertia forces and strain rate effects) rather than to the magnitude of applied strain which is generally small in dynamic conditions (earthquakes excluded). As inertia forces and strain rate have only little influence on the initial soil stiffness, dynamic soil stiffness and small-strain stiffness can in fact be considered as synonyms.

Probably the most frequently used model in soil dynamics is the Hardin-Drnevich relationship. From test data, sufficient agreement is found that the stress-strain curve for small strains can be adequately described by a simple hyperbolic law. The following analogy to the hyperbolic law for larger strains by Kondner (1963) (see previous Section) was proposed by Hardin & Drnevich (1972):

$$\frac{G_s}{G_0} = \frac{1}{1 + \left| \frac{\gamma}{\gamma_r} \right|} \quad (7.1)$$

where the threshold shear strain  $\gamma_r$  is quantified as:

$$\gamma_r = \frac{\tau_{max}}{G_0} \quad (7.2)$$

with  $\tau_{max}$  being the shear stress at failure. Essentially, Eqs. (7.1) and (7.2) relate large (failure) strains to small-strain properties which often work well.

More straightforward and less prone to error is the use of a smaller threshold shear strain. Santos & Correia (2001), for example suggest to use the shear strain  $\gamma_r = \gamma_{0.7}$  at which the secant shear modulus  $G_s$  is reduced to about 70 % of its initial value. Eq. (7.1) can then be rewritten as:

$$\frac{G_s}{G_0} = \frac{1}{1 + a \left| \frac{\gamma}{\gamma_{0.7}} \right|} \quad \text{where } a = 0.385 \quad (7.3)$$

In fact, using  $a = 0.385$  and  $\gamma = \gamma_{0.7}$  gives  $G_s/G_0 = 0.722$ . Hence, the formulation "about 70%" should be interpreted more accurately as 72.2%.

Figure 7.2 shows the fit of the modified Hardin-Drnevich relationship (Eq. 7.3) to normalized test data.

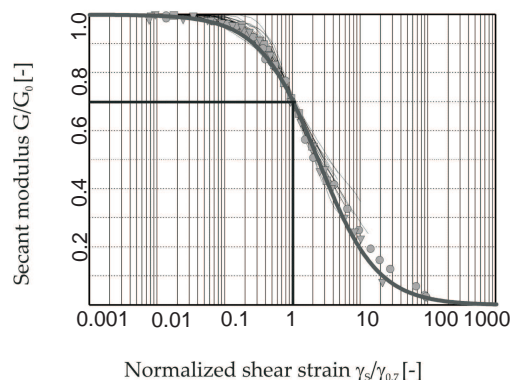


Figure 7.2 Results from the Hardin-Drnevich relationship compared to test data by Santos & Correia (2001)

## 7.2 APPLYING THE HARDIN-DRNEVICH RELATIONSHIP IN THE HS MODEL

The decay of soil stiffness from small strains to larger strains can be associated with loss of intermolecular and surface forces within the soil skeleton. Once the direction of loading is reversed, the stiffness regains a maximum recoverable value which is in the order of the initial soil stiffness. Then, while loading in the reversed direction is continued, the stiffness decreases again. A strain history dependent, multi-axial extension of the Hardin-Drnevich relationship is therefore needed in order to apply it in the Hardening Soil model. Such an extension has been proposed by Benz (2006) in the form of the small-strain overlay model. Benz derives a scalar valued shear strain  $\gamma_{hist}$  by the following projection:

$$\gamma_{hist} = \sqrt{3} \frac{\|\underline{\underline{H}}\Delta\mathbf{e}\|}{\|\Delta\mathbf{e}\|} \quad (7.4)$$

where  $\Delta\mathbf{e}$  is the actual deviatoric strain increment and  $\underline{\underline{H}}$  is a symmetric tensor that represents the deviatoric strain history of the material. Whenever a strain reversal is detected the tensor  $\underline{\underline{H}}$  is partially or fully reset before the actual strain increment  $\Delta\mathbf{e}$  is added. As the criterion for strain reversals serves a criterion similar as in Simpson's brick model (1992): All three principal deviatoric strain directions are checked for strain reversals separately which resembles three independent brick models. When there is no principal strain rotation, the criterion reduces to two independent brick-models. For further details on the strain tensor  $\underline{\underline{H}}$  and its transformation at changes in the load path it is referred to Benz (2006).

The scalar valued shear strain  $\gamma = \gamma_{hist}$  calculated in Eq. (7.4) is applied subsequently used in Eq. (7.3). Note that in both, Eqs. (7.3) and (7.4), the scalar valued shear strain is defined as:

$$\gamma = \frac{3}{2} \varepsilon_q \quad (7.5)$$

where  $\varepsilon_q$  is the second deviatoric strain invariant. In triaxial conditions  $\gamma$  can therefore be expressed as:

$$\gamma = \varepsilon_{axial} - \varepsilon_{lateral} \quad (7.6)$$

Within the HS small model, the stress-strain relationship can be simply formulated from the secant shear modulus (Eq. 7.3) as:

$$\tau = G_s \gamma = \frac{G_0 \gamma}{1 + 0.385 \frac{\gamma}{\gamma_{0.7}}} \quad (7.7)$$

Taking the derivative with respect to the shear strain gives the tangent shear modulus:

$$G_t = \frac{G_0}{\left(1 + 0.385 \frac{\gamma}{\gamma_{0.7}}\right)^2} \quad (7.8)$$

This stiffness reduction curve reaches far into the plastic material domain. In the Hardening Soil model and HS small model, stiffness degradation due to plastic straining is simulated with strain hardening. In the HS small model, the small-strain stiffness reduction curve is therefore bound by a certain lower limit, determined by conventional laboratory tests:

- The lower cut-off of the tangent shear modulus  $G_t$  is introduced at the unloading reloading stiffness  $G_{ur}$  which is defined by the material parameters  $E_{ur}$  and  $\nu_{ur}$ :

$$G_t \geq G_{ur} \quad \text{where} \quad G_{ur} = \frac{E_{ur}}{2(1 + \nu_{ur})} \quad \text{and} \quad G_t = \frac{E_t}{2(1 + \nu_{ur})} \quad (7.9)$$

- The cut-off shear strain  $\gamma_{cut-off}$  can be calculated as:

$$\gamma_{cut-off} = \frac{1}{0.385} \left( \sqrt{\frac{G_0}{G_{ur}}} - 1 \right) \gamma_{0.7} \quad (7.10)$$

Within the HS small model, the quasi-elastic tangent shear modulus is calculated by integrating the secant stiffness modulus reduction curve over the actual shear strain increment. An example of a stiffness reduction curve used in the HS small model is shown in Figure 7.3.

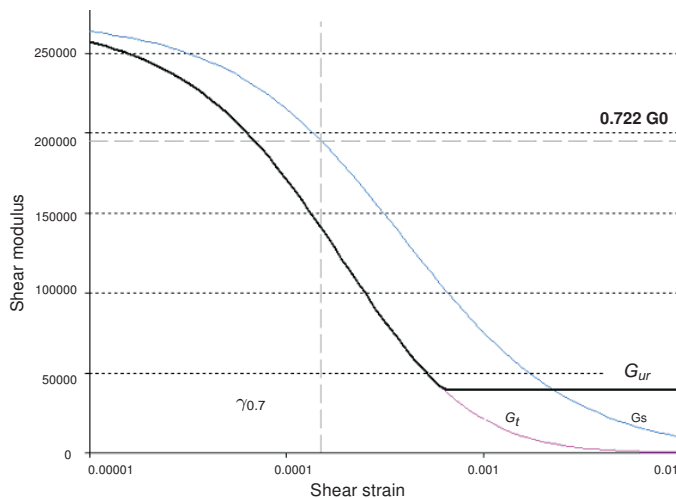


Figure 7.3 Secant and tangent shear modulus reduction curve

Moreover, the tangent shear modulus  $G_t$  and corresponding Young's modulus  $E_t$  (considering a constant Poisson's ratio  $\nu_{ur}$ ), is stress-dependent, and follows the same power law as formulated in Equation (6.5). For primary loading situations, the model uses the same hardening plasticity formulations as the Hardening Soil model, where  $E_{ur}$  is replaced by  $E_t$  as described above.

### 7.3 VIRGIN (INITIAL) LOADING VS. UNLOADING/RELOADING

Masing (1926) described the hysteretic behaviour of materials in unloading / reloading cycles in the form of the following rules:

- The shear modulus in unloading is equal to the initial tangent modulus for the initial loading curve.
- The shape of the unloading and reloading curves is equal to the initial loading curve, but twice its size. In terms of the above introduced threshold shear strain  $\gamma_{0.7}$ , Masing's rule can be fulfilled by the following setting in the Hardin-Drnevich relation:

$$\gamma_{0.7\text{re-loading}} = 2 \gamma_{0.7\text{virgin-loading}} \quad (7.11)$$

The HS small model adopts Masing's rule. However, instead of doubling the threshold shear strain, hardening plasticity accounts for more rapidly decaying small-strain stiffness during virgin loading. Figure 7.4 and Figure 7.5 illustrate Masing's rule and the secant stiffness reduction in virgin loading and unloading / reloading.

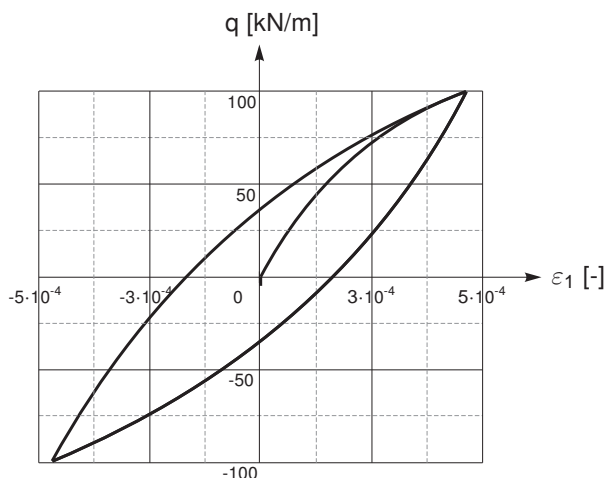


Figure 7.4 Hysteretic material behaviour

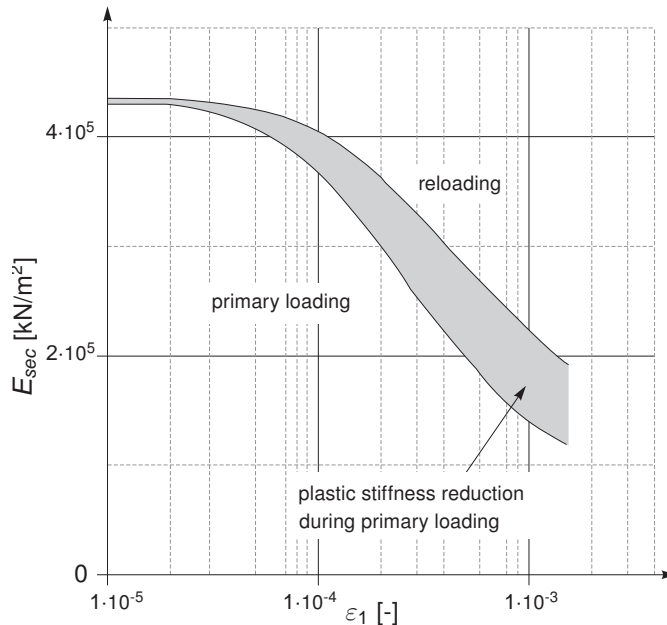


Figure 7.5 Stiffness reduction in initial- or primary loading and in unloading / reloading

## 7.4 MODEL PARAMETERS

Compared to the standard Hardening Soil model, the Hardening Soil model with small-strain stiffness requires two additional stiffness parameters as input:  $G_0^{ref}$  and  $\gamma_{0.7}$ . All other parameters, including the alternative stiffness parameters, remain the same as in the standard Hardening Soil model.  $G_0^{ref}$  defines the shear modulus at very small strains e.g.  $\varepsilon < 10^{-6}$  at a reference minor principal stress of  $-\sigma'_3 = p^{ref}$ .

Poisson's ratio  $\nu_{ur}$  is assumed a constant, as everywhere in PLAXIS, so that the shear modulus  $G_0^{ref}$  can also be calculated from the very small strain Young's modulus as  $G_0^{ref} = E_0^{ref} / (2(1 + \nu_{ur}))$ . The threshold shear strain  $\gamma_{0.7}$  is the shear strain at which the secant shear modulus  $G_s^{ref}$  is decayed to  $0.722G_0^{ref}$ . The threshold shear strain  $\gamma_{0.7}$  is to be supplied for virgin loading. In summary, the input stiffness parameters of the Hardening Soil model with small-strain stiffness are listed below:

$m$	: Power for stress-level dependency of stiffness	[-]
$E_{50}^{ref}$	: Secant stiffness in standard drained triaxial test	[kN/m <sup>2</sup> ]
$E_{oed}^{ref}$	: Tangent stiffness for primary oedometer loading	[kN/m <sup>2</sup> ]
$E_{ur}^{ref}$	: unloading / reloading stiffness from drained triaxial test	[kN/m <sup>2</sup> ]
$\nu_{ur}$	: Poisson's ratio for unloading-reloading	[-]
$G_0^{ref}$	: reference shear modulus at very small strains ( $\varepsilon < 10^{-6}$ )	[kN/m <sup>2</sup> ]
$\gamma_{0.7}$	: threshold shear strain at which $G_s = 0.722G_0$	[-]

Figure 7.6 illustrates the model's stiffness parameters in a drained triaxial test:  $E_{50}$ ,  $E_{ur}$ , and  $E_0 = 2G_0(1 + \nu_{ur})$ . For the order of strains at which  $E_{ur}$  and  $G_0$  are defined and

determined, one may refer to e.g. Figure 7.1 and Figure 7.3.

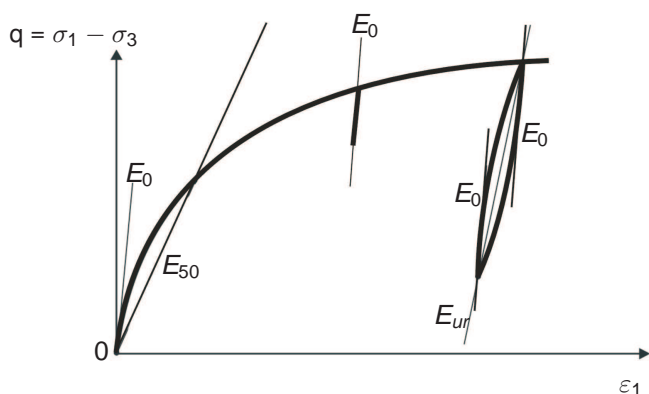


Figure 7.6 Stiffness parameters  $E_{50}$ ,  $E_{ur}$ , and  $E_0 = 2G_0(1 + \nu_{ur})$  of the Hardening Soil model with small-strain stiffness in a triaxial test

Figure 7.7 illustrates the model's stiffness parameters in a stress-controlled drained cyclic shear test.

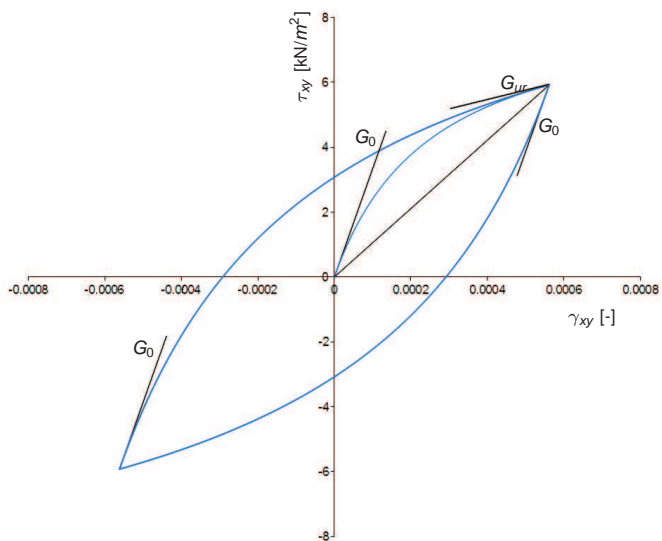


Figure 7.7 Stiffness parameters in cyclic shear test

A first estimation of the HSsmall parameters for quartz sand based on the relative density (RD) is given in Brinkgreve, Engin & Engin (2010).

**7.5 ON THE PARAMETERS  $G_0$  AND  $\gamma_{0.7}$**

A number of factors influence the small-strain parameters  $G_0$  and  $\gamma_{0.7}$ . Most importantly they are influenced by the material's actual state of stress and void ratio  $e$ . In the HSsmall model, the stress dependency of the shear modulus  $G_0$  is taken into account with the power law:

$$G_0 = G_0^{ref} \left( \frac{c \cos \varphi - \sigma'_3 \sin \varphi}{c \cos \varphi + p^{ref} \sin \varphi} \right)^m \tag{7.12}$$

which resembles the ones used for the other stiffness parameters. The threshold shear strain  $\gamma_{0.7}$  is taken independently of the mean stress.

Assuming that within a HSsmall (or HS) computation void ratio changes are rather small, the material parameters are not updated for changes in the void ratio. Knowledge of a material's initial void ratio can nevertheless be very helpful in deriving its small-strain shear stiffness  $G_0$ . Many correlations are offered in the literature (Benz, 2006). A good estimation for many soils is for example the relation given by Hardin & Black (1969):

$$G_0^{ref} = 33 \cdot \frac{(2.97 - e)^2}{1 + e} [MPa] \quad \text{for} \quad p^{ref} = 100 [kPa] \tag{7.13}$$

Alpan (1970) empirically related dynamic soil stiffness to static soil stiffness (Figure 7.8). The dynamic soil stiffness in Alpan's chart is equivalent to the small-strain stiffness  $G_0$  or  $E_0$ . Considering that the static stiffness  $E_{static}$  defined by Alpan equals approximately the unloading / reloading stiffness  $E_{ur}$  in the HS small model, Alpan's chart can be used to guess a soil's small-strain stiffness entirely based on its unloading / reloading stiffness  $E_{ur}$ . Although Alpan suggests that the ratio  $E_0/E_{ur}$  can exceed 10 for very soft clays, the maximum ratio  $E_0/E_{ur}$  or  $G_0/G_{ur}$  permitted in the HSsmall model is limited to 10.

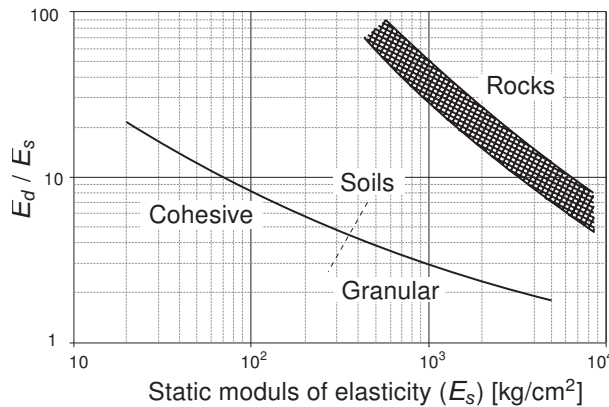


Figure 7.8 Relation between dynamic ( $E_d = E_0$ ) and static soil stiffness ( $E_s \approx E_{ur}$ ) after Alpan(1970)

In the absence of test data, correlations are also available for the threshold shear strain  $\gamma_{0.7}$ . Figure 7.9 for example gives a correlation between the threshold shear strain and the Plasticity Index. Using the original Hardin-Drnevich relationship, the threshold shear strain  $\gamma_{0.7}$  might be also related to the model's failure parameters. Applying the



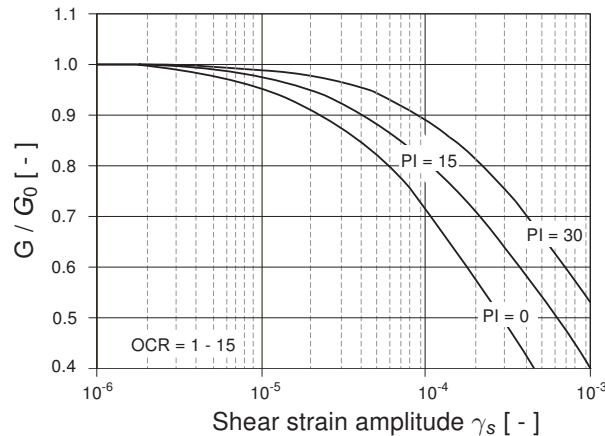


Figure 7.9 Influence of plasticity index (PI) on stiffness reduction after Vucetic & Dobry (1991)

Mohr-Coulomb failure criterion in Eqs. (7.2) and (7.3) yields:

$$\gamma_{0.7} \approx \frac{1}{9G_0} [2c'(1 + \cos(2\varphi')) - \sigma'_1(1 + K_0) \sin(2\varphi')] \quad (7.14)$$

where  $K_0$  is the earth pressure coefficient at rest and  $\sigma'_1$  is the effective vertical stress (pressure negative).

## 7.6 MODEL INITIALIZATION

Stress relaxation erases a soil's memory of previous applied stress. Soil aging in the form of particle (or assembly) reorganization during stress relaxation and formation of bonds between them can erase a soil's strain history. Considering that the second process in a naturally deposited soil develops relatively fast, the strain history should start from zero ( $\underline{H} = 0$ ) in most boundary value problems. This is the default setting in the HS small model.

However, sometimes an initial strain history may be desired. In this case the strain history can be adjusted by applying an extra load step before starting the actual analysis. Such an additional load step might also be used to model overconsolidated soils. Usually the over-consolidation's cause has vanished long before the start of calculation, so that the strain history should be reset afterwards. Unfortunately, strain history is already triggered by adding and removing a surcharge. In this case the strain history can be reset manually, by using the *Reset small strain* option in the calculation phases window. Also, when resetting displacements to zero, the strain history tensor is reset and the influence of strains from previous calculation phases is ignored.

When using the HS small model, caution should be given to nil-steps. The strain increments in nil-steps are purely derived from the small numerical unbalance in the system which is due to the accepted tolerated error in the computation. The strain increment direction in nil-steps is therefore arbitrary. Hence, a nil-step may function as randomly reverse load step which is in most cases not desired.

## 7.7 STATE PARAMETERS IN THE HS SMALL MODEL

In addition to the output of standard stress and strain quantities, the HS small model provides output on *State variables*. These parameters can be visualised by selecting the *State parameters* option from the *stresses* menu. An overview of available state parameters in addition to those listed for the Hardening Soil model is given below:

$\varepsilon_{xx} - \varepsilon_v$	: Strain history parameter used in strain-dependent stiffness formulation	[-]
$\varepsilon_{yy} - \varepsilon_v$	: Strain history parameter used in strain-dependent stiffness formulation	[-]
$\varepsilon_{zz} - \varepsilon_v$	: Strain history parameter used in strain-dependent stiffness formulation	[-]
$\varepsilon_{xy}$	: Strain history parameter used in strain-dependent stiffness formulation	[-]
$\varepsilon_{yz}$	: Strain history parameter used in strain-dependent stiffness formulation	[-]
$\varepsilon_{zx}$	: Strain history parameter used in strain-dependent stiffness formulation	[-]
$G_s^{ref}$	: Reference secant shear modulus at reference stress level	[-]
$G/G_{ur}$	: Ratio of elastic tangent shear modulus over unloading-reloading shear modulus	[-]

## 7.8 ON THE USE OF THE HS SMALL MODEL IN DYNAMIC CALCULATIONS

In contrast to the Hardening Soil model, the HS small model shows hysteresis in cyclic loading (Figure 7.6). The amount of hysteresis depends on the magnitude of the corresponding strain amplitude. However, note that the model does not generate accumulated strains with multiple loading cycles, nor does it generate pore pressures with undrained behaviour. When the HS small model is used wave velocities are not shown because they vary due to the stress-dependent stiffness.

When applied in dynamic calculations, the hysteretic behaviour of the HS small model leads to damping. The amount of hysteretic damping depends on the applied load amplitude and corresponding strain amplitudes. The maximum amount of hysteretic damping obtained with the HS small model depend on the ratio of  $G_0$  and  $G_{ur} = E_{ur}/2(1 + \nu_{ur})$ . A larger ratio leads to a larger maximum amount of hysteretic damping. For more information about the hysteretic damping in the HS small model reference is made to Brinkgreve, Kappert & Bonnier (2007).

## 7.9 OTHER DIFFERENCES WITH THE HARDENING SOIL MODEL

### *The mobilised dilatancy angle*

The shear hardening flow rule of both the Hardening Soil model and the Hardening Soil model with small-strain stiffness have the linear form:

$$\dot{\varepsilon}_V^p = \sin \psi_m \dot{\gamma}^p \quad (7.15)$$

The mobilised dilatancy angle  $\psi_m$  in compression however, is defined differently. The HS model assumes the following:

$$\begin{aligned} \text{For } \sin \varphi_m < 3/4 \sin \varphi & \quad \psi_m = 0 \\ \text{For } \sin \varphi_m \geq 3/4 \sin \varphi \text{ and } \psi > 0 & \quad \sin \psi_m = \max\left(\frac{\sin \varphi_m - \sin \varphi_{cv}}{1 - \sin \varphi_m \sin \varphi_{cv}}, 0\right) \\ \text{For } \sin \varphi_m \geq 3/4 \sin \varphi \text{ and } \psi \leq 0 & \quad \psi_m = \psi \\ \text{If } \varphi = 0 & \quad \psi_m = 0 \end{aligned} \quad (7.16)$$

where  $\varphi_{cv}$  is the critical state friction angle, being a material constant independent of density, and  $\varphi_m$  is the mobilised friction angle:

$$\sin \varphi_m = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3 - 2c \cot \varphi} \quad (7.17)$$

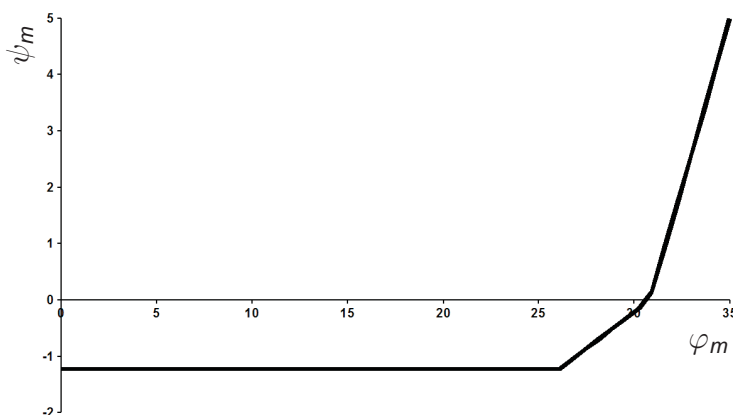


Figure 7.10 Plot of mobilized dilatancy angle  $\psi_m$  and mobilized friction angle  $\varphi_m$  for HS small model

For small mobilised friction angles and for negative values of  $\psi_m$ , as computed by Rowe's formula,  $\psi_m$  in the Hardening Soil model is taken zero. Bounding the lower value of  $\psi_m$  may sometimes yield too little plastic volumetric strains though. Therefore, the Hardening Soil model with small-strain stiffness adapts an approach by Li & Dafalias (2000) whenever  $\psi_m$ , as computed by Rowe's formula, is negative. In that case, the mobilised dilatancy in the Hardening Soil model with small-strain stiffness is calculated by the

following equation:

$$\sin \psi_m = \frac{1}{10} \left( -M \exp \left[ \frac{1}{15} \ln \left( \frac{\eta}{M} \frac{q}{q_a} \right) \right] + \eta \right) \quad (7.18)$$

where  $M$  is the stress ratio at failure and  $\eta = \max\left(\frac{q}{p}, \frac{1 - K_0}{3(1 + 2K_0)}\right)$ .

Eq. (7.18) is a simplified version of the void ratio dependent formulation by Li & Dafalias (2000).

The mobilised dilatancy as a function of  $\varphi_m$  for the HS small model is visualised in Figure 7.10.

## 8 THE SOFT SOIL MODEL

As soft soils we consider near-normally consolidated clays, clayey silts and peat. A special feature of such materials is their high degree of compressibility. This is best demonstrated by oedometer test data as reported for instance by Janbu in his Rankine lecture (1985). Considering tangent stiffness moduli at a reference oedometer pressure of 100 kPa, he reports for normally consolidated clays  $E_{oed} = 1$  to 4 MPa, depending on the particular type of clay considered. The differences between these values and stiffnesses for NC-sands are considerable as here we have values in the range of 10 to 50 MPa, at least for non-cemented laboratory samples. Hence, in oedometer testing normally consolidated clays behave ten times softer than normally consolidated sands. This illustrates the extreme compressibility of soft soils.

A feature of soft soils is the linear stress-dependency of soil stiffness. According to the Hardening Soil model we have:

$$E_{oed} = E_{oed}^{ref} \left( \frac{-\sigma'_1}{p^{ref}} \right)^m$$

at least for  $c = 0$  and  $\sigma'_3 = K_0^{nc} \sigma'_1$  and a linear relationship is obtained for  $m = 1$ . Indeed, on using an exponent equal to unity, the above stiffness law reduces to:

$$E_{oed} = \frac{-\sigma'_1}{\lambda^*} \text{ where } \lambda^* = \frac{p^{ref}}{E_{oed}^{ref}}$$

For this special case of  $m = 1$ , the Hardening Soil model yields  $\dot{\varepsilon} = \lambda^* \dot{\sigma}'_1 / \sigma'_1$ , which can be integrated to obtain the well-known logarithmic compression law  $\varepsilon = -\lambda^* \ln(-\sigma'_1)$  for primary oedometer loading.

For many practical soft-soil studies, the modified compression index  $\lambda^*$  will be known and the PLAXIS user can compute the oedometer modulus from the relationship:

$$E_{oed}^{ref} = \frac{p^{ref}}{\lambda^*}$$

From the above considerations it would seem that the Hardening Soil model is quite suitable for soft soils. Indeed, most soft soil problems can be analysed using this model, but the Hardening Soil model is not suitable when considering very soft soils with a high compressibility, i.e.  $E_{oed}^{ref} / E_{50}^{ref} < 0.5$ . For such soils, the Soft Soil model may be used.

Some features of the Soft Soil model are:

- Stress dependent stiffness (logarithmic compression behaviour).
- Distinction between primary loading and unloading-reloading.
- Memory for pre-consolidation stress.
- Failure behaviour according to the Mohr-Coulomb criterion.

### 8.1 ISOTROPIC STATES OF STRESS AND STRAIN ( $\sigma'_1 = \sigma'_2 = \sigma'_3$ )

In the Soft Soil model, it is assumed that there is a logarithmic relation between changes in volumetric strain,  $\varepsilon_v$ , and changes in mean effective stress,  $p'$ , which can be formulated as:

$$\varepsilon_v - \varepsilon_v^0 = -\lambda^* \ln \left( \frac{p' + c \cot \varphi}{p^0 + c \cot \varphi} \right) \quad (\text{virgin compression}) \quad (8.1)$$

In order to maintain the validity of Eq. (11.1) a minimum value of  $p'$  is set equal to a unit stress. The parameter  $\lambda^*$  is the modified compression index, which determines the compressibility of the material in primary loading. Note that  $\lambda^*$  differs from the index  $\lambda$  as used by Burland (1965). The difference is that Eq. (11.1) is a function of volumetric strain instead of void ratio. Plotting Eq. (11.1) gives a straight line as shown in Figure 11.3.

During isotropic unloading and reloading a different path (line) is followed, which can be formulated as:

$$\varepsilon_v^e - \varepsilon_v^{e0} = -\kappa^* \ln \left( \frac{p' + c \cot \varphi}{p^0 + c \cot \varphi} \right) \quad (\text{unloading and reloading}) \quad (8.2)$$

Again, a minimum value of  $p'$  is set equal to a unit stress. The parameter  $\kappa^*$  is the modified swelling index, which determines the compressibility of the material in unloading and subsequent reloading. Note that  $\kappa^*$  differs from the index  $\kappa$  as used by Burland. The ratio  $\lambda^*/\kappa^*$  is, however, equal to Burland's ratio  $\lambda/\kappa$ . The soil response during unloading and reloading is assumed to be elastic as denoted by the superscript  $e$  in Eq. (11.2). The elastic behaviour is described by Hooke's law. Eq. (11.2) implies linear stress dependency on the tangent bulk modulus such that:

$$K_{ur} \equiv \frac{E_{ur}}{3(1 - 2\nu_{ur})} = \frac{p' + c \cot \varphi}{\kappa^*} \quad (8.3)$$

in which the subscript  $ur$  denotes unloading / reloading. Note that effective parameters are considered rather than undrained soil properties, as might be suggested by the subscripts  $ur$ . Neither the elastic bulk modulus,  $K_{ur}$ , nor the elastic Young's modulus,  $E_{ur}$ , is used as an input parameter. Instead,  $\nu_{ur}$  and  $\kappa^*$  are used as input constants for the part of the model that computes the elastic strains.

An infinite number of unloading / reloading lines may exist in Figure 8.1, each corresponding to a particular value of the isotropic pre-consolidation stress  $p_p$ . The pre-consolidation stress represents the largest stress level experienced by the soil. During unloading and reloading, this pre-consolidation stress remains constant. In primary loading, however, the pre-consolidation stress increases with the stress level, causing irreversible (plastic) volumetric strains.

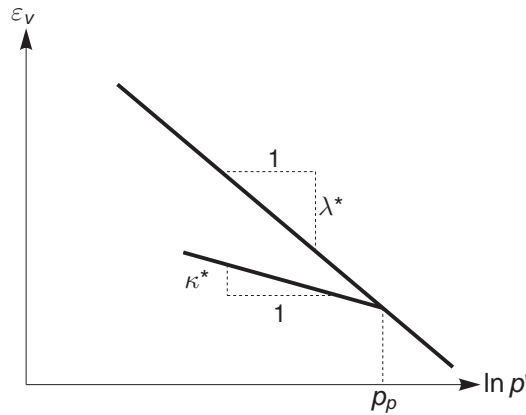


Figure 8.1 Logarithmic relation between volumetric strain and mean stress

## 8.2 YIELD FUNCTION

The yield function of the Soft Soil model is defined as:

$$f = \bar{f} - p_p \quad (8.4)$$

where  $\bar{f}$  is a function of the stress state ( $p'$ ,  $\tilde{q}$ ) and  $p_p$ , the pre-consolidation stress, is a function of plastic strain such that:

$$\bar{f} = \frac{\tilde{q}^2}{M^2(p' + c \cot \varphi)} + p' \quad (8.5)$$

$$p_p = p_p^0 \exp\left(\frac{-\varepsilon_v^p}{\lambda^* - \kappa^*}\right) \quad (8.6)$$

and  $\tilde{q}$  is a similar deviatoric stress quantity as defined for the cap yield surface in the Hardening Soil model:  $\tilde{q} = \sigma'_1 + (\alpha - 1)\sigma'_2 - \alpha\sigma'_3$  where  $\alpha = (3 + \sin \varphi)/(3 - \sin \varphi)$ .

The yield function ( $f = 0$ ) describes an ellipse in the  $p'$  -  $\tilde{q}$ -plane, as illustrated in Figure 8.2. The parameter  $M$  in Eq. (8.5) determines the height of the ellipse. The height of the ellipse is responsible for the ratio of horizontal to vertical stresses in primary one-dimensional compression.

As a result, the parameter  $M$  determines largely the coefficient of lateral earth pressure  $K_0^{nc}$ . In view of this, the value of  $M$  can be chosen such that a known value of  $K_0^{nc}$  is matched in primary one-dimensional compression. Such an interpretation and use of  $M$  differs from the original critical state line idea, but it ensures a proper matching of  $K_0^{nc}$ .

The tops of all ellipses are located on a line with slope  $M$  in the  $p'$  -  $\tilde{q}$ -plane. In (Burland, 1965) and (Burland, 1967) the  $M$ -line is referred to as the critical state line and represents stress states at post peak failure. The parameter  $M$  is then based on the critical state friction angle. In the Soft Soil model, however, failure is not necessarily related to critical state. The Mohr-Coulomb failure criterion is a function of the strength parameters  $\varphi$  and  $c$ , which might not correspond to the  $M$ -line. The isotropic pre-consolidation stress  $p_p$  determines the extent of the ellipse along  $p'$  axis. During loading, an infinite number of ellipses may exist (see Figure 8.2) each corresponding to a

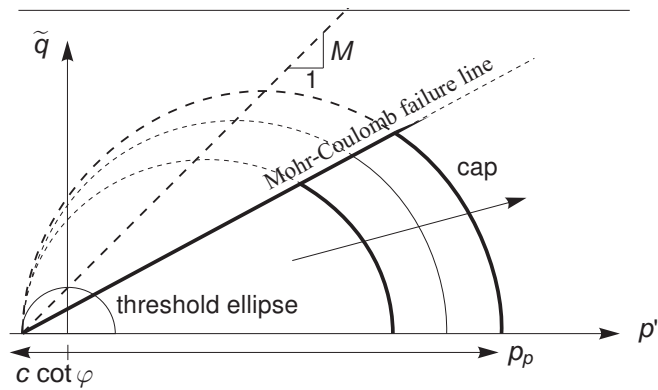


Figure 8.2 Yield surface of the Soft Soil model in  $p'$  -  $q$ -plane

particular value of  $p_p$ . In tension ( $p' < 0$ ), the ellipse extends to  $c \cot \varphi$  (Eq. (8.5) and Figure 8.2). In order to make sure that the right hand side of the ellipse (i.e. the 'cap') will remain in the 'compression' zone ( $p' > 0$ ) a minimum value of  $c \cot \varphi$  is adopted for  $p_p$ . For  $c = 0$ , a minimum value of  $p_p$  equal to a stress unit is adopted. Hence, there is a 'threshold' ellipse as illustrated in Figure 8.2.

The value of  $p_p$  is determined by volumetric plastic strain following the hardening relation, Eq. (8.6). This equation reflects the principle that the pre-consolidation stress increases exponentially with decreasing volumetric plastic strain (compaction).  $p_p^0$  can be regarded as the initial value of the pre-consolidation stress. The determination of  $p_p^0$  is treated in Section 2.8. According to Eq. (8.6) the initial volumetric plastic strain is assumed to be zero.

In the Soft Soil model, the yield function, Eq. (8.4), describes the irreversible volumetric strain in primary compression, and forms the cap of the yield contour. To model the failure state, a perfectly-plastic Mohr-Coulomb type yield function is used. This yield function represents a straight line in  $p'$  -  $q$ -plane as shown in Figure 8.2. The slope of the failure line is smaller than the slope of the  $M$ -line.

The total yield contour, as shown by the bold lines in Figure 8.2, is the boundary of the elastic stress area. The failure line is fixed, but the cap may increase in primary compression. Stress paths within this boundary give only elastic strain increments, whereas stress paths that tend to cross the boundary generally give both elastic and plastic strain increments.

For general states of stress  $(p', \tilde{q})$ , the plastic behaviour of the Soft Soil model is defined by the combination of the cap yield function and the Mohr-Coloumb yield functions. The total yield contour in principal stress space is indicated in Figure 8.3.



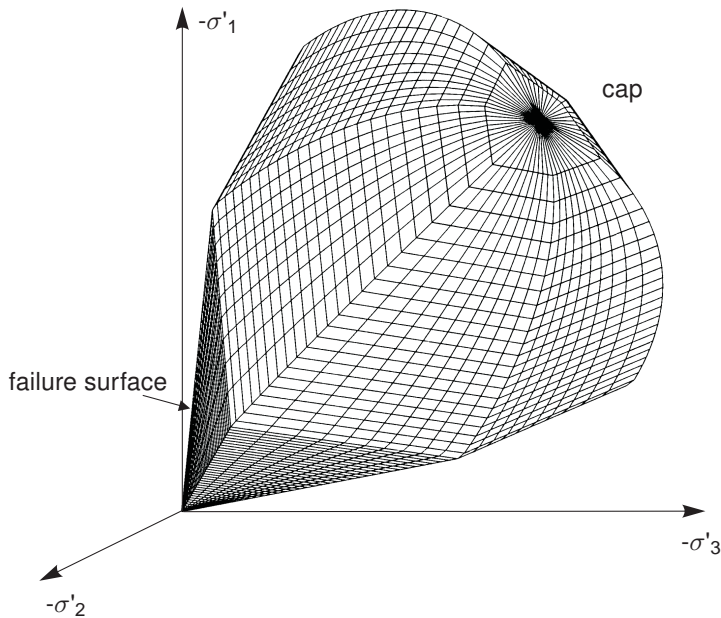


Figure 8.3 Representation of total yield contour of the Soft Soil model in principal stress space

### 8.3 PARAMETERS OF THE SOFT SOIL MODEL

The parameters of the Soft Soil model include compression and swelling indices, which are typical for soft soils, as well as the Mohr-Coulomb model failure parameters. In total, the Soft Soil model requires the following parameters to be determined:

Basic parameters:

$\lambda^*$	: Modified compression index	[-]
$\kappa^*$	: Modified swelling index	[-]
$c$	: Effective cohesion	[kN/m <sup>2</sup> ]
$\varphi$	: Friction angle	[°]
$\psi$	: Dilatancy angle	[°]
$\sigma_t$	: Tensile strength	[kN/m <sup>2</sup> ]

Advanced parameters (use default settings):

$\nu_{ur}$	: Poisson's ratio for unloading / reloading	[-]
$K_0^{nc}$	: Coefficient of lateral stress in normal consolidation	[-]
$M$	: $K_0^{nc}$ -parameter	[-]

Figure 8.4 shows the PLAXIS window for inputting the values of the model parameters.  $M$  is calculated automatically from the coefficient of the lateral earth pressure,  $K_0^{nc}$ , by means of Eq. (8.8). Note that, physically, in the current model  $M$  differs from the same parameter in the Modified Cam-Clay model where it is related to the material friction.

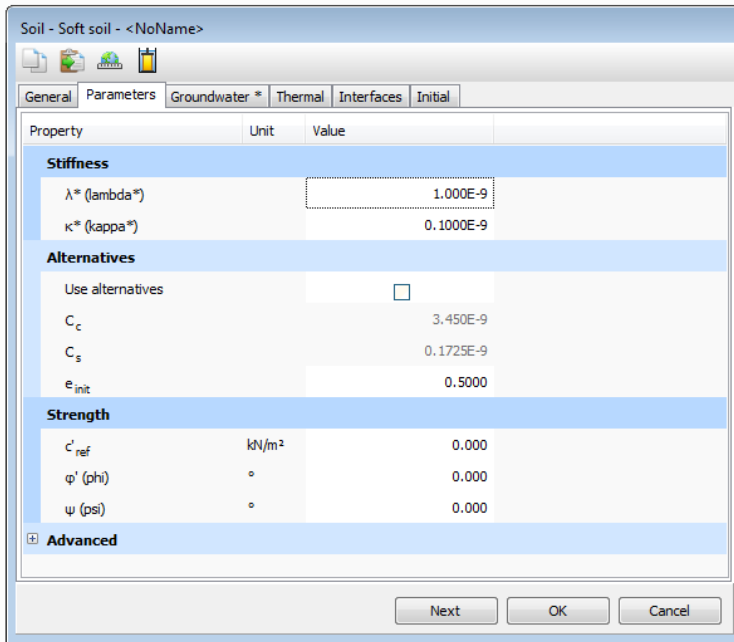


Figure 8.4 Parameters tabsheet for the Soft Soil model

**Modified swelling index and modified compression index**

These parameters can be obtained from an isotropic compression test including isotropic unloading. When plotting the logarithm of the mean stress as a function of the volumetric strain for clay-type materials, the plot can be approximated by two straight lines (see Figure 8.1). The slope of the primary loading line gives the modified compression index, and the slope of the unloading (or swelling) line gives the modified swelling index. Note that there is a difference between the modified indices  $\kappa^*$  and  $\lambda^*$  and the original Cam-Clay parameters  $\kappa$  and  $\lambda$ . The latter parameters are defined in terms of the void ratio  $e$  instead of the volumetric strain  $\varepsilon_v$ .

Apart from the isotropic compression test, the parameters  $\kappa^*$  and  $\lambda^*$  can be obtained from a one-dimensional compression test. Here a relationship exists with the internationally recognized parameters for one-dimensional compression and swelling,  $C_c$  and  $C_s$ . These relationships are summarized in Table 8.1.

Table 8.1a Relationship to Cam-Clay parameters

1. $\lambda^* = \frac{\lambda}{1 + e}$	2. $\kappa^* = \frac{\kappa}{1 + e}$
----------------------------------------	--------------------------------------

Table 8.1b Relationship to internationally normalized parameters

3. $\lambda^* = \frac{C_c}{2.3(1 + e)}$	4. $\kappa^* \approx \frac{2C_s}{2.3(1 + e)}$
-----------------------------------------	-----------------------------------------------

Remarks on Table 8.1:

- In relations 1 and 2, the void ratio,  $e$ , is assumed to be constant. In fact,  $e$  will change during a compression test, but this will give a relatively small difference in void ratio. For  $e$  one can use the average void ratio that occurs during the test or just

the initial value.

- In relation 4 there is no exact relation between  $\kappa^*$  and the one-dimensional swelling index  $C_s$ , because the ratio of horizontal and vertical stresses changes during one-dimensional unloading. For this approximation it is assumed that the average stress state during unloading is an isotropic stress state, i.e. the horizontal and vertical stresses are equal.
- In practice, swelling is often assumed to be equivalent to recompression behaviour, which, may not be right. Hence  $\kappa^*$  should be based on  $C_s$  rather than the recompression index  $C_r$ .
- The factor 2.3 in relation 3 is obtained from the ratio between the logarithm of base 10 and the natural logarithm.
- The ratio  $\lambda^*/\kappa^*$  ( $=\lambda/\kappa$ ) ranges, in general, between 2.5 and 7.

### **Cohesion**

The cohesion has the dimension of stresses. A small effective cohesion may be used, including a cohesion of zero. Entering a cohesion will result in an elastic region that is partly located in the 'tension' zone, as illustrated in Figure 8.2. The left hand side of the ellipse crosses the  $p'$ -axis at a value of  $c \cot \varphi$ . In order to maintain the right hand side of the ellipse (i.e. the cap) in the 'pressure' zone of the stress space, the isotropic pre-consolidation stress  $p_p$  has a minimum value of  $c \cot \varphi$ . This means that entering a cohesion larger than zero may result in a state of 'over-consolidation', depending on the magnitude of the cohesion and the initial stress state. As a result, a stiffer behaviour is obtained during the onset of loading. It is not possible to specify undrained shear strength by means of high cohesion and a friction angle of zero. Input of model parameters should always be based on effective values. The PLAXIS option to model undrained behaviour using effective parameters may be used (*Undrained (A)*). Please note that the resulting effective stress path may not be accurate, which may lead to an unrealistic undrained shear strength. Hence, when using *Undrained (A)* as drainage type, the resulting stress state must be checked against a known undrained shear strength profile.

### **Friction angle**

The effective angle of internal friction represents the increase of shear strength with effective stress level. It is specified in degrees. Zero friction angle is not allowed. On the other hand, care should be taken with the use of high friction angles. It is often recommended to use  $\varphi_{cv}$ , i.e. the critical state friction angle, rather than a higher value based on small strains. Moreover, using a high friction angle will substantially increase the computational requirements.

### **Dilatancy angle**

For the type of materials, which can be described by the Soft Soil model, the dilatancy can generally be neglected. A dilatancy angle of zero degrees is considered in the standard settings of the Soft Soil model.

### Poisson's ratio

In the Soft Soil model, the Poisson's ratio  $\nu$  is the well known pure elastic constant rather than the pseudo-elasticity constant as used in the linear elastic perfectly-plastic model. Its value will usually be in the range between 0.1 and 0.2. If the standard setting for the Soft Soil model parameters is selected, then  $\nu_{ur} = 0.15$  is automatically used. For loading of normally consolidated materials, Poisson's ratio plays a minor role, but it becomes important in unloading problems. For example, for unloading in a one-dimensional compression test (oedometer), the relatively small Poisson's ratio will result in a small decrease of the lateral stress compared with the decrease in vertical stress. As a result, the ratio of horizontal and vertical stress increases, which is a well-known phenomenon in overconsolidated materials. Hence, Poisson's ratio should not be based on the normally consolidated  $K_0^{nc}$ -value, but on the ratio of the horizontal stress increment to the vertical stress increment in oedometer unloading and reloading test such that:

$$\frac{\nu_{ur}}{1 - \nu_{ur}} = \frac{\Delta\sigma_{xx}}{\Delta\sigma_{yy}} \quad (\text{unloading and reloading}) \quad (8.7)$$

### $K_0^{nc}$ -parameter

The parameter  $M$  is automatically determined based on the coefficient of lateral earth pressure in normally consolidated condition,  $K_0^{nc}$ , as entered by the user. The exact relation between  $M$  and  $K_0^{nc}$  gives (Brinkgreve, 1994):

$$M = 3 \sqrt{\frac{(1 - K_0^{nc})^2}{(1 + 2K_0^{nc})^2} + \frac{(1 - K_0^{nc})(1 - 2\nu_{ur})(\lambda^*/\kappa^* - 1)}{(1 + 2K_0^{nc})(1 - 2\nu_{ur})\lambda^*/\kappa^* - (1 - K_0^{nc})(1 + \nu_{ur})}} \quad (8.8)$$

The value of  $M$  is indicated in the input window. As can be seen from Eq. (8.8),  $M$  is also influenced by the Poisson's ratio  $\nu_{ur}$  and by the ratio  $\lambda^*/\kappa^*$ . However, the influence of  $K_0^{nc}$  is dominant. Eq. (8.8) can be approximated by:

$$M \approx 3.0 - 2.8K_0^{nc} \quad (8.9)$$

## 8.4 STATE PARAMETERS IN THE SOFT SOIL MODEL

In addition to the output of standard stress and strain, the Soft Soil model provides output (when being used) on state variables such as the hardening parameter  $\gamma^p$  and the isotropic pre-consolidation stress  $p_p$ . These parameters can be visualised by selecting the *State parameters* option from the *stresses* menu. An overview of available state parameters is given below:

$p^{eq}$  : Equivalent isotropic stress [kN/m<sup>2</sup>]

$$p^{eq} = \sqrt{\frac{\tilde{q}^2}{M^2} + (p')^2}$$

$P_p$  : Isotropic preconsolidation stress [kN/m<sup>2</sup>]

$OCR$  : Isotropic over-consolidation ratio ( $OCR = P_p/p^{eq}$ ) [-]

$\gamma^p$  : Hardening parameter (equivalent mobilised plastic shear strain) [-]

$E_{ur}$	: Current stress-dependent elastic Young's modulus	[-]
$c$	: Current depth-dependent cohesion	[-]

## 8.5 ON THE USE OF THE SOFT SOIL MODEL IN DYNAMIC CALCULATIONS

When using the Soft Soil model in dynamic calculations, the modified swelling index  $\kappa^*$  needs to be selected such that the model correctly predicts wave velocities in the soil. This generally requires a smaller value than just an unloading-reloading index.

When subjected to dynamic or cyclic loading, the Soft Soil model will generate plastic strains when the preconsolidation stress is increased. However, it should be noted that stress cycles within the current hardening contour will only generate elastic strains and no (hysteretic) damping, nor accumulation of strains or pore pressure, nor liquefaction. In order to account for the soil damping in cyclic loading, Rayleigh damping may be defined.



## 9 SOFT SOIL CREEP MODEL (TIME DEPENDENT BEHAVIOUR)

### 9.1 INTRODUCTION

Both the Hardening Soil model and the Soft Soil model can be used to model the behaviour of compressible soft soils, but none of these models are suitable when considering creep, i.e. secondary compression. All soils exhibit some creep, and primary compression is thus always followed by a certain amount of secondary compression. Assuming the secondary compression (for instance during a period of 10 or 30 years) to be a certain percentage of the primary compression, it is clear that creep is important for problems involving large primary compression. This is for instance the case when constructing embankments on soft soils. Indeed, large primary settlements of footings and embankments are usually followed by substantial creep settlements in later years. In such cases it is desirable to estimate the creep from FEM-computations.

Foundations may also be founded on initially overconsolidated soil layers that yield relatively small primary settlements. Then, as a consequence of the loading, a state of normal consolidation may be reached and significant creep may follow. This is a treacherous situation as considerable secondary compression is not preceded by the warning sign of large primary compression. Again, computations with a creep model are desirable.

Buisman (1936) was probably the first to propose a creep law for clay after observing that soft-soil settlements could not be fully explained by classical consolidation theory. This work on 1D-secondary compression was continued by other researchers including, for example, Bjerrum (1967), Garlanger (1972), Mesri & Godlewski (1977) and Leroueil (1977). More mathematical lines of research on creep were followed by, for example, Sekiguchi (1977), Adachi & Oka (1982) and Borja & Kavazanjian (1985). This mathematical 3D-creep modelling was influenced by the more experimental line of 1D-creep modelling, but conflicts exist.

3D-creep should be a straight forward extension of 1D-creep, but this is hampered by the fact that present 1D-models have not been formulated as differential equations. For the presentation of the Soft Soil Creep model we will first complete the line of 1D-modelling by conversion to a differential form. From this 1D differential equation an extension was made to a 3D-model. This chapter gives a full description of the formulation of the Soft Soil Creep model. In addition, attention is focused on the model parameters. Finally, a validation of the 3D model is presented by considering both model predictions and data from triaxial tests. Here, attention is focused on constant strain rate triaxial tests and undrained triaxial creep tests. For more applications of the model the reader is referred to Vermeer, Stolle & Bonnier (1998), Vermeer & Neher (1999) and Brinkgreve (2004).

Some basic characteristics of the Soft Soil Creep model are:

- Stress-dependent stiffness (logarithmic compression behaviour)
- Distinction between primary loading and unloading-reloading
- Secondary (time-dependent) compression
- Ageing of pre-consolidation stress
- Failure behaviour according to the Mohr-Coulomb criterion

## 9.2 BASICS OF ONE-DIMENSIONAL CREEP

When reviewing previous literature on secondary compression in oedometer tests, one is struck by the fact that it concentrates on behaviour related to step loading, even though natural loading processes tend to be continuous or transient in nature. Buisman (1936) was probably the first to consider such a classical creep test. He proposed the following equation to describe creep behaviour under constant effective stress:

$$\varepsilon = \varepsilon_c - C_B \log \left( \frac{t}{t_c} \right) \quad \text{for: } t > t_c \quad (9.1)$$

where  $\varepsilon_c$  is the strain up to the end of consolidation,  $t$  the time measured from the beginning of loading,  $t_c$  the time to the end of primary consolidation and  $C_B$  is a material constant.

Please note that we do not follow the soil mechanics convention that compression is considered positive. Instead, compressive stresses and strains are taken to be negative. For further consideration, it is convenient to rewrite this equation as:

$$\varepsilon = \varepsilon_c - C_B \log \left( \frac{t_c + t'}{t_c} \right) \quad \text{for: } t' > 0 \quad (9.2)$$

with  $t' = t - t_c$  being the effective creep time.

Based on the work by Bjerrum on creep, as published for instance in 1967, Garlanger (1972) proposed a creep equation of the form:

$$e = e_c - C_\alpha \log \left( \frac{\tau_c + t'}{\tau_c} \right) \quad \text{with: } C_\alpha = C_B(1 + e_0) \quad \text{for: } t' > 0 \quad (9.3)$$

Differences between Garlanger's and Buisman's forms are modest. The engineering strain  $\varepsilon$  is replaced by void ratio  $e$  and the consolidation time  $t_c$  is replaced by the parameter  $\tau_c$ . Eqs. (9.2) and (9.3) are identical when choosing  $\tau_c = t_c$ . For the case that  $\tau_c \neq t_c$ , differences between both formulations will vanish when the effective creep time  $t'$  increases.

For practical consulting, oedometer tests are usually interpreted by assuming  $t_c = 24h$ . Indeed, the standard oedometer test is a *Multiple Stage Loading Test* with loading periods of precisely one day. Due to the special assumption that this loading period coincides to the consolidation time  $t_c$ , it follows that such tests have no effective creep time.

Hence one obtains  $t' = 0$  and the log-term drops out of Eq. (9.3). It would thus seem that there is no creep in this standard oedometer test, but this suggestion is false. Even highly impermeable oedometer samples need less than one hour for primary consolidation.

Then all excess pore pressures are zero and one observes pure creep for the other 23 hours of the day. Therefore we will not make any assumptions about the precise values of  $\tau_c$  and  $t_c$ .

Another slightly different possibility to describe secondary compression is the form adopted by Butterfield (1979):

$$\varepsilon^H = \varepsilon_c^H - c \ln \left( \frac{\tau_c + t'}{\tau_c} \right) \quad (9.4)$$



where  $\varepsilon^H$  is the logarithmic strain defined as:

$$\varepsilon^H = \ln \left( \frac{V}{V_0} \right) = \ln \left( \frac{1 + e}{1 + e_0} \right) \quad (9.5)$$

with the subscript '0' denoting the initial values. The superscript 'H' is used to denote logarithmic strain, as the logarithmic strain measure was originally used by Hencky. For small strains it is possible to show that:

$$c = \frac{C_\alpha}{(1 + e_0) \cdot \ln 10} = \frac{C_B}{\ln 10} \quad (9.6)$$

because then logarithmic strain is approximately equal to the engineering strain. Both Butterfield (1979) and den Haan (1994) showed that for cases involving large strain, the logarithmic small strain supersedes the traditional engineering strain.

### 9.3 ON THE VARIABLES $\tau_c$ AND $c$

In this section attention will first be focused on the variable  $\tau_c$ . Here a procedure is to be described for an experimental determination of this variable. In order to do so we depart from Eq. (9.4). By differentiating this equation with respect to time and dropping the superscript 'H' to simplify the notation, one finds:

$$-\dot{\varepsilon} = \frac{c}{\tau_c + t'} \quad \text{or inversely:} \quad -\frac{1}{\dot{\varepsilon}} = \frac{\tau_c + t'}{c} \quad (9.7)$$

which allows one to make use of the construction developed by Janbu (1969) for evaluating the parameters  $c$  and  $\tau_c$  from experimental data. Both the traditional way, being indicated in Figure 9.1a, as well as the Janbu method of Figure 9.1b can be used to determine the parameter  $c$  from an oedometer test with constant load.

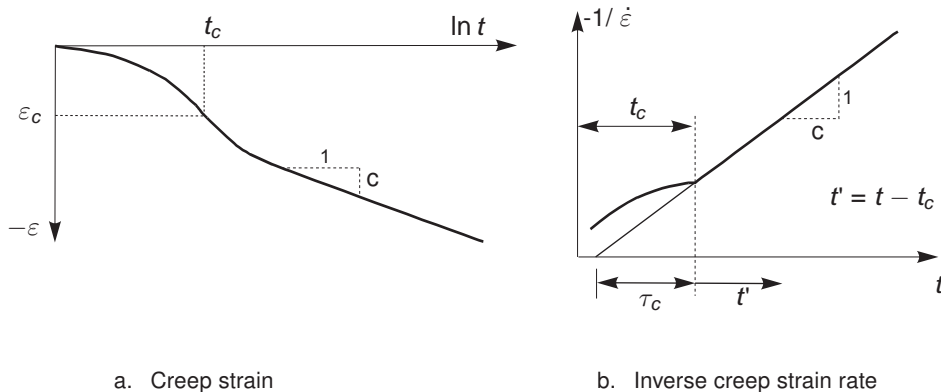


Figure 9.1 Standard oedometer test

The use of the Janbu method is attractive, because both  $\tau_c$  and  $c$  follow directly when fitting a straight line through the data. In Janbu's representation of Figure 9.1b,  $\tau_c$  is the intercept with the (non-logarithmic) time axis of the straight creep line. The deviation from a linear relation for  $t < t_c$  is due to consolidation.

Considering the classical literature it is possible to describe the end-of-consolidation strain  $\varepsilon_c$ , by an equation of the form:

$$\varepsilon_c = \varepsilon_c^e + \varepsilon_c^c = -a \ln\left(\frac{\sigma'}{\sigma'_{0'}}\right) - (b - a) \ln\left(\frac{\sigma_{pc}}{\sigma_{p0}}\right) \quad (9.8)$$

Note that  $\varepsilon$  is a logarithmic strain, rather than a classical small strain although we conveniently omit the subscript 'H'. In the above equation  $\sigma'_{0'}$  represents the initial effective pressure before loading and  $\sigma'$  is the final effective loading pressure. The values  $\sigma_{p0}$  and  $\sigma_{pc}$  represent the pre-consolidation pressure corresponding to before-loading and end-of-consolidation states respectively. In most literature on oedometer testing, one adopts the void ratio  $e$  instead of  $\varepsilon$ , and  $\log$  instead of  $\ln$ , the swelling (recompression) index  $C_s$  instead of  $a$ , and the compression index  $C_c$  instead of  $b$ . The above constants  $a$  and  $b$  relate to  $C_s$  and  $C_c$  as:

$$a = \frac{C_s}{(1 + e_0) \cdot \ln 10} \qquad b = \frac{C_c}{(1 + e_0) \cdot \ln 10} \quad (9.9)$$

Combining Eqs. (9.4) and (9.8) it follows that:

$$\varepsilon = \varepsilon^e + \varepsilon^c = -a \ln\left(\frac{\sigma'}{\sigma'_{0'}}\right) - (b - a) \ln\left(\frac{\sigma_{pc}}{\sigma_{p0}}\right) - c \ln\left(\frac{\tau_c + t'}{\tau_c}\right) \quad (9.10)$$

where  $\varepsilon$  is the total logarithmic strain due to an increase in effective stress from  $\sigma'_{0'}$  to  $\sigma'$  and a time period of  $t_c + t'$ . In Figure 9.2 the terms of Eq. (9.10) are depicted in an  $\varepsilon - \ln \sigma$  diagram.

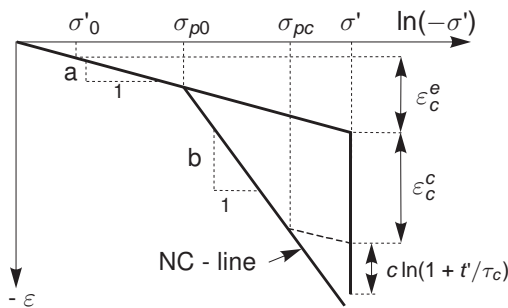


Figure 9.2 Idealised stress-strain curve from oedometer test with division of strain increments into an elastic and a creep component. For  $t' + t_c = 1$  day, one arrives precisely on the NC-line

Up to this point, the more general problem of creep under transient loading conditions has not yet been addressed, as it should be recalled that restrictions have been made to creep under constant load. For generalising the model, a differential form of the creep model is needed. No doubt, such a general equation may not contain  $t'$  and neither  $\tau_c$  as the consolidation time is not clearly defined for transient loading conditions.

#### 9.4 DIFFERENTIAL LAW FOR 1D-CREEP

The previous equations emphasize the relation between accumulated creep and time, for a given constant effective stress. For solving transient or continuous loading problems, it

is necessary to formulate a constitutive law in differential form, as will be described in this section. In a first step we will derive an equation for  $\tau_c$ . Indeed, despite the use of logarithmic strain and  $\ln$  instead of  $\log$ , equation (Eq. 9.10) is classical without adding new knowledge. Moreover, the question on the physical meaning of  $\tau_c$  is still open. In fact, we have not been able to find precise information on  $\tau_c$  in the literature, apart from Janbu's method of experimental determination.

In order to find an analytical expression for the quantity  $\tau_c$ , we adopt the basic idea that all inelastic strains are time dependent. Hence total strain is the sum of an elastic part  $\varepsilon^e$  and a time-dependent creep part  $\varepsilon^c$ . For non-failure situations as met in oedometer loading conditions, we do not assume an instantaneous plastic strain component, as used in traditional elastoplastic modelling. In addition to this basic concept, we adopt Bjerrum's idea that the pre-consolidation stress depends entirely on the amount of creep strain being accumulated in the course of time. In addition to (9.10) we therefore introduce the expression:

$$\varepsilon = \varepsilon^e + \varepsilon^c = -a \ln \left( \frac{\sigma'}{\sigma'_{p0}} \right) - (b - a) \ln \left( \frac{\sigma_{pc}}{\sigma_{p0}} \right) \quad (9.11)$$

where

$$\sigma_p = \sigma_{p0} \exp \left( \frac{-\varepsilon^c}{b - a} \right)$$

Please note that  $\varepsilon^c$  is negative, so that  $\sigma_p$  exceeds  $\sigma_{p0}$ . The longer a soil sample is left to creep the larger  $\sigma_p$  grows. The time-dependency of the pre-consolidation pressure  $\sigma_p$  is now found by combining Eqs. (9.10) and (9.11) to obtain:

$$\varepsilon^c - \varepsilon_c^c = -(b - a) \ln \left( \frac{\sigma_p}{\sigma_{pc}} \right) = -c \ln \left( \frac{\tau_c + t'}{\tau_c} \right) \quad (9.12)$$

This equation can now be used for a better understanding of  $\tau_c$ , at least when adding knowledge from standard oedometer loading. In conventional oedometer testing the load is stepwise increased and each load step is maintained for a constant period of  $t_c + t' = \tau$ , where  $\tau$  is precisely one day.

In this way of stepwise loading the so-called normal consolidation line (NC-line) with  $\sigma_p = \sigma'$  is obtained. On entering  $\sigma_p = \sigma'$  and  $t' = \tau - t_c$  into Eq. (9.12) it is found that:

$$(b - a) \ln \left( \frac{\sigma'}{\sigma_{pc}} \right) = c \ln \left( \frac{\tau_c + \tau_c}{\tau_c} \right) \quad \text{for: } \text{OCR} = 1 \quad (9.13)$$

It is now assumed that  $(\tau_c - t_c) \ll \tau$ . This quantity can thus be disregarded with respect to  $\tau$  and it follows that:

$$\frac{\tau}{\tau_c} = \left( \frac{\sigma'}{\sigma_{pc}} \right)^{\frac{b-a}{c}} \quad \text{or:} \quad \tau_c = \tau \left( \frac{\sigma_{pc}}{\sigma'} \right)^{\frac{b-a}{c}} \quad (9.14)$$

Hence  $\tau_c$  depends both on the effective stress  $\sigma'$  and the end-of-consolidation pre-consolidation stress  $\sigma_{pc}$ . In order to verify the assumption  $(\tau_c - t_c) \ll \tau$ , it should be realised that usual oedometer samples consolidate for relatively short periods of less than one hour. Considering load steps on the normal consolidation line, we have  $\text{OCR}=1$  both in the beginning and at the end of the load step. During such a load step  $\sigma_p$

increases from  $\sigma_{p0}$  up to  $\sigma_{pc}$  during the short period of (primary) consolidation. Hereafter  $\sigma_p$  increases further from  $\sigma_{pc}$  up to  $\sigma'$  during a relatively long creep period. Hence, at the end of the day the sample is again in a state of normal consolidation, but directly after the short consolidation period the sample is under-consolidated with  $\sigma_p < \sigma'$ . For the usually very high ratios of  $(b - a)/c \geq 15$ , we thus find very small  $\tau_c$ -values from Eq. (9.14). Hence not only  $t_c$  but also  $\tau_c$  tends to be small with respect to  $\tau$ . It thus follows that the assumption  $(\tau_c - t_c) \ll \tau$  is certainly correct.

Having derived the simple expression Eq. (9.14) for  $\tau_c$ , it is now possible to formulate the differential creep equation. To this end Eq. (9.10) is differentiated to obtain:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = -a \frac{\dot{\sigma}'}{\sigma'} - \frac{c}{\tau_c + t'} \quad (9.15)$$

where  $\tau_c + t'$  can be eliminated by means of Eq. (9.12) to obtain:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = -a \frac{\dot{\sigma}'}{\sigma'} - \frac{c}{\tau_c} \left( \frac{\sigma_{pc}}{\sigma_p} \right) \frac{b-a}{c}$$

with:

$$\sigma_p = \sigma_{p0} \exp\left(\frac{-\varepsilon^c}{b-a}\right) \quad (9.16)$$

Again it is recalled that  $\varepsilon^c$  is a compressive strain, being considered negative in this manual. Eq. (9.14) can now be introduced to eliminate  $\tau_c$  and  $\sigma_{pc}$  and to obtain:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = -a \frac{\dot{\sigma}'}{\sigma'} - \frac{c}{\tau} \left( \frac{\sigma'}{\sigma_p} \right) \frac{b-a}{c} \quad (9.17)$$

## 9.5 THREE-DIMENSIONAL-MODEL

On extending the 1D-model to general states of stress and strain, the well-known stress invariants for isotropic stress  $p$  and deviatoric stress  $q$  are adopted. These invariants are used to define a new stress measure named  $p^{eq}$ :

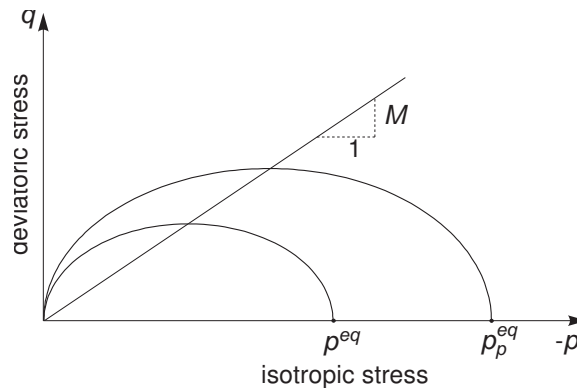
$$p^{eq} = p' + \frac{\tilde{q}^2}{M^2(p' + c \cot \varphi)} \quad (9.18)$$

and  $\tilde{q}$  is a similar deviatoric stress quantity as defined in the Hardening Soil model and Soft Soil model. In Figure 9.3 it is shown that the stress measure  $p^{eq}$  is constant on ellipses in  $p - \tilde{q}$ -plane. In fact we have the ellipses from the Modified Cam-Clay model as introduced by Roscoe & Burland (1968).

The soil parameter  $M$  represents the slope of the so-called '*critical state line*' as also indicated in Figure 9.3. We use:

$$M = \frac{6 \sin \varphi_{cv}}{3 - \sin \varphi_{cv}} \quad (9.19)$$

where  $\varphi_{cv}$  is the critical-void friction angle, also referred to as critical-state friction angle. The equivalent pressure  $p^{eq}$  is constant along ellipsoids in principal stress space. To


 Figure 9.3 Diagram of  $p^{eq}$ -ellipse in a  $p$ - $\tilde{q}$ -plane

extend the 1D-theory to a general 3D-theory, attention is now focused on normally consolidated states of stress and strain as met in oedometer testing. In such situations it yields  $\sigma'_2 = \sigma'_3 = K_0^{nc} \sigma'_1$ , and it follows from Eq. (9.18) that:

$$\begin{aligned} p^{eq} &= \sigma'_1 \left[ \frac{1 + 2K_0^{nc}}{3} + \frac{3(1 - K_0^{nc})^2}{M^2(1 + 2K_0^{nc})} \right], \\ p_p^{eq} &= \sigma_p \left[ \frac{1 + 2K_0^{nc}}{3} + \frac{3(1 - K_0^{nc})^2}{M^2(1 + 2K_0^{nc})} \right] \end{aligned} \quad (9.20)$$

where  $p_p^{eq}$  is a generalised pre-consolidation pressure, being simply proportional to the one-dimensional one  $\sigma_p$ . For known values of  $K_0^{nc}$ ,  $p^{eq}$  can thus be computed from  $\sigma'_1$ , and  $p_p^{eq}$  can thus be computed from  $\sigma_p$ . Omitting the elastic strain in the 1D-equation (Eq. 9.17), introducing the above expressions for  $p^{eq}$  and  $p_p^{eq}$  and writing  $\varepsilon_v$  instead of  $\varepsilon$  it is found that:

$$-\dot{\varepsilon}_v^c = \frac{c}{\tau} \left( \frac{p^{eq}}{p_p^{eq}} \right)^{\frac{b-a}{c}} \quad \text{where} \quad p_p^{eq} = p_{p0}^{eq} \exp\left(\frac{-\varepsilon_v^c}{b-a}\right) \quad (9.21)$$

For one-dimensional oedometer conditions, this equation reduces to Eq. (9.17), so that one has a true extension of the 1D-creep model. It should be noted that the subscript '0' is once again used in the equations to denote initial conditions and that  $\varepsilon_v^c = 0$  for time  $t = 0$ .

Instead of the parameters  $a$ ,  $b$  and  $c$  of the 1D-model, we will now change to the material parameters  $\kappa^*$ ,  $\lambda^*$  and  $\mu^*$ , who fit into the framework of critical-state soil mechanics. Conversion between constants follows the rules:

$$\kappa^* \approx 2a \qquad b = \lambda^* \qquad \mu^* = c \quad (9.22)$$

The expression for  $\kappa^*$  is an approximation. This is a result of the fact that during unloading and reloading under oedometer conditions the ratio of normal stress components changes and therefore the changes of  $p'$  and  $\sigma'_1$  deviate.

On using these new parameters, Eq. (9.21) changes to become:

$$-\dot{\varepsilon}_v^c = \frac{\mu^*}{\tau} \left( \frac{p^{eq}}{p_p^{eq}} \right)^{\frac{\lambda^* - \kappa^*}{\mu^*}} \quad \text{with} \quad p_p^{eq} = P_{p0}^{eq} \exp\left(\frac{-\varepsilon_v^c}{\lambda^* - \kappa^*}\right) \quad (9.23)$$

As yet the 3D-creep model is incomplete, as we have only considered a volumetric creep strain  $\varepsilon_v^c$ , whilst soft soils also exhibit deviatoric creep strains.

To introduce general creep strains, we adopt the view that creep strain is simply a time-dependent plastic strain. It is thus logic to assume a flow rule for the rate of creep strain, as usually done in plasticity theory. For formulating such a flow rule, it is convenient to adopt the vector notation and considering principal directions:

$$\underline{\sigma} = \begin{pmatrix} \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix}^T \quad \text{and} \quad \underline{\varepsilon} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{pmatrix}^T$$

where  $T$  is used to denote a transpose. Similar to the 1D-model we have both elastic and creep strains in the 3D-model. Using Hooke's law for the elastic part, and a flow rule for the creep part, one obtains:

$$\dot{\underline{\varepsilon}} = \dot{\underline{\varepsilon}}^e + \dot{\underline{\varepsilon}}^c = \underline{\underline{D}}^{-1} \dot{\underline{\sigma}}' + \lambda \frac{\partial g^c}{\partial \underline{\sigma}'} \quad (9.24)$$

where the elasticity matrix and the plastic potential function are defined as:

$$\underline{\underline{D}}^{-1} = \frac{1}{E_{ur}} \begin{bmatrix} 1 & -\nu_{ur} & -\nu_{ur} \\ -\nu_{ur} & 1 & -\nu_{ur} \\ -\nu_{ur} & -\nu_{ur} & 1 \end{bmatrix} \quad \text{and} \quad g^c = p^{eq}$$

where  $E_{ur}$  relates to the modified swelling index (Eq. (9.27)) Hence we use the equivalent pressure  $p^{eq}$  as a plastic potential function for deriving the individual creep strain-rate components. The subscripts 'ur' are introduced to emphasize that both the elasticity modulus and Poisson's ratio will determine unloading-reloading behaviour. Now it follows from the above equations that:

$$\dot{\varepsilon}_v^c = \dot{\varepsilon}_1^c + \dot{\varepsilon}_2^c + \dot{\varepsilon}_3^c = \lambda \cdot \left( \frac{\partial p^{eq}}{\partial \sigma'_1} + \frac{\partial p^{eq}}{\partial \sigma'_2} + \frac{\partial p^{eq}}{\partial \sigma'_3} \right) = \lambda \cdot \frac{\partial p^{eq}}{\partial p'} = \lambda \cdot \alpha \quad (9.25)$$

Hence we define  $\alpha = \partial p^{eq} / \partial p'$ . Together with Eqs. (9.23) and (9.24) this leads to:

$$\dot{\underline{\varepsilon}} = \underline{\underline{D}}^{-1} \dot{\underline{\sigma}}' + \frac{\dot{\varepsilon}_v^c}{\alpha} \frac{\partial p^{eq}}{\partial \underline{\sigma}'} = \underline{\underline{D}}^{-1} \dot{\underline{\sigma}}' - \frac{1}{\alpha} \frac{\mu^*}{\tau} \left( \frac{p^{eq}}{p_p^{eq}} \right)^{\frac{\lambda^* - \kappa^*}{\mu^*}} \frac{\partial p^{eq}}{\partial \underline{\sigma}'} \quad (9.26)$$

where:

$$p_p^{eq} = p_{p0}^{eq} \exp\left(\frac{-\varepsilon_v^c}{\lambda^* - \kappa^*}\right)$$

or inversely:

$$-\varepsilon_v^c = (\lambda^* - \kappa^*) \ln\left(\frac{p_p^{eq}}{p_{p0}^{eq}}\right)$$

## 9.6 FORMULATION OF ELASTIC 3D-STRAINS

Considering creep strains, it has been shown that the 1D-model can be extended to obtain the 3D-model, but as yet this has not been done for the elastic strains.

To get a proper 3D-model for the elastic strains as well, the elastic modulus  $E_{ur}$  has to be defined as a stress-dependent tangent stiffness according to:

$$E_{ur} = 3(1 - 2\nu_{ur})K_{ur} = -3(1 - 2\nu_{ur})\left(\frac{p' + c \cot \varphi}{\kappa^*}\right) \quad (9.27)$$

Hence,  $E_{ur}$  is not a new input parameter, but simply a variable quantity that relates to the input parameter  $\kappa^*$ . On the other hand  $\nu_{ur}$  is an additional true material constant.

Hence similar to  $E_{ur}$ , the bulk modulus  $K_{ur}$  is stress dependent according to the rule  $K_{ur} = -(p' + c \cot \varphi)^*/\kappa^*$  where in this context  $c$  is again the effective cohesion rather than the creep parameter. Now it can be derived for the volumetric elastic strain that:

$$\dot{\varepsilon}_V^e = \frac{\dot{p}'}{K_{ur}} = -\kappa^* \frac{\dot{p}'}{p' + c \cot \varphi} \quad \text{or by integration: } -\varepsilon_V^e = \kappa^* \ln\left(\frac{p' + c \cot \varphi}{p'_0 + c \cot \varphi}\right) \quad (9.28)$$

Hence in the 3D-model the elastic strain is controlled by the mean stress  $p'$ , rather than by principal stress  $\sigma'$  as in the 1D-model. However mean stress can be converted into principal stress. For one-dimensional compression on the normal consolidation line, we have both  $3p' = (1 + 2K_0^{nc})\sigma'$  and  $3p'_0 = (1 + 2K_0^{nc})\sigma'_0$  and it follows that  $p'/p'_0 = \sigma'/\sigma'_0$ . As a consequence, for  $c = 0$ , we derive the simple rule  $-\varepsilon_V^e = \kappa^* \ln \sigma'/\sigma'_0$ , whereas the 1D-model involves  $-\varepsilon_V^e = a \ln \sigma'/\sigma'_0$ . It would thus seem that  $\kappa^*$  coincides with  $a$ . Unfortunately this line of thinking cannot be extended toward overconsolidated states of stress and strain. For such situations, it can be derived that:

$$\frac{\dot{p}'}{p'} = \frac{1 + \nu_{ur}}{1 - \nu_{ur}} \frac{1}{1 + 2K_0} \frac{\dot{\sigma}'}{\sigma'} \quad (9.29)$$

and it follows that:

$$-\dot{\varepsilon}_V^e = \kappa^* \frac{\dot{p}'}{p'} = \frac{1 + \nu_{ur}}{1 - \nu_{ur}} \frac{\kappa^*}{1 + 2K_0} \frac{\dot{\sigma}'}{\sigma'} \quad (9.30)$$

where  $K_0$  depends to a great extent on the degree of over-consolidation. For many situations, it is reasonable to assume  $K_0 \approx 1$  and together with  $\nu_{ur} \approx 0.2$  one obtains  $-2\varepsilon_V^e \approx \kappa^* \ln(\sigma'/\sigma'_0)$ . Good agreement with the 1D-model is thus found by taking  $\kappa^* \approx 2a$ .

## 9.7 FORMULATION OF FAILURE CONDITION

The creep formulation does not include failure. Therefore, a Mohr-Coulomb type failure criterion, formulated in a perfect-plasticity framework, is added to the Soft Soil Creep model, generating plastic strains as soon as the failure condition is met. As soon as the Mohr-Coulomb failure yield criterion  $f(\underline{\sigma}', c, \varphi) = 0$  is met, instantaneous plastic strain rates develop according to the flow rule  $\dot{\underline{\varepsilon}}^p = \lambda \partial g / \partial \underline{\sigma}'$  with  $g = g(\sigma', \psi)$ . For details see Chapter 3 on the Mohr-Coulomb model.

In each stress point, the stresses are calculated according to the creep formulation before considering the failure criterion. Subsequently, the new stress state is checked against the failure criterion and corrected, if applicable.

### 9.8 PARAMETERS OF THE SOFT SOIL CREEP MODEL

In addition to the parameters of the Soft Soil Creep model, the Soft Soil Creep model involves a creep parameter in the form of the *Modified Creep index*  $\mu^*$ . In total, the Soft Soil Creep model requires the following parameters to be determined:

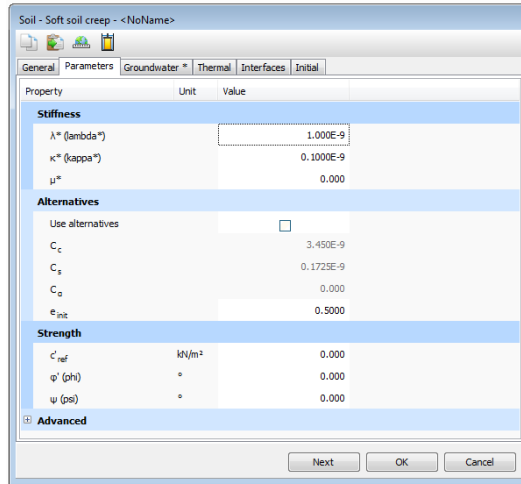


Figure 9.4 Parameters tabsheet for the Soft Soil Creep model

In conclusion, the Soft Soil Creep model requires the following material constants:

*Failure parameters as in the Mohr-Coulomb model:*

- $c$  : Effective cohesion [kN/m<sup>2</sup>]
- $\varphi$  : Friction angle [°]
- $\psi$  : Dilatancy angle [°]

*Basic stiffness parameters:*

- $\kappa^*$  : Modified swelling index [-]
- $\lambda^*$  : Modified compression index [-]
- $\mu^*$  : Modified creep index [-]

*Advanced parameters (it is advised to use the default setting):*

- $\nu_{ur}$  : Poisson's ratio for unloading-reloading (default 0.15) [-]
- $K_0^{nc}$  :  $\sigma'_{xx}/\sigma'_{yy}$  stress ratio in a state of normal consolidation [-]
- $M$  :  $K_0^{nc}$ -related parameter (see below) [-]



Instead of defining the stiffness by the basic stiffness parameters, alternative stiffness parameters can be used. These material constants are given by:

$C_c$	: Compression index	[-]
$C_s$	: Swelling index	[-]
$C_\alpha$	: Creep index for secondary compression	[-]
$e_{init}$	: Initial void ratio	[-]

### **Modified swelling index, modified compression index and modified creep index**

These parameters can be obtained from an isotropic compression test including isotropic unloading. When plotting the logarithm of the mean stress as a function of the volumetric strain for clay-type materials, the plot can be approximated by two straight lines (see Figure 8.1). The slope of the primary loading line gives the modified compression index, and the slope of the unloading (or swelling) line gives the modified swelling index. Note that there is a difference between the modified indices  $\kappa^*$  and  $\lambda^*$  and the original Cam-Clay parameters  $\kappa$  and  $\lambda$ . The latter parameters are defined in terms of the void ratio  $e$  instead of the volumetric strain  $\varepsilon_v$ . The parameter  $\mu^*$  (Table 9.1b) can be obtained by measuring the volumetric strain on the long term and plotting it against the logarithm of time (see Figure 9.1).

In Table 9.1c, the value 2.3 is in fact  $\ln 10$  and stems from the conversion from  $^{10}\log$  to natural logarithm. The alternative stiffness parameters can also be calculated from this table. Since the void ratio  $e$  is not a constant, in the conversion from the alternative parameters to the original model parameters in PLAXIS the void ratio  $e$  is defined as the initial void ratio  $e_{init}$ . Entering a particular value for one of the alternatives  $C_c$ ,  $C_s$  or  $C_\alpha$  results in a change of  $\lambda^*$ ,  $\kappa^*$  or  $\mu^*$  respectively.

As already indicated in Section 9.6, there is no exact relation between the isotropic compression index  $\kappa^*$  and the one-dimensional swelling indices  $A$  and  $C_s$ , because the ratio of horizontal and vertical stress changes during one-dimensional unloading. For the approximation it is assumed that the average stress state during unloading is an isotropic stress state, i.e. the horizontal and vertical stresses are equal.

For a rough estimate of the model parameters, one might use the correlation  $\lambda^* \approx I_p(\%)/500$ , the fact that  $\lambda^*/\mu^*$  is in the range between 15 to 25 and the general observation  $\lambda^*/\kappa^*$  is between 2.5 and 7.

Table 9.1a Relationship to Cam-Clay parameters

$\lambda^* = \frac{\lambda}{1+e}$	$\kappa^* = \frac{\kappa}{1+e}$	---
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Table 9.1b Relationship to  $A, B, C$  parameters

$\lambda^* = b$	$\kappa^* \approx 2a$	$\mu^* = c$
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Table 9.1c Relationship to internationally normalized parameters

$\lambda^* = \frac{C_c}{2.3(1+e)}$	$\kappa^* \approx \frac{2 C_s}{2.3(1+e)}$	$\mu^* = \frac{C_\alpha}{2.3(1+e)}$
------------------------------------	-------------------------------------------	-------------------------------------

For characterising a particular layer of soft soil, it is also necessary to know the initial

pre-consolidation pressure  $\sigma_{p0}$ . This pressure may, for example, be computed from a given value of the over-consolidation ratio (OCR). Subsequently  $\sigma_{p0}$  can be used to compute the initial value of the generalised pre-consolidation pressure  $p_p^{eq}$  (see Section 2.8).

### Poisson's ratio

In the case of the Soft Soil Creep model, Poisson's ratio is purely an elasticity constant rather than a pseudo-elasticity constant as used in the Mohr-Coulomb model. Its value will usually be in the range between 0.1 and 0.2. If the standard setting for the Soft Soil Creep model parameters is selected, then the value  $\nu_{ur} = 0.15$  is automatically adopted. For loading of normally consolidated materials, Poisson's ratio plays a minor role, but it becomes important in unloading problems. For example, for unloading in a one-dimensional compression test (oedometer), the relatively small Poisson's ratio will result in a small decrease of the lateral stress compared with the decrease in vertical stress. As a result, the ratio of horizontal and vertical stress increases, which is a well-known phenomenon for overconsolidated materials. Hence, Poisson's ratio should not be based on the normally consolidated  $K_0^{nc}$ -value, but on the ratio of difference in horizontal stress to difference in vertical stress in oedometer unloading and reloading:

$$\frac{\nu_{ur}}{1 - \nu_{ur}} = \frac{\Delta\sigma_{xx}}{\Delta\sigma_{yy}} \quad (\text{unloading and reloading}) \quad (9.31)$$

### $K_0^{nc}$ - parameter

By default,  $M$  is automatically determined based on the coefficient of lateral earth pressure in normally consolidated condition,  $K_0^{nc}$ , as entered by the user. The exact relationship between  $M$  and  $K_0^{nc}$  can be formulated as (Brinkgreve, 1994):

$$M = 3 \sqrt{\frac{(1 - K_0^{nc})^2}{(1 + 2K_0^{nc})^2} + \frac{(1 - K_0^{nc})(1 - 2\nu_{ur})(\lambda^*/\kappa^* - 1)}{(1 + 2K_0^{nc})(1 - 2\nu_{ur})\lambda^*/\kappa^* - (1 - K_0^{nc})(1 + \nu_{ur})}} \quad (9.32)$$

Hence the user cannot enter directly a particular value of  $M$ . Instead he can choose values for  $K_0^{nc}$ . Note that the particular selection of  $M$  has an influence on lateral deformation of pseudo-vertical loading problems. For details, see Brinkgreve (2004).

## 9.9 STATE PARAMETERS IN THE SOFT SOIL CREEP MODELMODEL

In addition to the output of standard stress and strain quantities, the Soft Soil Creep modelmodel provides output (when being used) on state variables such as the isotropic pre-consolidation stress  $p_p$ . These parameters can be viewed by selecting the *State parameters* option from the *Stresses* menu. An overview of available state parameters is given below:

$p_{eq}$	: Equivalent isotropic stress	[-]
	$p_{eq} = p' + q^2 / M^2 (p' + ccot\phi)$	
$p_p$	: Isotropic pre-consolidation stress	[-]
OCR	: Isotropic over-consolidation	[-]

$$OCR = p_p / p_{eq}$$

## 9.10 ON THE USE OF THE SOFT SOIL CREEP MODEL IN DYNAMIC CALCULATIONS

When using the Soft Soil Creep model in dynamic calculations, the modified swelling index  $\kappa^*$  needs to be selected such that the model correctly predicts wave velocities in the soil. This generally requires a smaller value than just an unloading-reloading index.

When subjected to dynamic or cyclic loading, the Soft Soil Creep model will generate plastic strains when the preconsolidation stress is increased. However, it should be noted that stress cycles within the current creep contour will only generate elastic strains and not (hysteretic) damping, nor accumulation of strains or pore pressure, nor liquefaction. In order to account for the soil damping in cyclic loading, Rayleigh damping may be defined.

## 9.11 ON THE USE OF THE SOFT SOIL CREEP MODEL IN PRACTICAL APPLICATIONS

In the Soft Soil Creep model, creep strains are generated as long as there is effective stress. In oedometer tests and other lab tests, self-weight stresses of the soil sample are negligible and the effective stress in the sample is dominated by external loading conditions. However, when it comes to practical applications, the effective stress in the soil is generally dominated by the initial self-weight stresses. As a consequence, creep will occur without additional loading.

Following the formulation of the model, the rate at which creep strains occur highly depends on the over-consolidation ratio as well as the ratio of the (modified) compression index over the (modified) creep index. Regarding the latter, it should be considered that natural clays may involve structure (bonding) whilst the Soft Soil Creep model does not include such effects. This requires the effective stress range in the application to be taken into account when determining the compression and creep indices from one-dimensional compression tests. This may also have an effect on the pre-consolidation stress to be used in the application. Moreover, considering 'normally-consolidated' soft soil deposits in practice, it would seem logical to set the initial *OCR*-value equal to 1.0. However, this would lead to unrealistic large creep strain rates due to the initial stresses, without even considering additional loading.

In order to avoid these unrealistic creep strain rates, it is recommended to set the initial *OCR*-value larger than 1.0. A value in the order of 1.2-1.4 will generally work, but in some cases even higher values might be needed. Please keep in mind that this changes the pre-consolidation pressure, such that the material becomes lightly over-consolidated and the pre-consolidation stress may not correspond anymore to what is observed in compression tests.

The above may be validated from a practical viewpoint by considering that the layer has aged since its deposition in geological history. Moreover, in particular the top few metres of an existing soft soil layer have been subjected to various possible external influences (traffic, weather, temperature changes, changes in saturation, etc.). Accurate measurements of the pre-consolidation pressure on soil samples that are assumed to be

'normally-consolidated' would typically show a pre-consolidation pressure that is noticeably larger than the initial effective stress. However, lab tests are often performed on soil samples that have been disturbed to a certain extent, and therefore the measurement of the pre-consolidation stress in practice is often inaccurate.

In conclusion, the determination of model parameters for the Soft Soil Creep model requires a cautious interpretation of test data, in view of the envisioned practical application. It is recommended, before commencing any 2D or 3D analysis, to perform a careful calibration of model parameters. This can be done by performing simulations using the PLAXIS Soil Test facility in combination with an analysis of a one-dimensional soil column (based on the in-situ soil layering), in which a realistic time interval is considered.

## 10 MODIFIED CAM-CLAY MODEL

The Modified Cam-Clay model is described in several textbooks on critical state soil mechanics (for example Muir Wood (1990)). In this chapter a short overview is given of the basic equations.

### 10.1 FORMULATION OF THE MODIFIED CAM-CLAY MODEL

In the Modified Cam-Clay model, a logarithmic relation is assumed between void ratio  $e$  and the mean effective stress  $p'$  in virgin isotropic compression, which can be formulated as:

$$e - e^0 = -\lambda \ln \left( \frac{p'}{p^0} \right) \quad (\text{virgin isotropic compression}) \quad (10.1)$$

The parameter  $\lambda$  is the Cam-Clay isotropic compression index, which determines the compressibility of the material in primary loading. When plotting relation (Eq. 10.1) in a  $e - \ln p'$  diagram one obtains a straight line. During unloading and reloading, a different line is followed, which can be formulated as:

$$e - e^0 = -\kappa \ln \left( \frac{p'}{p^0} \right) \quad (\text{isotropic unloading and reloading}) \quad (10.2)$$

The parameter  $\kappa$  is the Cam-Clay isotropic swelling index, which determines the compressibility of material in unloading and reloading. In fact, an infinite number of unloading and reloading lines exists in  $p' - e$ -plane each corresponding to a particular value of the preconsolidation stress  $p_p$ .

The yield function of the Modified Cam-Clay model is defined as:

$$f = \frac{q^2}{M^2} + p'(p' - p_p) \quad (10.3)$$

The yield surface ( $f = 0$ ) represents an ellipse in  $p' - q$ -plane as indicated in Figure 10.1. The yield surface is the boundary of the elastic stress states. Stress paths within this boundary only give elastic strain increments, whereas stress paths that tend to cross the boundary generally give both elastic and plastic strain increments.

In  $p' - q$ -plane, the top of the ellipse intersects a line that we can be written as:

$$q = Mp' \quad (10.4)$$

This line is called the critical state line (CSL) and gives the relation between  $p'$  and  $q$  in a state of failure (i.e. the critical state). The constant  $M$  is the tangent of the critical state line and determines the extent to which the ultimate deviatoric stress,  $q$ , depends on the mean effective stress,  $p'$ . Hence,  $M$  can be regarded as a friction constant. Moreover,  $M$  determines the shape of the yield surface (height of the ellipse) and influences the coefficient of lateral earth pressure  $K_0^{nc}$  in a normally consolidated stress state under conditions of one-dimensional compression.

The preconsolidation stress,  $p_p$ , determines the size of the ellipse. In fact, an infinite

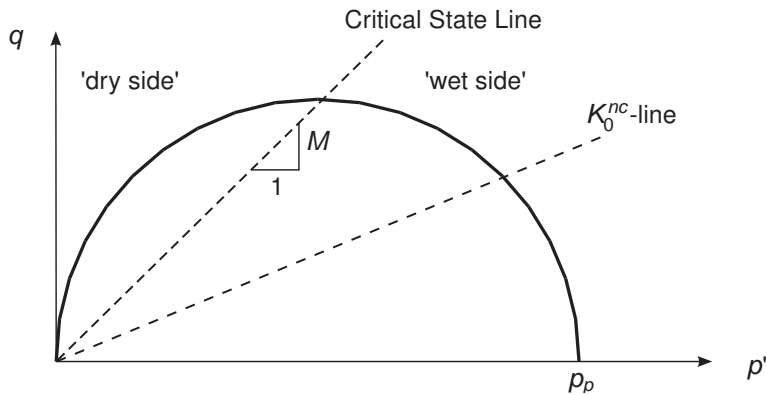


Figure 10.1 Yield surface of the Modified Cam-Clay model in  $p'$  -  $q$ -plane

number of ellipses exist, each corresponding to a particular value of  $p_p$ .

The left hand side of the yield ellipse (often described as the 'dry side' of the critical state line) may be thought of as a failure surface. In this region plastic yielding is associated with softening, and therefore failure. The values of  $q$  can become unrealistically large in this region.

For more detailed information on Cam-Clay type models, the reader is referred to Muir Wood (1990).

**Hint:** Note that the Modified Cam-Clay model as implemented in PLAXIS gives a Drucker-Prager failure state instead of a Mohr-Coulomb type of failure.

## 10.2 PARAMETERS OF THE MODIFIED CAM-CLAY MODEL

The Modified Cam-Clay model is based on five parameters:

- $\nu_{ur}$ : Poisson's ratio
- $\kappa$ : Cam-Clay swelling index
- $\lambda$ : Cam-Clay compression index
- $M$ : Tangent of the critical state line
- $e_{init}$ : Initial void ratio

### **Poisson's ratio**

Poisson's ratio  $\nu_{ur}$  is a real elastic parameter and not a pseudo-elasticity constant as used in the Mohr-Coulomb model. Its value will usually be in the range between 0.1 and 0.2.

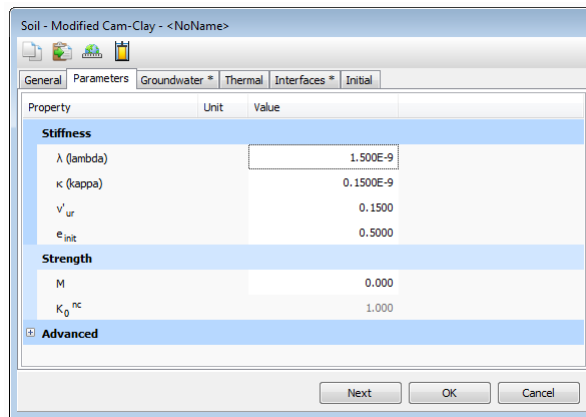


Figure 10.2 Parameters for the Modified Cam-Clay model

### Compression index and swelling index

These parameters can be obtained from an isotropic compression test including isotropic unloading. When plotting the natural logarithm of the mean stress as a function of the void ratio for clay-type materials, the plot can be approximated by two straight lines. The slope of the primary loading line gives the compression index and the slope of the unloading line gives the swelling index. These parameters can be obtained from a one-dimensional compression test, as discussed in Section 8.3.

### Tangent of the critical state line

In order to obtain the correct shear strength, the parameter  $M$  should be based on the friction angle  $\varphi$ . The critical state line is comparable with the Drucker-Prager failure line, and represents a (circular) cone in principal stress space. Hence, the value of  $M$  can be obtained from  $\varphi$ :

$$M = \frac{6 \sin \varphi}{3 - \sin \varphi} \quad (\text{for initial compression stress states}) \quad (\sigma_1' \leq \sigma_2' = \sigma_3')$$

$$M = \frac{6 \sin \varphi}{3 + \sin \varphi} \quad (\text{for triaxial extension stress states}) \quad (\sigma_1' = \sigma_2' \leq \sigma_3')$$

$$M \approx \sqrt{3} \sin \varphi \quad (\text{for plain strain stress states})$$

In addition to determining the shear strength, the parameter  $M$  has an important influence on the value of the coefficient of lateral earth pressure,  $K_0^{nc}$ , in a state of normal consolidation. In general, when  $M$  is chosen such that the model predicts the correct shearing strength, the resulting value of  $K_0^{nc}$  is too high.

## 10.3 STATE PARAMETERS IN THE MODIFIED CAM-CLAY MODEL

In addition to the output of standard stress and strain, the Modified Cam-Clay model provides output (when being used) on state variables such as the isotropic pre-consolidation stress  $p_p$  and the isotropic over-consolidation ratio  $OCR$ . These parameters can be visualised by selecting the *State parameters* option from the *Stresses* menu. An overview of available state parameters is given below:

$p^{eq}$  : Equivalent isotropic stress [kN/m<sup>2</sup>]

$$p^{eq} = \sqrt{\frac{\tilde{q}^2}{M^2} + (p')^2}$$

$p^p$  : Isotropic preconsolidation stress [kN/m<sup>2</sup>]

$OCR$  : Isotropic over-consolidation ratio ( $OCR = p^p/p^{eq}$ ) [-]

#### 10.4 ON THE USE OF THE MODIFIED CAM-CLAY MODEL IN DYNAMIC CALCULATIONS

When using the Modified Cam-Clay model in dynamic calculations, the swelling index  $\kappa$  needs to be selected such that the model correctly predicts wave velocities in the soil. This generally requires a smaller value than just an unloading-reloading index.

When subjected to dynamic or cyclic loading, the Modified Cam-Clay model will generate plastic strains when the preconsolidation stress is increased. However, it should be noted that stress cycles within the current creep contour will only generate elastic strains and no (hysteretic) damping, nor accumulation of strains or pore pressure, nor liquefaction. In order to account for the soil damping in cyclic loading, Rayleigh damping may be defined.

#### 10.5 WARNING

The Modified Cam-Clay model may allow for extremely large shear stresses. This is particularly the case for stress paths that cross the critical state line. Furthermore, the Modified Cam-Clay model may give softening behaviour for particular stress paths. Without special regularization techniques, softening behaviour may lead to mesh dependency and convergence problems of iterative procedures. Moreover, the Modified Cam-Clay model cannot be used in combination with *Safety* analysis by means of phi-c reduction. The use of the Modified Cam-Clay model in practical applications is not recommended.





## 11 THE SEKIGUCHI-OHTA MODEL

The Sekiguchi-Ohta model has been developed to formulate a constitutive law for normally consolidated clay. Particular emphasis is placed on taking the effect of time and stress-induced anisotropy into consideration. A complete description of the model has been presented in Sekiguchi & Ohta (1977) and Iizuka & Ohta (1987).

### 11.1 FORMULATION OF THE SEKIGUCHI-OHTA MODEL

The Sekiguchi-Ohta model combines the concepts lying behind the well known Cam Clay model (Roscoe, Schofield & Thurairajah (1963)) and the rheological model developed by Murayama & Shibata (1966). The Cam Clay model was further developed by Ohta & Hata (1973) counting for the stress induced anisotropy for anisotropically consolidated clays. However due to the fact that this model deals with the stress-strain behaviour of the soil in equilibrium, the time effect is not considered. The rheological model is further developed by Sekiguchi (1977) to describe the time-dependent and elastoplastic behaviour for normally consolidated clays.

#### 11.1.1 ISOTROPIC STATES OF STRESS AND STRAIN ( $\sigma'_1 = \sigma'_2 = \sigma'_3$ )

In the Sekiguchi-Ohta model, it is assumed that there is a logarithmic relation between changes in volumetric strain,  $\varepsilon_v$ , and changes in mean effective stress,  $p'$ , which can be formulated as:

$$\varepsilon_v - \varepsilon_v^0 = -\lambda^* \ln\left(\frac{p'}{p^0}\right) \quad (\text{virgin compression}) \quad (11.1)$$

The parameter  $\lambda^*$  is the modified compression index, which determines the compressibility of the material in primary loading. Note that  $\lambda^*$  differs from the index  $\lambda$  as used by Burland (1965). The difference is that Eq. (11.1) is a function of volumetric strain instead of void ratio. Plotting Eq. (11.1) gives a straight line as shown in Figure 11.3.

During isotropic unloading and reloading a different path (line) is followed, which can be formulated as:

$$\varepsilon_v^e - \varepsilon_v^{e0} = -\kappa^* \ln\left(\frac{p'}{p^0}\right) \quad (\text{unloading and reloading}) \quad (11.2)$$

The parameter  $\kappa^*$  is the modified swelling index, which determines the compressibility of the material in unloading and subsequent reloading. Note that  $\kappa^*$  differs from the index  $\kappa$  as used in the Cam-Clay models. The ratio  $\lambda^*/\kappa^*$  is, however, equal to the ratio  $\lambda/\kappa$ . The soil response during unloading and reloading is assumed to be elastic as denoted by the superscript  $e$  in Eq. (11.2). The elastic behaviour is described by Hooke's law. Eq. (11.2) implies linear stress dependency on the tangent bulk modulus such that:

$$K_{ur} \equiv \frac{E_{ur}}{3(1 - 2\nu_{ur})} = \frac{p'}{\kappa^*} \quad (11.3)$$

in which the subscript  $ur$  denotes unloading / reloading. Neither the elastic bulk modulus,  $K_{ur}$ , nor the elastic Young's modulus,  $E_{ur}$ , is used as an input parameter. Instead,  $\nu_{ur}$  and

$\kappa^*$  are used as input constants for the part of the model that computes the elastic strains.

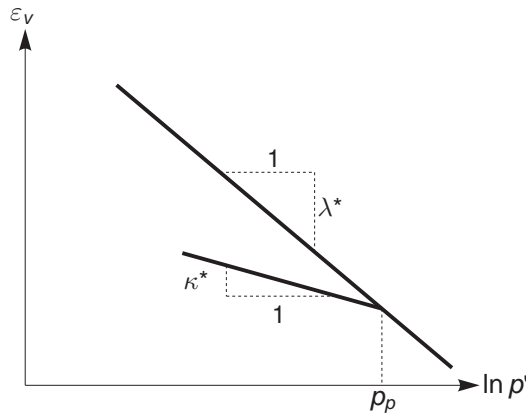


Figure 11.1 Logarithmic relation between volumetric strain and mean stress

An infinite number of unloading / reloading lines may exist in Figure 11.3, each corresponding to a particular value of the isotropic pre-consolidation stress  $p_p$ . The pre-consolidation stress represents the largest stress level experienced by the soil. During unloading and reloading, this pre-consolidation stress remains constant. In primary loading, however, the pre-consolidation stress increases with the stress level, causing irreversible (plastic) volumetric strains.

### 11.1.2 INVISCID (TIME-INDEPENDENT) FORMULATION

The yield function of the inviscid model in triaxial space is expressed by the following equation:

$$f = MD \ln\left(\frac{p'}{p_p}\right) + D \frac{\bar{q}}{p'} \quad (11.4)$$

where  $M$  is the critical state frictional parameter,  $p'$  is the mean effective stress,  $p_p$  is the isotropic hardening stress parameter,  $\bar{q}$  is the relative deviatoric stress and  $D$  is the coefficient of dilatancy.

The isotropic hardening stress parameter of the model is defined as:

$$p_p = p_{p0} e^{\frac{(\varepsilon_v^p - \varepsilon_{v0}^p)}{MD}} \quad (11.5)$$

where  $MD = \lambda^* - \kappa^*$ ,  $\varepsilon_v^p$  is the current plastic volumetric strain and  $\varepsilon_{v0}^p$  is the initial plastic volumetric strain. Hence, the parameter  $D$  is an auxiliary parameter implicitly defined as  $D = (\lambda^* - \kappa^*)/M$ .

### 11.1.3 VISCID (TIME-DEPENDENT) FORMULATION

The flow function of the visco-plastic (viscid) model is written as:

$$F = \alpha^* \ln \left( 1 + \frac{\dot{\nu}_0 t}{\alpha^*} \exp\left(\frac{f(\sigma)}{\alpha^*}\right) \right) - \varepsilon_V^{vp} = 0 \quad (11.6)$$

where

$$f(\sigma) = MD \ln \frac{p'}{p'_0} + D \frac{\bar{q}}{p'} \quad (11.7)$$

The flow function  $F$  can be transformed to a function of stress and hardening parameter  $g$  as follows:

$$g(\sigma', h) = f(\sigma') - h(\varepsilon_V^{vp}, t) = 0 \quad (11.8)$$

where the hardening parameter  $h$  is defined as:

$$h(\varepsilon_V^{vp}, t) = \alpha^* \ln \left\{ \frac{\alpha^*}{\dot{\nu}_0 t} \left[ \exp\left(\frac{\varepsilon_V^{vp}}{\alpha^*}\right) - 1 \right] \right\} \quad (11.9)$$

The initial time to calculate the hardening parameter  $h$  should not be zero, because it is not determined due to  $1/t$  in Eq. (11.9). To be able to calculate the initial volumetric visco-plastic strain, it is assumed that the hardening parameter  $h$  is equal to zero. Hence, the initial visco-plastic volumetric strain can be calculated as follows:

$$h(\varepsilon_V^{vp}, t) = \alpha^* \ln \left\{ \frac{\alpha^*}{\dot{\nu}_0 t} \left[ \exp\left(\frac{\varepsilon_V^{vp}}{\alpha^*}\right) - 1 \right] \right\} = 0 \quad (11.10)$$

$$\varepsilon_{V0}^{vp} = \alpha^* \ln \left\{ \frac{\dot{\nu}_0 t}{\alpha^*} + 1 \right\} = 0 \quad (11.11)$$

$\varepsilon_{V0}^{vp}$  is used as the initial visco-plastic volumetric strain to calculate the current visco-plastic volumetric strain.

## 11.2 PARAMETERS OF THE SEKIGUCHI-OHTA MODEL

### 11.2.1 INVISCID MODEL

The inviscid Sekiguchi-Ohta model requires a total of seven parameters (Figure 11.2).

*Basic parameters for soil stiffness:*

$\lambda^*$	: Modified compression index	[-]
$\kappa^*$	: Modified swelling index	[-]

Instead of entering the basic parameters for soil stiffness, alternative parameters can be entered. These parameters are listed below:

$C_c$	: Compression index	[-]
$C_r$	: Recompression index	[-]

$e_{init}$  : Initial void ratio [-]

The relationship of the alternative and the basic parameters for soil stiffness is given in Table 8.1.

*Parameters for soil strength:*

$M$  : Slope of the critical state line [-]

Advanced parameters (use default settings):

$\nu_{ur}$  : Poisson's ratio for unloading / reloading [-]

$K_0^{nc}$  : Coefficient of lateral stress in normal consolidation [-]

Figure 11.2 shows the PLAXIS window for inputting the values of the model parameters.

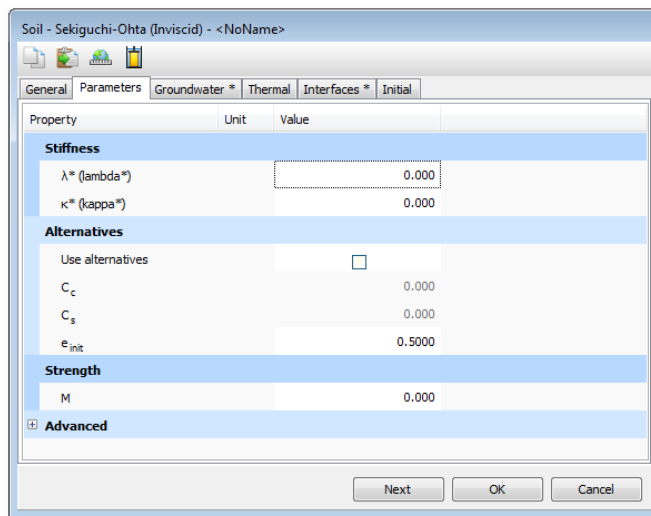


Figure 11.2 Parameters for the Sekiguchi-Ohta model

**Modified compression index and modified swelling index ( $\lambda^*$  and  $\kappa^*$ )**

These parameters can be obtained from an isotropic compression test including isotropic unloading. When plotting the logarithm of the mean effective stress as a function of the volumetric strain for clay type materials, the plot can be approximated by two straight lines, see Figure 11.3. The slope of the primary loading line gives the modified compression index  $\lambda^*$ , and the slope of the unloading (or swelling) line gives the modified swelling index  $\kappa^*$ .

**Poisson's ratio ( $\nu_{ur}$ )**

The poisson's ratio  $\nu_{ur}$  is a real elastic parameter and not a pseudo-elasticity constant as used in the Mohr-Coulomb model. Its value will usually be in the range between 0.1 and 0.2.

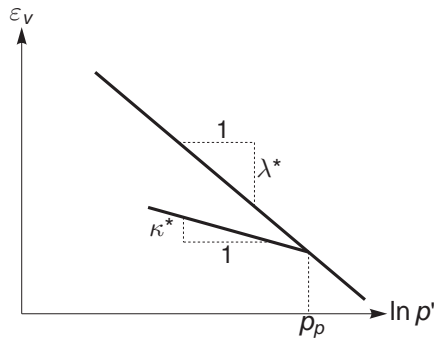


Figure 11.3 Logarithmic relation between volumetric strain and mean stress

**Earth pressure coefficient at rest ( $K_0^{nc}$ )**

The  $K_0^{nc}$  parameter is defined as the stress ratio in one-dimensional compression in a state of normal consolidation:

$$K_0^{nc} = \frac{\sigma'_{xx}}{\sigma'_{yy}}$$

The  $K_0^{nc}$  -parameter determines the singular point in the Sekiguchi-Ohta model yield contour. Hence,  $K_0^{nc}$  relates to the inclination of the stress path in one-dimensional compression where  $\alpha$ , as described in the Eq. (11.12) is the slope of the  $K_0^{nc}$  line in the  $p$  -  $q$  plane.

$$\alpha = \frac{3(1 - K_0^{nc})}{1 + 2K_0^{nc}} \tag{11.12}$$

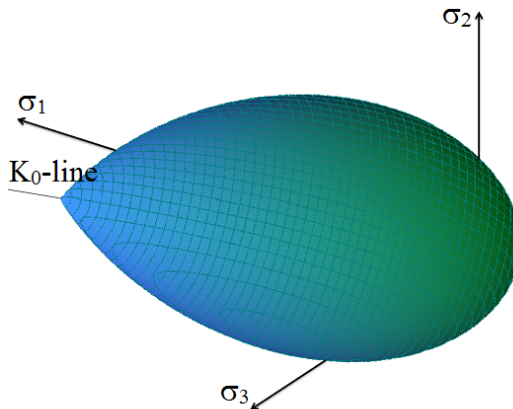


Figure 11.4 The Sekiguchi-Ohta model yield surface in triaxial stress space

**Slope of the critical state line ( $M$ )**

In order to obtain the correct shear strength, the parameter  $M$  should be based on the friction angle  $\phi$ . The critical state line is comparable with the Drucker-Prager failure criteria, and represents a (circular) cone in the principal stress space. Hence, the value of

$M$  can be obtained from  $\phi$  in a similar way as the Drucker-Prager friction constant  $\alpha$  is obtained from  $\phi$ . For details see also the Modified Cam-Clay model in the Material Models Manual.

**Initial overconsolidation ratio ( $OCR_0$ )**

The initial overconsolidation ratio  $OCR_0$  is defined as the highest vertical effective stress experienced in the past,  $\sigma'_p$ , divided by the current stress  $\sigma'_{yy}$ . A soil which is currently experiencing its highest stress is said to be normally consolidated and to have an  $OCR$  of 1.

**Initial pre-overburden pressure ( $POP_0$ )**

The initial pre-overburden pressure  $POP_0$ , expressed in the unit of stress, is defined as:

$$POP_0 = \sigma'_p - \sigma'_{yy}$$

where  $\sigma'_p$  is the vertical pre-consolidation stress (the greatest vertical stress reached previously) and  $\sigma'_{yy}$  is the in situ effective vertical stress.

**11.2.2 MODEL PARAMETERS OF THE VISCID MODEL**

Compared to the inviscid Sekiguchi-Ohta model, the viscid Sekiguchi-Ohta model requires the coefficient of secondary compression  $\alpha^*$  and the initial volumetric strain rate  $\dot{v}_0$  as two additional parameters of input. All other parameters remain the same as in the inviscid Sekiguchi-Ohta model.

$\lambda^*$	: Modified compression index	[-]
$\kappa^*$	: Modified swelling index	[-]
$\nu_{ur}$	: Poisson's ratio for unloading / reloading	[-]
$K_0^{nc}$	: Coefficient of lateral stress in normal consolidation	[-]
$M$	: Slope of the critical state line	[-]
$OCR_0$	: Initial overconsolidation ratio	[-]
$POP_0$	: Initial pre-overburden pressure	[kN/m <sup>2</sup> ]
$\alpha^*$	: Coefficient of secondary compression	[-]
$\dot{v}_0$	: Initial volumetric strain rate	[day <sup>-1</sup> ]

**Coefficient of secondary compression ( $\alpha^*$ )**

The coefficient of secondary compression  $\alpha^*$  is defined as:

$$\alpha^* = \frac{d\varepsilon_v}{d(\ln t)}$$

at time  $t_c$  (the end of primary consolidation).

**Initial volumetric strain rate ( $\dot{v}_0$ )**

The initial volumetric strain rate  $\dot{v}_0$  at reference state is expressed as:

$$\dot{v}_0 = \frac{\alpha^*}{t_c^*}$$







## 12 THE NGI-ADP MODEL (ANISOTROPIC UNDRAINED SHEAR STRENGTH)

The NGI-ADP model may be used for capacity, deformation and soil-structure interaction analyses involving undrained loading of clay. The basis of the material model is:

- Input parameters for (undrained) shear strength for three different stress paths/states (Active, Direct Simple Shear, Passive).
- A yield criterion based on a translated approximated Tresca criterion.
- Elliptical interpolation functions for plastic failure strains and for shear strengths in arbitrary stress paths.
- Isotropic elasticity, given by the unloading/reloading shear modulus,  $G_{ur}$ .

### 12.1 FORMULATION OF THE NGI-ADP MODEL

The NGI-ADP model is formulated for a general stress state, matching both undrained failure shear strengths and strains to that of selected design profiles (Andresen & Jostad (1999), Andresen (2002), Grimstad, Andresen & Jostad (2010)). The model formulation is presented in steps, starting with 1D anisotropy in triaxial test condition. In Section 12.1.2 a simplified expression for plane strain is presented. Thereafter the formulation is extended to full 3D stress state. In this formulation compressive stresses are positive.

In the NGI-ADP model the Tresca approximation after Billington (1988) together with a modified von Mises plastic potential function (von Mises (1913)) is used to circumvent the possible corner problems. The yield and plastic potential function are independent of the mean stress hence zero plastic volume strain develops.

#### 12.1.1 1D MODEL PRESENTATION

Under triaxial tests condition two undrained shear strengths can be determined, i.e.  $s_u^C$  and  $s_u^E$ . The test measures the response in vertical stress  $\sigma'_v$  and horizontal stress  $\sigma'_h$  for applied shear strain  $\gamma$ . The Tresca yield criteria can be modified, Eq. (12.1), to account for the difference in undrained shear strength in compression and extension:

$$f = \left| \tau - (1 - \kappa)\tau_0 - \kappa \frac{s_u^C - s_u^E}{2} \right| - \kappa \frac{s_u^C + s_u^E}{2} = 0 \quad (12.1)$$

where  $\tau = 0.5(\sigma'_v - \sigma'_h)$  and the initial in situ maximum shear stress  $\tau_0$  is then defined as  $\tau_0 = 0.5(\sigma'_{v0} - \sigma'_{h0}) = 0.5\sigma'_{v0}(1 - K_0)$ .

To account for difference in failure shear strain a stress path dependent hardening parameter is introduced. The stress path dependent hardening is made possible by different plastic failure shear strain  $\gamma_f^p$  in compression and extension. The hardening function is given by:

$$\kappa = 2 \frac{\sqrt{\gamma^p / \gamma_f^p}}{1 + \gamma^p / \gamma_f^p} \quad \text{when } \gamma^p < \gamma_f^p \quad \text{else } \kappa = 1 \quad (12.2)$$

where  $\gamma^p$  and  $\gamma_f^p$  are the plastic shear strain and the failure (peak) plastic shear strain

respectively.

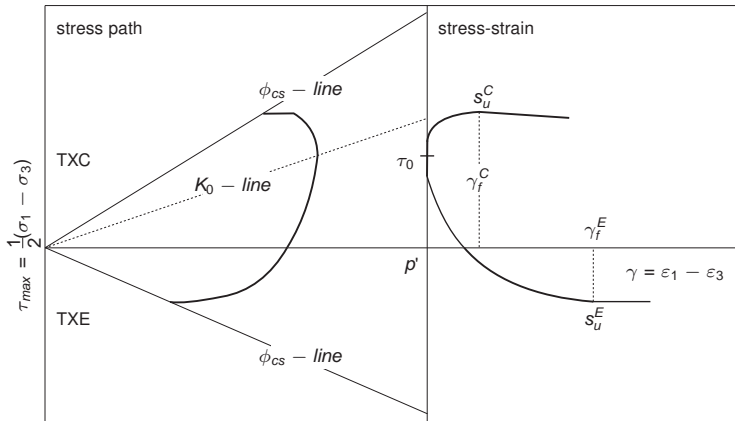


Figure 12.1 Typical stress paths and stress strain curves for triaxial compression and triaxial extension

### 12.1.2 THE NGI-ADP MODEL IN PLANE STRAIN

The yield criterion for the NGI-ADP model in plane strain is defined by:

$$f = \sqrt{\left(\frac{\sigma_{yy} - \sigma_{xx}}{2} - (1 - \kappa)\tau_0 - \kappa \frac{s_u^A - s_u^P}{2}\right)^2 + \left(\tau_{xy} \frac{s_u^A + s_u^P}{2s_u^{DSS}}\right)^2} - \kappa \frac{s_u^A + s_u^P}{2} = 0 \quad (12.3)$$

Restriction to clays with horizontal surfaces are made to simplify the presentation. Further  $y$  is taken as the vertical (depositional) direction. For isotropy in hardening (i.e.  $\kappa$  independent of stress orientation) Eq. (12.3) plots as an elliptical shaped curve in a plane strain deviatoric stress plot. When  $\kappa$  equals 1.0, the criterion in Eq. (12.3) reduces to the formulation given by Davis & Christian (1971). While hardening the yield curves are characterized by slightly distorted elliptical shapes. The shape is dependent on the interpolation function used and values of failure strain. The NGI-ADP model uses elliptical interpolation between failure strain in passive stress state, direct simple shear and active stress state. In the implementation of the NGI-ADP model the yield surface is ensured to remain convex by restricting the input.

### 12.1.3 THE NGI-ADP MODEL IN 3D STRESS SPACE

This section describes the actual implementation of the NGI-ADP model in PLAXIS, whereas the previous sections should be regarded as an introduction using simplified formulation. For the general stress condition a modified deviatoric stress vector is defined

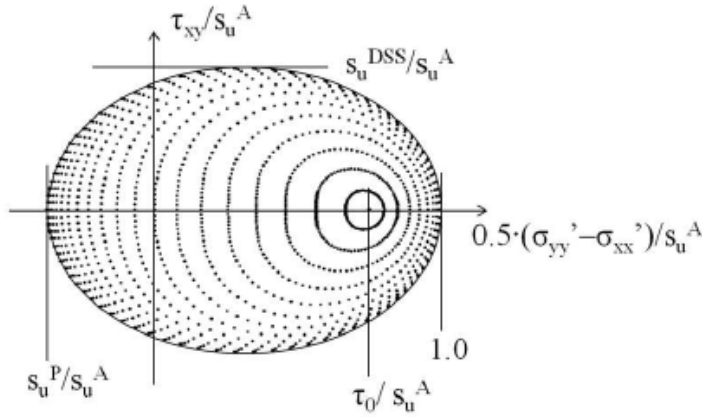


Figure 12.2 Typical deviatoric plane strain plot of equal shear strain contours for the NGI-ADP model

as:

$$\begin{bmatrix} \hat{s}_{xx} \\ \hat{s}_{yy} \\ \hat{s}_{zz} \\ \hat{s}_{xy} \\ \hat{s}_{xz} \\ \hat{s}_{yz} \end{bmatrix} = \begin{bmatrix} \sigma'_{xx} - \sigma'_{xx0}(1 - \kappa) + \kappa \frac{1}{3}(s_u^A - s_u^P) - \hat{p} \\ \sigma'_{yy} - \sigma'_{yy0}(1 - \kappa) - \kappa \frac{2}{3}(s_u^A - s_u^P) - \hat{p} \\ \sigma'_{zz} - \sigma'_{zz0}(1 - \kappa) + \kappa \frac{1}{3}(s_u^A - s_u^P) - \hat{p} \\ \tau_{xy} \frac{s_u^A + s_u^P}{2s_u^{DSS}} \\ \tau_{xz} \\ \tau_{yz} \frac{s_u^A + s_u^P}{2s_u^{DSS}} \end{bmatrix} \quad (12.4)$$

where  $\sigma'_{xx0}$ ,  $\sigma'_{yy0}$  and  $\sigma'_{zz0}$  are the initial stresses and  $\hat{p}$  is the modified mean stress. The modified mean stress is defined as:

$$\begin{aligned} \hat{p} &= \frac{(\sigma'_{xx} - \sigma'_{xx0}(1 - \kappa)) + (\sigma'_{yy} - \sigma'_{yy0}(1 - \kappa)) + (\sigma'_{zz} - \sigma'_{zz0}(1 - \kappa))}{3} \\ &= p' - (1 - \kappa)p'_0 \end{aligned} \quad (12.5)$$

where  $p'$  is the mean stress. Modified second and third deviatoric invariants are defined accordingly in Eqs. (12.6) and (12.7).

$$\hat{J}_2 = -\hat{s}_{xx}\hat{s}_{yy} - \hat{s}_{xx}\hat{s}_{zz} - \hat{s}_{yy}\hat{s}_{zz} + \hat{s}_{xz}^2 + \hat{s}_{yz}^2 \quad (12.6)$$

$$\hat{J}_3 = \hat{s}_{xx}\hat{s}_{yy}\hat{s}_{zz} + 2\hat{s}_{xy}\hat{s}_{yz}\hat{s}_{xz} - \hat{s}_{xx}\hat{s}_{yz}^2 - \hat{s}_{yy}\hat{s}_{xz}^2 - \hat{s}_{zz}\hat{s}_{xy}^2 \quad (12.7)$$

The yield criterion is expressed as:

$$f = \sqrt{H(\omega)\hat{J}_2} - \kappa \frac{s_u^A + s_u^P}{2} = 0 \quad (12.8)$$

where, to approximate the Tresca criterion, the term  $H(\omega)$  is defined as:

$$H(\omega) = \cos^2 \left( \frac{1}{6} \arccos(1 - 2a_1\omega) \right) \quad \text{with} \quad \omega = \frac{27}{4} \frac{\hat{J}_3^2}{\hat{J}_2^3} \quad (12.9)$$

By letting the value of  $a_1$  go to 1.0 an exact Tresca criterion is obtained. The parameter  $a_1$  can be directly linked to the rounding ratio  $s_u^C/s_u^A$ . This ratio takes typically a value just below 1.0 and a value of 0.99 is chosen as an appropriate default. Figure 12.3 shows the failure criterion of the NGI-ADP model in the  $\pi$ -plane (for Cartesian stresses) with default rounding ratio. This criterion is continuous and differentiable and it is described by a single function.

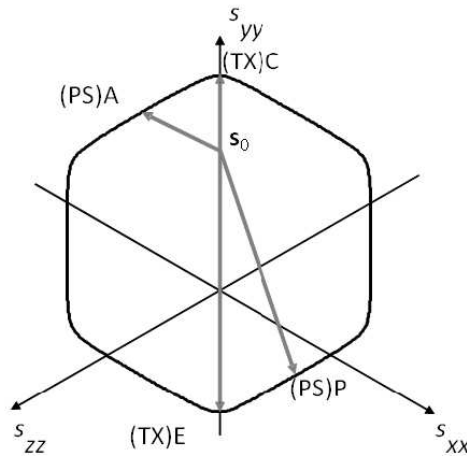


Figure 12.3 Failure criterion of the NGI-ADP model in the  $\pi$ -plane

The combinations of strength ratios are limited by lower limit for combinations of  $s_u^C/s_u^A$  and  $s_u^P/s_u^A$ .

The value of  $\gamma_f^p$  is given by elliptical interpolation:

$$\gamma_f^p(\hat{\theta}) = \frac{\hat{R}_B \hat{R}_D \sqrt{(\hat{R}_D^2 - \hat{R}_C^2) \cos^2(2\hat{\theta}) + \hat{R}_C^2 - \hat{R}_D^2 \hat{R}_A \cos(2\hat{\theta})}}{\hat{R}_B^2 - (\hat{R}_B^2 - \hat{R}_D^2) \cos^2(2\hat{\theta})} \quad (12.10)$$

where

$$\hat{R}_A = \frac{\gamma_{f,E}^p - \gamma_{f,C}^p}{2} \quad (12.11)$$

$$\hat{R}_B = \frac{\gamma_{f,E}^p + \gamma_{f,C}^p}{2} \quad (12.12)$$

$$\hat{R}_C = \sqrt{\gamma_{f,E}^p \gamma_{f,C}^p} \quad (12.13)$$

$$\hat{R}_D = \frac{\gamma_{f,DSS}^p \hat{R}_B}{\hat{R}_C} \quad (12.14)$$

and  $\gamma_{f,C}^p$ ,  $\gamma_{f,DSS}^p$  and  $\gamma_{f,E}^p$  are the failure plastic maximum shear strain in triaxial



$\sigma_{tt}$  and  $\tau_{tn}$  by:

$$\begin{bmatrix} \sigma_{tt} \\ \sigma_{nn} \\ \tau_{tn} \end{bmatrix} = A \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \quad (12.20)$$

where  $A$  is the transformation matrix.

In a local coordinate system three strains  $\varepsilon_{nn}$ ,  $\varepsilon_{tt}$  and  $\gamma_{tn}$  are defined in the plane strain condition. Due to the requirement of no volume change for a perfect plastic mechanism with shearing in tangential direction, will give that  $\varepsilon_{nn} = \varepsilon_{tt} = 0$ , resulting in:

$$\frac{\partial f}{\partial(\sigma_{nn} - \sigma_{tt})} = 0 \quad (12.21)$$

The plain strain formulation for the NGI-ADP model is defined as follows:

$$f = \sqrt{\left(\frac{\sigma_{nn} - \sigma_{tt}}{2} \cos(2\beta) - \tau_{tn} \sin(2\beta) - R_A\right)^2 + \left(\frac{R_B}{R_D} \left(\frac{\sigma_{nn} - \sigma_{tt}}{2} \sin(2\beta) + \tau_{tn} \cos(2\beta)\right)\right)^2} - R_B = 0 \quad (12.22)$$

where

$$R_A = \frac{s_u^A - s_u^P}{2} \quad (12.23)$$

$$R_B = \frac{s_u^A + s_u^P}{2} \quad (12.24)$$

$$R_D = s_u^{DSS} \quad (12.25)$$

## 12.2 PARAMETERS OF THE NGI-ADP MODEL

*Stiffness parameters:*

$G_{ur}/s_u^A$	: Ratio unloading/reloading shear modulus over (plane strain) active shear strength	[-]
$\gamma_f^C$	: Shear strain at failure in triaxial compression	[%]
$\gamma_f^E$	: Shear strain at failure in triaxial extension	[%]
$\gamma_f^{DSS}$	: Shear strain at failure in direct simple shear	[%]

**Hint:** Note that the input value of shear strains is in percent

*Strength parameters:*

$s_{u,ref}^A$	: Reference (plane strain) active shear strength	[kN/m <sup>2</sup> /m]
$s_u^{C,TX} / s_u^A$	: Ratio triaxial compressive shear strength over (plane strain) active shear strength (default = 0.99)	[-]
$y_{ref}$	: Reference depth	[m]
$s_{u,inc}^A$	: Increase of shear strength with depth	[kN/m <sup>2</sup> /m]
$s_u^P / s_u^A$	: Ratio of (plane strain) passive shear strength over (plane strain) active shear strength	[-]
$\tau_0 / s_u^A$	: Initial mobilization (default = 0.7)	[-]
$s_u^{DSS} / s_u^A$	: Ratio of direct simple shear strength over (plane strain) active shear strength	[-]

*Advanced parameter:*

$\nu'$	: Poisson's ratio	[-]
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### **Ratio unloading / reloading shear modulus over plane strain active shear strength ( $G_{ur} / s_u^A$ )**

Ratio unloading / reloading shear stiffness as a ratio of the plane strain active shear strength. If the shear strength is increasing with depth the constant ratio for  $G_{ur} / s_u^A$  gives a shear stiffness increasing linearly with depth.

### **Shear strain at failure in triaxial compression ( $\gamma_f^C$ )**

This parameter  $\gamma_f^C$  (%) defines the shear strain at which failure is obtained in undrained triaxial compression mode of loading, i.e.  $\gamma_f^C = \frac{3}{2} \varepsilon_1^C$  from triaxial testing.

### **Shear strain at failure in triaxial extension ( $\gamma_f^E$ )**

This parameter  $\gamma_f^E$  (%) defines the shear strain at which failure is obtained in undrained triaxial extension mode of loading, i.e.  $\gamma_f^E = \frac{3}{2} \varepsilon_1^E$  from triaxial testing.

### **Shear strain at failure in direct simple shear ( $\gamma_f^{DSS}$ )**

This parameter  $\gamma_f^{DSS}$  (%) defines the shear strain at which failure is obtained in undrained direct simple shear mode of loading (DSS device).

For near normally consolidated clays, the failure strain in compression loading  $\gamma_f^C$  is generally the lowest value and the failure strain in extension loading  $\gamma_f^E$  is the highest value. The failure strain from direct simple shear loading takes an intermediate value, i.e.  $\gamma_f^C < \gamma_f^{DSS} < \gamma_f^E$ . From laboratory test results reported in literature one find typically  $\gamma_f^E$  in the range 3-8 %,  $\gamma_f^{DSS}$  in the range 2-8 % and  $\gamma_f^C$  in the range 0.5 - 4 %.

If stress-strain curves from undrained triaxial and/or DSS laboratory tests are available it is recommended to choose the elastic shear modulus and failure strains such that a good fit to the curves are obtained. This is in particular important for deformation and SLS

assessments. However, for pure capacity and stability (e.g. factor of safety) analyses the values for shear strains at failure is not important and one may set all three values equal to e.g. 5 % for simplicity.

Note that it is the failure strains from triaxial loading that is input because they are the most readily available. When the NGI-ADP model is used for plane strain conditions the failure strains will automatically be slightly adjusted for that loading condition. See Grimstad, Andresen & Jostad (2010) for more details.

### **Reference active shear strength ( $s_{u,ref}^A$ )**

The reference active shear strength is the shear strength obtained in (plane strain) undrained active stress paths for the reference depth  $y_{ref}$ , expressed in the unit of stress.

### **Ratio triaxial compressive shear strength over active shear strength ( $s_u^{C,TX} / s_u^A$ )**

This ratio  $s_u^{C,TX} / s_u^A$  defines the shear strength in undrained triaxial compression mode of loading in relation to the shear strength in plane strain undrained active mode of loading. The value cannot be changed by the user and is predefined at 0.99 giving practically the same strengths in triaxial and plane strain conditions.

### **Reference depth ( $y_{ref}$ )**

This is the reference depth  $y_{ref}$  at which the reference active shear strength  $s_{u,ref}^A$  is defined. Below this depth the shear strength and stiffness may increase linearly with increasing depth. Above the reference depth the shear strength is equal to  $s_{u,ref}^A$ .

### **Increase of shear strength with depth ( $s_{u,inc}^A$ )**

This parameter  $s_{u,inc}^A$  defines the increase (positive) or decrease (negative) of the undrained active shear strength with depth, expressed in the unit of stress per unit of depth. Above the reference depth the shear strength is equal to  $s_{u,ref}^A$ , below the reference depth the shear strength is defined as:

$$s_u^A(y) = s_{u,ref}^A + (y_{ref} - y) s_{u,inc}^A \quad (12.26)$$

### **Ratio of passive shear strength over active shear strength ( $s_u^P / s_u^A$ )**

This ratio  $s_u^P / s_u^A$  defines the undrained shear strength for (plane strain) passive mode of loading.

### **Ratio of direct simple shear strength over active shear strength ( $s_u^{DSS} / s_u^A$ )**

This ratio  $s_u^{DSS} / s_u^A$  defines the undrained shear strength for direct simple shear mode of loading. Please note that active / passive strength input is defined for plane strain conditions. However, it is generally acceptable and only slightly conservative to use the strength obtained from a triaxial compression test as input for the active plane strain condition (i.e.  $s_u^A = s_u^{C,TX}$ ) and the strength obtained from a triaxial extension test as input for the passive plane strain condition (i.e.  $s_u^P = s_u^{E,TX}$ ). More control over the strength difference between triaxial and plane strain loading conditions can be obtained by using the advanced parameter  $s_u^{C,TX} / s_u^A$ .



For near normally consolidated clays, the passive strength  $s_u^P$  is generally the lowest strength value, while the direct simple shear strength takes an intermediate value, i.e.  $s_u^P < s_u^{DSS} < s_u^A$ . From laboratory results reported in literature one find typically  $s_u^P/s_u^A$  in the range 0.2 - 0.5 and  $s_u^{DSS}/s_u^A$  in the range 0.3 - 0.8. If direct simple shear strengths are not available  $s_u^{DSS}$  can be estimated from:  $s_u^{DSS}/s_u^A = (1 + s_u^P/s_u^A)/2$ .

### Initial mobilization ( $\tau_0/s_u^A$ )

The initial mobilization  $\tau_0/s_u^A$  is clearly defined for nearly horizontally deposited normally consolidated or lightly overconsolidated clay layers where the vertical stress is the major principle stress  $\sigma'_1$ . As defined in Figure 12.4, the initial mobilization can be calculated from the earth pressure coefficient at rest  $K_0$  by the following equation:

$\tau_0/s_u^A = -0.5(1 - K_0)\sigma'_{yy0}/s_u^A$ , where  $\sigma'_{yy0}$  is the initial (in situ) vertical effective stress (compression negative). A default value 0.7 of  $\tau_0/s_u^A$  is given which represent a typical value for a near normally consolidated clay deposit (e.g.  $K_0 = 0.55$  and  $-\sigma'_{yy0}/s_u^A = 3.11$ , or  $K_0 = 0.6$  and  $-\sigma'_{yy0}/s_u^A = 3.5$ ).

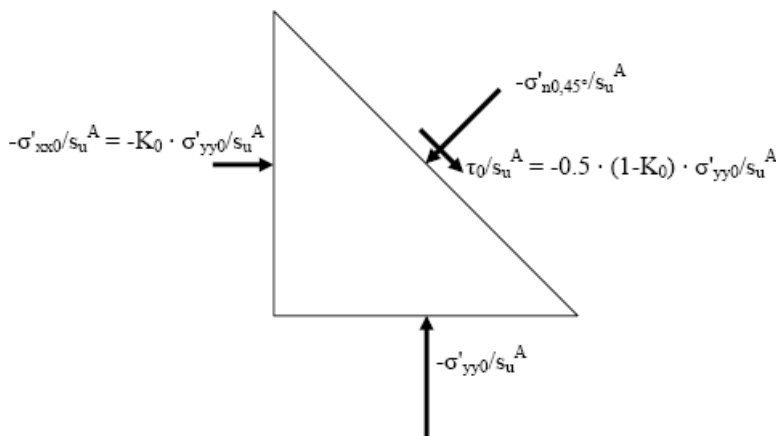


Figure 12.4 Definition of initial mobilized maximum shear stress  $\tau_0 = 1/2 |\sigma'_{yy0} - \sigma'_{xx0}|$  for a soil element in a horizontal deposited layer.

A more detailed evaluation of the initial (in situ) mobilization can be done by assessing the in situ  $K_0$  value and use the relationship:  $\tau_0/s_u^A = -0.5(1 - K_0)\sigma'_{v0}/s_u^A$ . Changing the default value for the initial mobilization should be considered in particular for over-consolidated materials where  $K_0$  generally is higher than 0.6, however the NGI-ADP model is not intended used for heavily overconsolidated clays and should be used with care for  $K_0 > 1.0$  (i.e. negative  $\tau_0/s_u^A$ ).

For non-horizontal layering (e.g. sloping ground) a  $K_0$  procedure is normally not recommended. In such cases it is recommended to establish the initial stress condition by gravity loading using a material model suited for such a purpose (e.g. drained behavior with the Mohr-Coulomb model or the Hardening Soil model). After the equilibrium initial stresses are established for gravity loading in the first phase, one should switch to the NGI-ADP model in the relevant clusters for the next phase and run a NIL step (i.e. without changing the external loads). The hardening parameter of the NGI-ADP model will then be adjusted such that equilibrium is obtained ( $f=0$ ). Then in the third phase the external loading can be applied.

**Poisson's ratio ( $\nu'$ )**

Similar as in the Mohr-Coulomb model, Poisson's ratio is generally between 0.3 and 0.4 for loading conditions considering effective stress analysis. For unloading conditions a lower value is more appropriate. See the linear elastic perfectly plastic model (Mohr-Coulomb model) for more details.

When the *Drainage type = Undrained (B)* option is used an effective Poisson's ratio should be entered that is less than 0.35. Excess pore pressures are generated using the bulk modulus of water as described in Section 2.4. In this case, *Undrained (B)*, the model will calculate excess pore pressures due to mean stress changes according to (2.37).

When the *Undrained (C)* drainage option is used a pure total stress analysis is carried out where no distinction between effective stresses and pore pressures is made and all stress changes should be considered as changes in total stress. A Poisson's ratio close to 0.5 should be entered.  $\nu = 0.495$  is given as default.

**12.3 STATE PARAMETERS IN THE NGI-ADP MODEL**

In addition to the output of standard stress and strain quantities, the NGI-ADP model provides output (when being used) on state variables such as plastic shear strain  $\gamma_p$  and the hardening function  $r_\kappa$ . These parameters can be visualised by selecting the *State parameters* option from the *Stresses* menu. An overview of available state parameters is given below:

$\gamma_p$  : Plastic shear strain [-]

$$\gamma_p =$$

$r_\kappa$  : Hardening function [-]

$$r_\kappa = 2 \frac{\sqrt{\gamma^p / \gamma_f^p}}{1 + \gamma^p / \gamma_f^p}$$



## 13 APPLICATION OF ADVANCED SOIL MODELS

In this chapter, advanced soil models will be utilised in various applications in order to illustrate the particular features of these models.

### 13.1 HARDENING SOIL MODEL: RESPONSE IN DRAINED AND UNDRAINED TRIAXIAL TESTS

In this section, the Hardening Soil model is utilised for the simulations of drained and undrained triaxial tests. Arbitrary sets of model parameters, Table 13.1, representing sands of different properties, are considered.

A triaxial test can simply be modelled by means of an axisymmetric geometry of unit dimensions (1m x 1m), that represent a quarter of the soil specimen, Figure 13.1. These dimensions are not realistic, but they are selected for simplicity. The dimension of the model does not influence the results, provided that the soil weight is not taken into account. In this configuration the stresses and strains are uniformly distributed over the geometry. The deformation magnitudes in  $x$ - and  $y$ -direction of the top right hand corner correspond to the horizontal and vertical strains respectively.

The left hand side and the bottom of the geometry are axes of symmetry. At these boundaries the displacements normal to the boundary are fixed and the tangential displacements are kept free to allow for 'smooth' movements. The remaining boundaries are fully free to move.

The value of the applied loads can be controlled by the load multipliers such as  $\Sigma MloadA$  and  $\Sigma MloadB$ . However, in PLAXIS 2D, and as described in the Reference Manual, the load configurations and magnitudes can be specified in the Input program. Then in the calculation program these loads can be activated or deactivated by means of the *Staged construction* option. For this case, and to simulate the confining pressure  $p'$ , distributed loads of  $-100 \text{ kN/m}^2$  representing the principal stresses  $\sigma'_1$  (load A) and  $\sigma'_3$  (load B) are applied in the *Input program*, as shown in Figure 13.1.

Table 13.1 Arbitrary Hardening Soil parameters for sands of different densities

Parameter	Loose	Medium	Dense	Unit
$E_{50}^{ref}$ (for $p_{ref} = 100 \text{ kPa}$ )	20000	30000	40000	$\text{kN/m}^2$
$E_{ur}^{ref}$ (for $p_{ref} = 100 \text{ kPa}$ )	60000	90000	120000	$\text{kN/m}^2$
$E_{oed}^{ref}$ (for $p_{ref} = 100 \text{ kPa}$ )	20000	30000	40000	$\text{kN/m}^2$
Cohesion $c$	0.0	0.0	0.0	$\text{kN/m}^2$
Friction angle $\varphi$	30	35	40	°
Dilatancy angle $\psi$	0	5	10	°
Poisson's ratio $\nu_{ur}$	0.2	0.2	0.2	-
Power $m$	0.5	0.5	0.5	-
$K_0^{nc}$ (using Cap)	0.5	0.43	0.36	-
Tensile strength	0.0	0.0	0.0	$\text{kN/m}^2$
Failure ratio	0.9	0.9	0.9	-

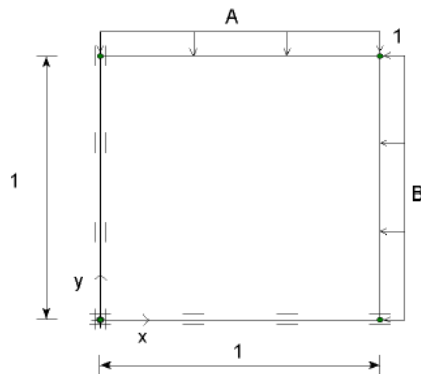


Figure 13.1 Simplified configuration of a triaxial test

A very coarse mesh is sufficient for this simple geometry. Initial stresses and steady pore pressures are not taken into account.

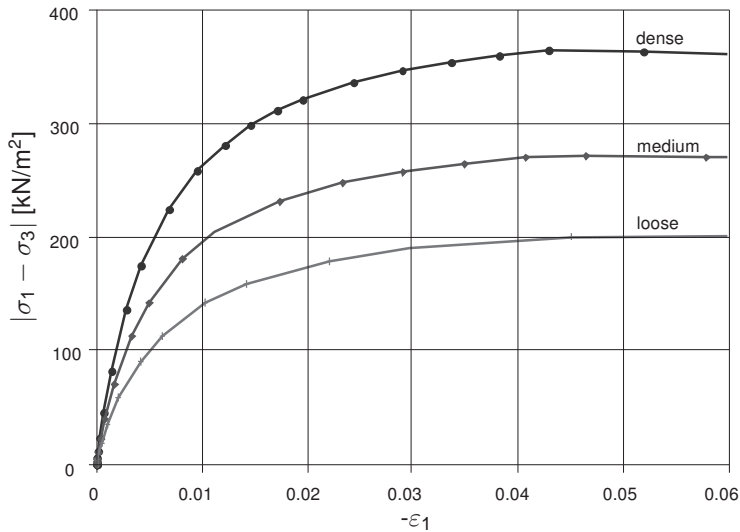


Figure 13.2 Results of drained triaxial tests using the Hardening Soil model, Principal stress difference versus axial strain

In the *Calculation program*, the calculation of all phases can be done by means of the *Staged construction* process. In the first phase, the confinement pressure  $p'$  is applied by activating load A and B. In the second phase the displacements are reset to zero and the sample is vertically loaded up to failure while the horizontal load is kept constant. This implies modification of load A by double clicking the load in the geometry model. As a result a *load* window appears in which the input values of the load can be changed. (Details of the procedure can be found in the Reference and Tutorial manuals.) The latter phase is carried out for drained as well as undrained conditions.

These calculations are performed for the three different sets of material parameters, Table 13.1. The computational results are presented in the figures on the following pages.

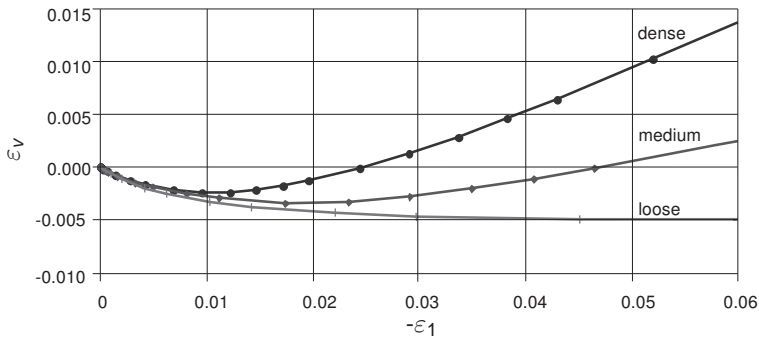


Figure 13.3 Results of drained triaxial tests using the Hardening Soil model, Volumetric strain versus axial strain

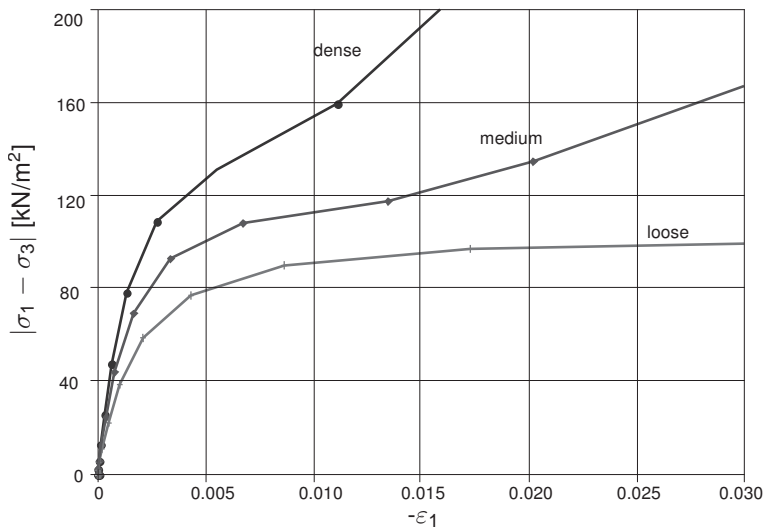


Figure 13.4 Results of undrained triaxial tests using the Hardening Soil model, Principal stress difference versus axial strain

Figure 13.2 shows the principal stress difference versus the axial strain for the drained condition. This shows a hyperbolic relationship between the stress and the strain, which is typical for the Hardening Soil model. Obviously, the failure level is higher when the sand is denser. The Hardening Soil model does not include softening behaviour, so after reaching failure the stress level does not reduce, at least in the drained tests.

Figure 13.3 shows the axial strain versus the volumetric strain for the drained test. This graph clearly shows the influence of dilatancy in the denser sands. In contrast to the Mohr-Coulomb model, the transition from elastic behaviour to failure is much more gradual when using the Hardening Soil model. In fact, in the Hardening Soil model, plastic strain occurs immediately after load application.

In the undrained tests, Figure 13.4, the failure level is, in principle, lower than that of the drained tests. However, for the medium and dense sands the stress level continues to increase after reaching the failure level due to the fact that dilatancy occurs which causes reduction of excess pore pressures and thus increase of the effective stresses. This can be seen in Figure 13.5.

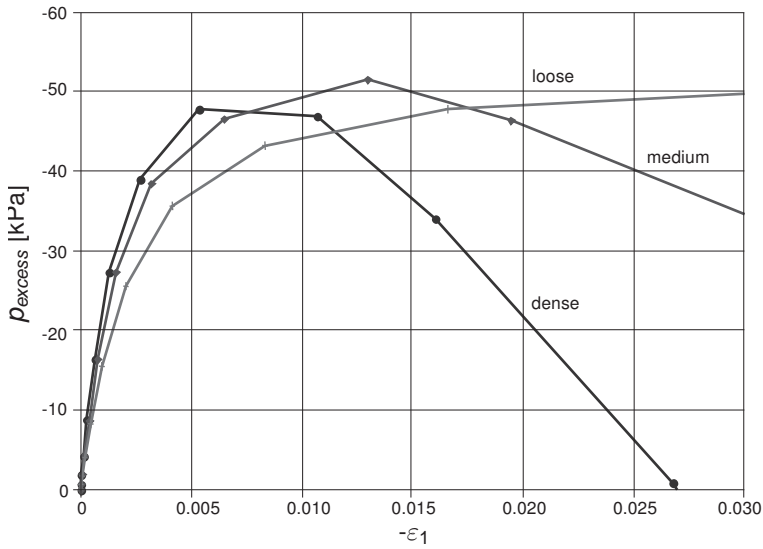


Figure 13.5 Results of undrained triaxial tests using the Hardening Soil model, Excess pore pressure vs axial strain

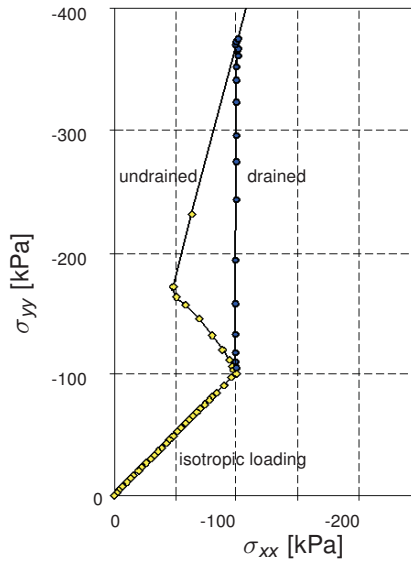


Figure 13.6 Stress paths for drained and undrained triaxial tests using the Hardening Soil model

Figure 13.6 shows the effective stress paths, for the medium sand, during both the drained and undrained tests. During first phase (isotropic loading), both tests were drained. In the second phase there is a clear distinction between the two tests. In the undrained test the effective horizontal stress reduces while the vertical stress increases due to the development of excess pore pressures. The decrease in horizontal effective stress is more than when if the Mohr-Coulomb model would have been used. This is attributed to the plastic compaction (Cap hardening) that occurs in the Hardening Soil model.

### 13.2 APPLICATION OF THE HARDENING SOIL MODEL ON REAL SOIL TESTS

In this section the ability of the Hardening Soil model to simulate laboratory tests on sand is examined by comparing PLAXIS calculation results with those obtained from laboratory tests provided by Prof. J. Desrues. Extensive laboratory tests were conducted on loose and dense Hostun sand. On the basis of these tests the model parameters for the Hardening Soil model were determined, Table 13.2.

Table 13.2 Hardening Soil parameters for loose and dense Hostun sand

Parameter	Loose sand	Dense sand	Unit
Volumetric weight $\gamma$	17	17.5	kN/m <sup>3</sup>
$E_{50}^{ref}$ ( $p_{ref} = 100$ kPa)	20000	37000	kN/m <sup>2</sup>
$E_{ur}^{ref}$ ( $p_{ref} = 100$ kPa)	60000	90000	kN/m <sup>2</sup>
$E_{oed}^{ref}$ ( $p_{ref} = 100$ kPa)	16000	29600	kN/m <sup>2</sup>
Cohesion $c$	0.0	0.0	kN/m <sup>2</sup>
Friction angle $\varphi$	34	41	°
Dilatancy angle $\psi$	0	14	°
Poisson's ratio $\nu_{ur}$	0.20	0.20	-
Power $m$	0.65	0.50	-
$K_0^{nc}$	0.44	0.34	-
Tensile strength	0.0	0.0	kN/m <sup>2</sup>
Failure ratio	0.9	0.9	-

#### **Triaxial test**

Standard drained triaxial tests were performed on loose and dense sand specimens. In PLAXIS the procedure for the simulation of the triaxial tests has been described in Section 13.1. In the first phase the sample is isotropically compressed up to a confining pressure of  $p' = -300$  kN/m<sup>2</sup>. In the second phase the sample is vertically loaded up to failure while the horizontal stress (confining pressure) is kept constant. The computational results and the measured data are presented in Figure 13.7, Figure 13.8, Figure 13.9 and Figure 13.10.

The figures show that the computational results match reasonably with the test data. It can be seen that the material response (measured and computed) show gradual transition from elastic to plastic behaviour. As such the relation between the deviatoric stress and the axial strain can be approximated by a hyperbola.

The failure level is fully controlled by the friction angle (the cohesion is zero). The test results on dense sand show softening behaviour after the peak load has been reached. Modelling of the softening behaviour, however, is not incorporated in the Hardening Soil model, and thus, the deviatoric stress remains constant. It can also be seen from the test data that the dilatancy reduces during softening. However, in the Hardening Soil model the dilatancy continues to infinity, unless the dilatancy cut-off option has been used.

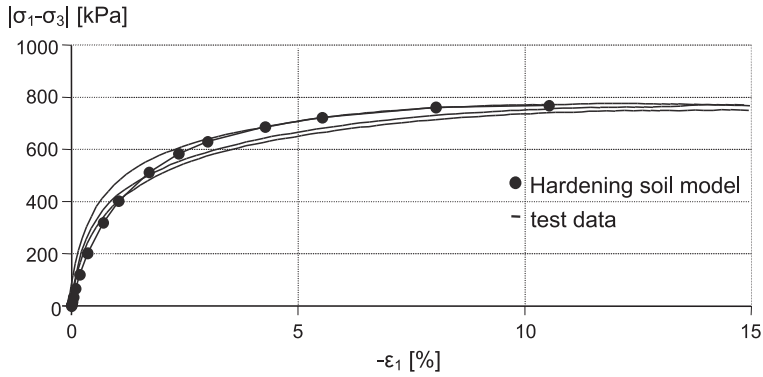


Figure 13.7 Results of drained triaxial tests on loose Hostun sand, principal stress ratio versus axial strain

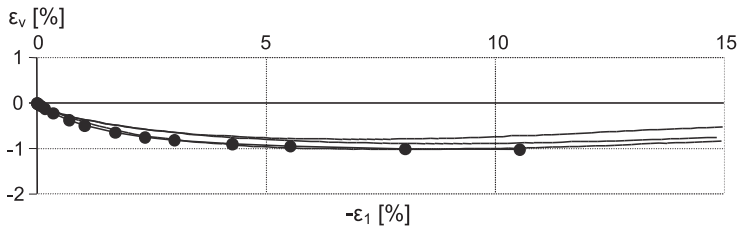


Figure 13.8 Results of drained triaxial tests on loose Hostun sand, volumetric strain versus axial strain

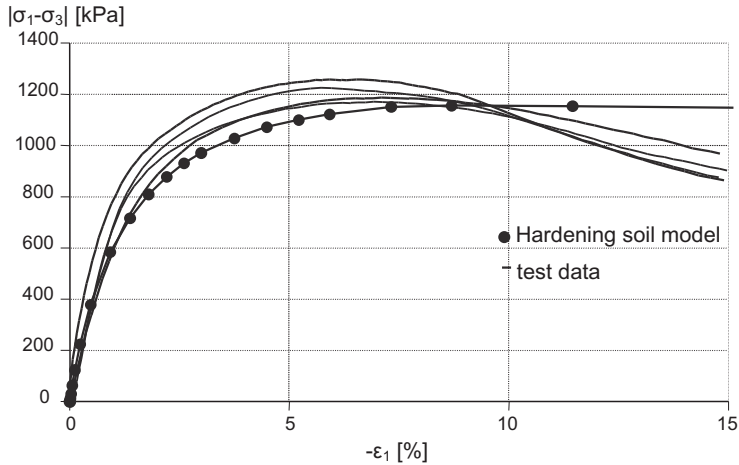


Figure 13.9 Results of drained triaxial tests on dense Hostun sand, principal stress ratio versus axial strain

**Oedometer test**

As for the triaxial test, a set of oedometer test on both loose and dense sands, Table 13.2, was conducted. In PLAXIS the oedometer test is simulated as an axisymmetric geometry with unit dimensions, Figure 13.11. A coarse mesh is sufficient for this case.

The computational results as compared with those obtained from the laboratory tests are



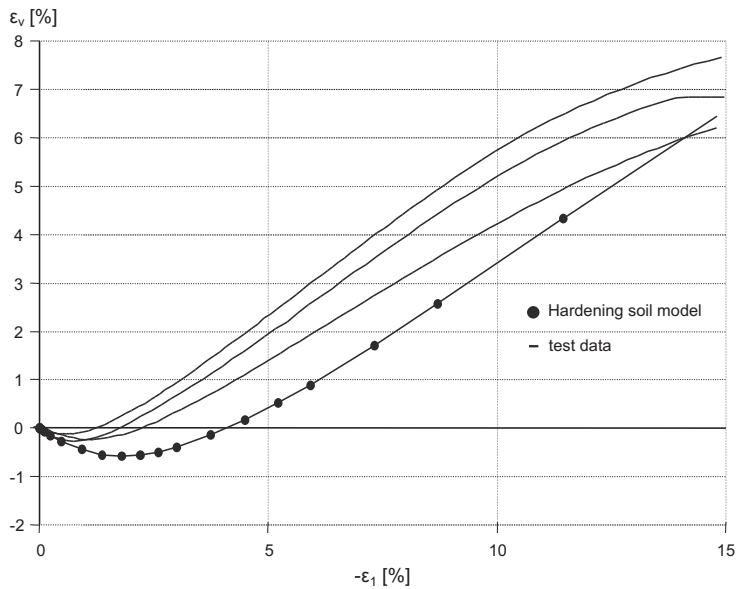


Figure 13.10 Results of drained triaxial tests on dense Hostun sand, volumetric strain versus axial strain

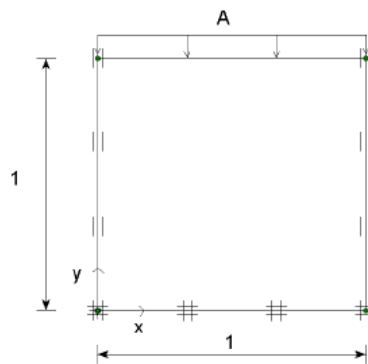


Figure 13.11 Simplified configuration of an oedometer test

shown in Figure 13.12 and Figure 13.13.

From a stress free state the loose sand sample is loaded consecutively to 25 kPa, 50 kPa, 100 kPa and 200 kPa with intermediate unloading. The dense sand sample is loaded to 50 kPa, 100 kPa, 200 kPa and 400 kPa with intermediate unloading.

As it can be seen, the computational results show a reasonable agreement with the test data. No doubt, distinction should be made between loose and dense soil, but it seems that for a soil with a certain density the stiffness behaviour under different stress paths can be well captured with a single set of model parameters. (A small offset of 0.15% has been applied to the computational results of the loose sample in order to account for the relative soft response at the beginning of the test.)

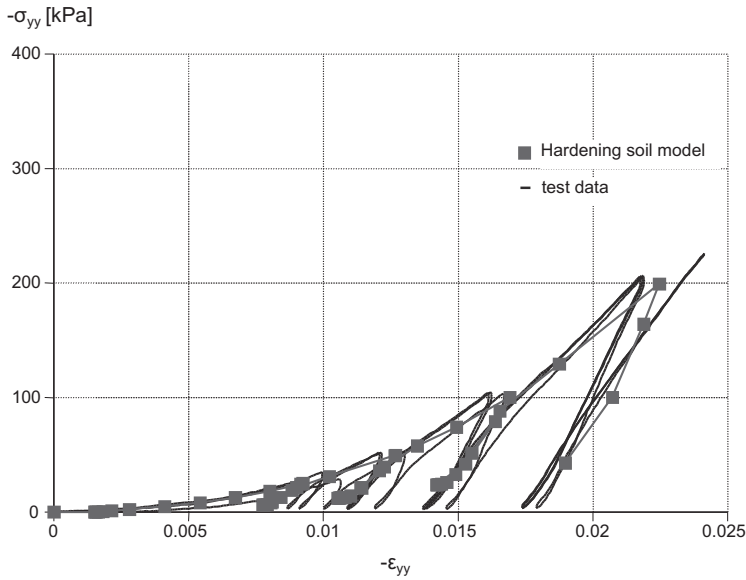


Figure 13.12 Results of oedometer test on loose Hostun sand, axial stress versus axial strain

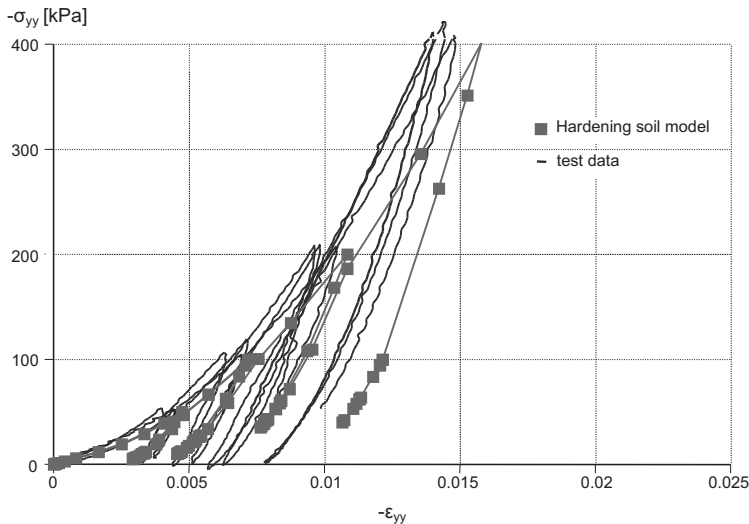


Figure 13.13 Results of oedometer test on dense Hostun sand, axial stress versus axial strain

**Pressiometer test**

In this section the Pressiometer test is simulated and results from PLAXIS and laboratory experimentation are compared. Laboratory testing results on dense sand with material parameters listed in Table 13.2 are used.

In the field, the pressiometer with 44 mm in diameter covered with a membrane with 160 mm in height is attached to the Cone penetration shaft. In the laboratory, the pressiometer is attached to a 44 mm pipe and placed in a circular calibration chamber with a diameter of 1.2 m and a height of 0.75 m. A large overburden pressure of 500 kPa is applied at the surface to simulate the stress state at larger depths. In PLAXIS only half

of the geometry is simulated by an axisymmetric model, Figure 13.14. The overburden pressure is simulated by load A, and the expansion of the pressiometer is simulated by imposing a horizontal distributed load, load B. Therefore the initial standard boundary conditions have to be changed near the pressiometer in order to allow for free horizontal displacements.

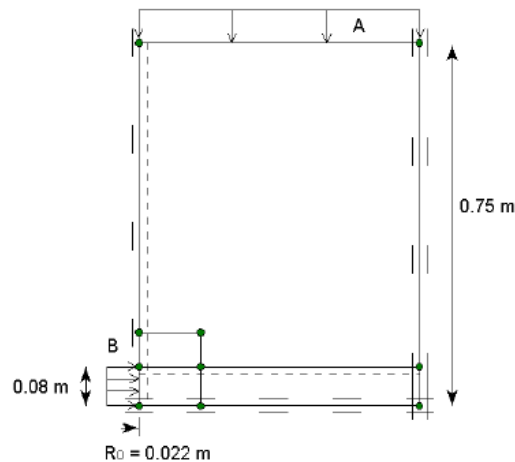


Figure 13.14 Geometry model for pressiometer test

To allow for a discontinuity in horizontal displacements, a vertical interface along the shaft of the pressiometer borehole and a horizontal interface just above the pressiometer are introduced. Both interfaces are set rigid ( $R_{inter} = 1.0$ ). Extra geometry lines are created around the pressiometer to locally generate a finer mesh.

After the generation of initial stresses, the vertical overburden load (load A) is applied using the standard boundary fixities. From the calculations, the lateral stress around the pressiometer appears to be 180 kPa. Subsequently, the horizontal fixity near the pressiometer is removed, in the *Input program*, and replaced by Load B with a magnitude of 180 kPa. In the next calculation the pressure (load B) is further increased by use of *Staged construction* in an *Updated mesh analysis*. The results of this calculation are presented in Figure 13.15 and Figure 13.16.

Figure 13.15 shows details of the deformations and the stress distribution when the pressure in the pressiometer was 2350 kPa. The high passive stresses appear very locally near the pressiometer. Just above the pressiometer the vertical stress is very low due to arching effects. Away from the pressiometer, a normal  $K_0$ -like stress state exists.

Figure 13.16 shows a comparison of the numerical results with those obtained from the laboratory test. In the figure the pressiometer pressure is presented as a function of the relative volume change. The latter quantity cannot directly be obtained from PLAXIS and was calculated from the original radius  $R_0$  and the lateral expansion  $u_x$  of the pressiometer:

$$\frac{\Delta V}{V_0} = \frac{(R_0 + u_x)^2 - R_0^2}{R_0^2}$$

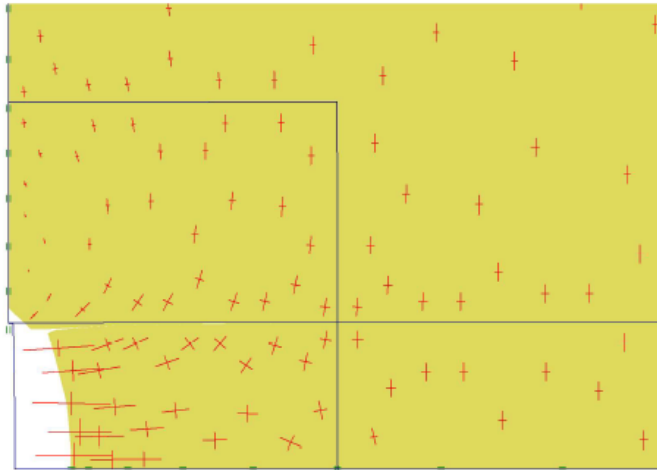


Figure 13.15 Stress distribution in deformed geometry around the pressimeter at a pressure of 2350 kPa

Up to a pressure of 1600 kPa the results match quite well. Above 1600 kPa there is a sudden decrease in stiffness in the real test data, which cannot be explained. Nevertheless, the original set of parameters for the dense sand that were derived from triaxial testing also seem to match the pressimeter data quite well.

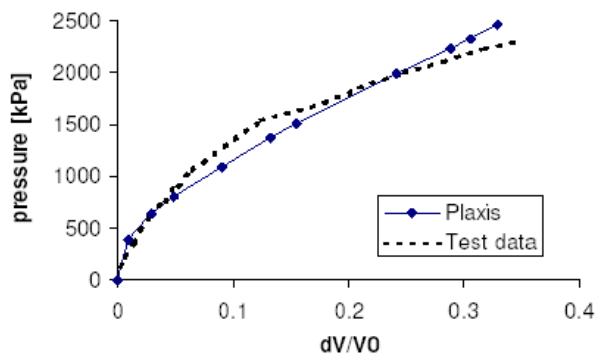


Figure 13.16 Comparison of numerical results and pressimeter test data

## Conclusion

The above results indicate that by use of the Hardening Soil model it is possible to simulate different laboratory tests with different stress paths. This cannot be obtained with simple models such as Mohr-Coulomb without changing input parameters. Hence, the parameters in the Hardening Soil model are consistent and more or less independent from the particular stress path. This makes the Hardening Soil model a powerful and an accurate model, which can be used in many applications.

### 13.3 APPLICATION OF THE HS SMALL MODEL ON REAL SOIL TESTS

In this section, the ability of the HS small model to simulate laboratory tests is examined. Both, the laboratory test data and the basic HS parameters are identical to those presented in the previous section. The two additional small strain parameters used in the Hardening Soil model are quantified in Table 13.3.

Table 13.3 Additional HS small model parameters for loose and dense Hostun sand

Parameter	Loose sand	Dense sand	Unit
$G_0^{ef}$ ( $p_{ref} = 100$ kPa)	70000	112500	kN/m <sup>2</sup>
Shear strain $\gamma_{0.7}$	0.0001	0.0002	-

Triaxial tests on loose and dense Hostun sand are presented in Figure 13.17 and Figure 13.18 respectively. As a reference, the previously obtained results from the Hardening Soil model are plotted as well.

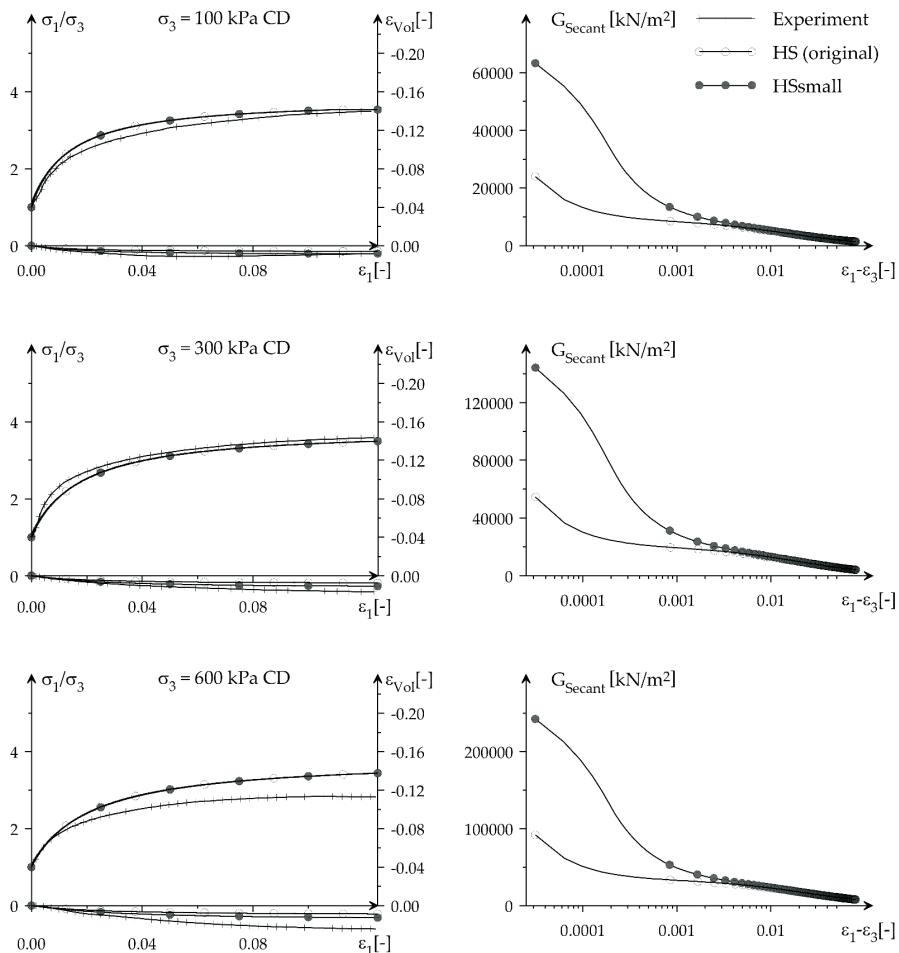


Figure 13.17 Excavation Drained triaxial tests on loose Hostun sand at confining pressures of 100, 300, and 600 kPa. Left: Stress-strain data. Right: Shear modulus reduction.

The overall stress-strain data obtained from both models seems almost identical. Only a closer look at the small-strain domain shows a clear difference: The HS small model follows a S-shaped stiffness reduction curve with much higher initial stiffness than the one of the Hardening Soil model. Generally, both models match the test data at different confining pressures reasonably well.

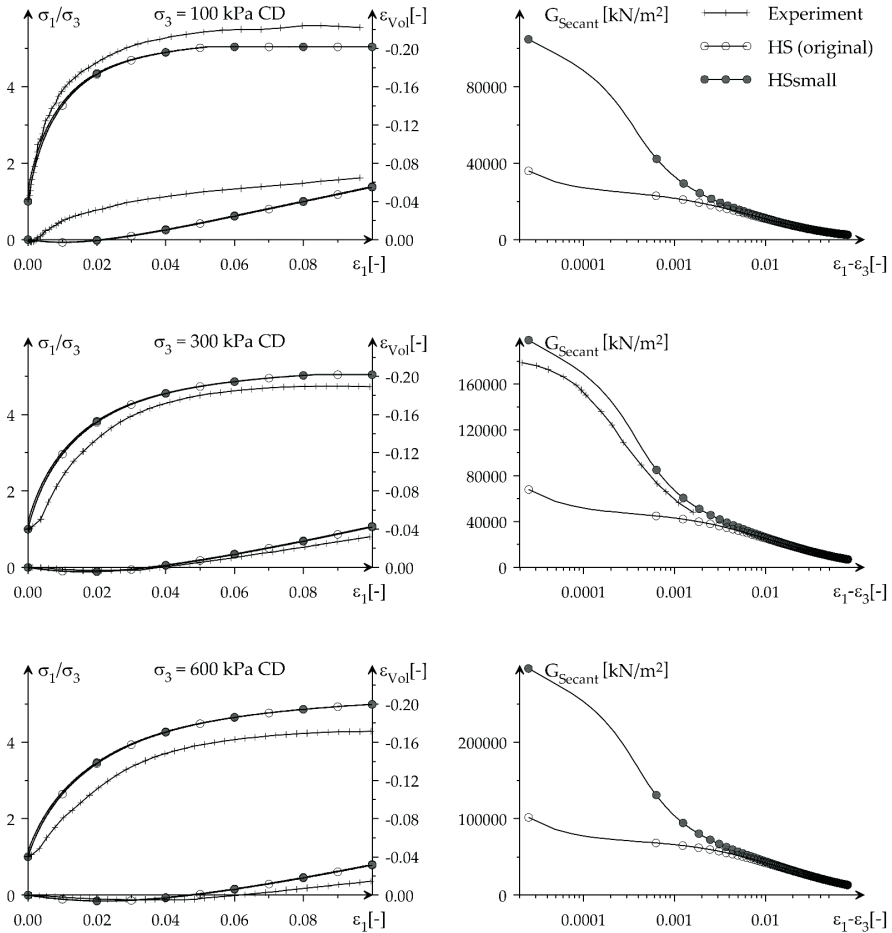


Figure 13.18 Drained triaxial tests on dense Hostun sand at confining pressures of 100, 300, and 600 kPa. Left: Stress-strain data. Right: Shear modulus reduction.

Figure 13.19 presents results from a cyclic triaxial test by Rivera & Bard on dense sand. The HS small model simulation of the test shows material damping which could not be obtained when simulating the test with the Hardening Soil model. As virgin loading is conducted in triaxial compression, the unloading sequence in triaxial extension gives some plasticity. Therefore the first unloading / reloading loop is not closed.

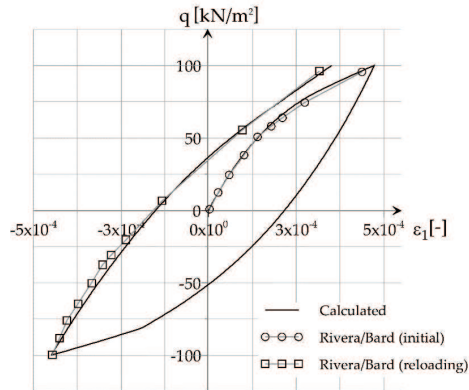


Figure 13.19 Hysteresis loop in a drained triaxial test on dense Hostun sand. Test data published in Biarez & Hicher (1994).

### 13.4 SOFT SOIL CREEP MODEL : UNDRAINED TRIAXIAL TESTS AT DIFFERENT LOADING RATES

In this section the Soft Soil Creep model (see Chapter 9) is utilised for the simulation of clay in an undrained triaxial test at different strain rates. The model parameters are obtained from test results on Haney Clay and are listed in Table 13.4.

The initial isotropic preconsolidation pressure  $P_p = 373 \text{ kN/m}^2$ , as reported in the literature, is obtained by specifying a POP of  $433 \text{ kN/m}^2$  in the initial conditions.

Table 13.4 Soft Soil Creep model parameters for Haney clay

Parameter	Symbol	Value	Unit
Modified compression index	$\lambda^*$	0.105	-
Modified swelling index	$\kappa^*$	0.016	-
Secondary compression index	$\mu^*$	0.004	-
Poisson's ratio	$\nu_{ur}$	0.15	-
Cohesion	$c$	0.0	$\text{kN/m}^2$
Friction angle	$\varphi$	32	$^\circ$
Dilatancy angle	$\psi$	0.0	$^\circ$
Coefficient of lateral stress	$K_0^{nc}$	0.61	-
Permeability	$k_x, k_y$	0.0001	m/day
Pre overburden pressure	$POP$	433	$\text{kN/m}^2$

Modelling of the triaxial test is as described in Section 13.1. However, here, the real dimension of the test set-up is simulated ( $17.5 \times 17.5 \text{ mm}^2$ ), Figure 13.20. The specimen surfaces (top and right hand side in Figure 13.20) are assumed drained whereas the other boundaries are assumed closed.

The global mesh coarseness is set to coarse. A local refinement of 0.25 is used at the upper and right-hand side of the model.

In addition to isotropic loading, prescribed displacements are also applied. Both types of loading are simulated using the *Staged construction* option. During isotropic loading,

horizontal and vertical loads are applied. The calculation phases for isotropic loading consist of undrained plastic and consolidation analyses.

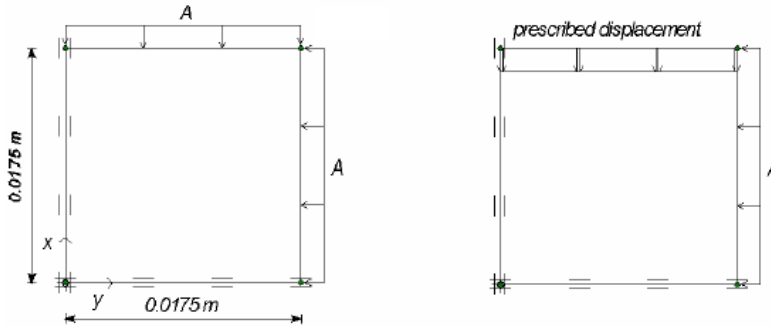


Figure 13.20 Modelling of triaxial test on Haney clay. Left, Initial configuration. Right, configuration for phase 9 - 11

After the isotropic loading phases, the displacements are reset to zero. The vertical load is deactivated and the prescribed displacement is activated. Rate of loading is simulated by applying prescribed displacements at different velocities. As such, a total of 12% axial strain (2.1 mm, vertical displacement) is applied in 8.865 days (0.00094%/minute), 0.0556 days (0.15%/minute) and 0.00758 days (1.10%/minute) respectively. Each of the prescribed displacement loading phase starts from the end of the isotropic loading phase. The calculation scheme is listed in Table 13.5.

Table 13.5 Loading scheme for triaxial tests at different loading rates

Phase	Start from:	Calculation	Load [kPa]	Displacement [mm]	Time interval [day]
1	0	Plastic	65	Inactive	0.00
2	1	Consolidation	65	Inactive	0.01
3	2	Plastic	130	Inactive	0.00
4	3	Consolidation	130	Inactive	0.01
5	4	Plastic	260	Inactive	0.00
6	5	Consolidation	260	Inactive	0.01
7	6	Plastic	520	Inactive	0.00
8	7	Consolidation	520	Inactive	0.01
9	8	Plastic	520	0.0021	8.865
10	8	Plastic	520	0.0021	0.0556
11	8	Plastic	520	0.0021	0.00758

The computational results are presented in Figure 13.21 and Figure 13.22. Figure 13.21 shows the stress-strain curves of the prescribed displacement loading phases. This figure is obtained from  $Force_y$  and  $u_y$  of the upper side of the model:

$$q_{average} = \frac{2Force_y}{R^2} - \sigma_3 \tag{13.1}$$



where:  $R = 0.0175$ ; and  $\sigma_3 = 520 \text{ kN/m}^2$  (13.2)

It can be seen that the shear strength highly depends on the strain rate; the higher strain rate the higher the shear strength.

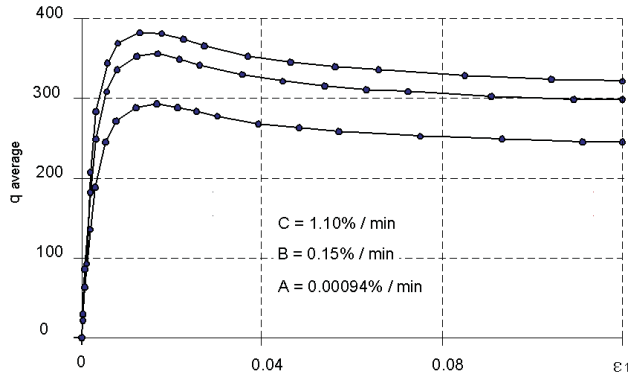


Figure 13.21 Average deviatoric stress versus axial strain for different rates of straining

Figure 13.22 shows the  $p$ - $q$  stress paths from the prescribed displacement loading phases. For higher strain rates there is a smaller reduction of the mean effective stress, which allows for a larger ultimate deviatoric stress. It should be noted that the stress state is not homogeneous at all, because of the inhomogeneous (excess) pore pressure distribution. This is due to the fact that points close to draining boundaries consolidate faster than points at a larger distance.

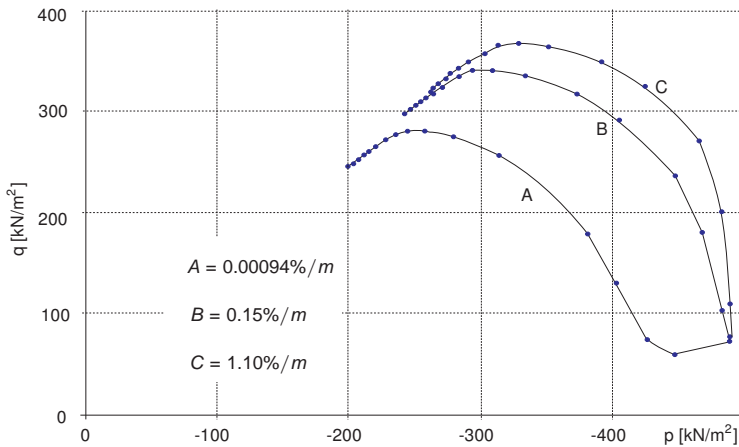


Figure 13.22  $p'$ - $q$  stress paths for different rates of straining for a point at position (0.01, 0.01)

In addition to the full tests as described before, the last part of the test can also be done in a simplified way using the *Soil Test* facility. Since the *Soil Test* facility operates on a single stress point, it is not possible to start the undrained triaxial tests from an inhomogeneous stress state, as considered in the full finite element based model. Instead, we start from an isotropic effective stress of  $500 \text{ kN/m}^2$ .

Table 13.6 Test conditions in the *Soil Test* facility

Test	A(0.00094%/min)	B(0.15%/min)	C(1.10%/min)
Type of test	Triaxial-Undrained, compression, isotropic	Triaxial-Undrained, compression, isotropic	Triaxial-Undrained, compression, isotropic
Initial effect. stress $ \sigma_3' $	520 kN/m <sup>2</sup>	520 kN/m <sup>2</sup>	520 kN/m <sup>2</sup>
Maximum strain $ \varepsilon_1 $	12.0 %	12.0 %	12.0 %
Time $\Delta t$	8.865 days	0.0556 days	0.00758 days
Number of steps	500	500	500
Vert. precons. stress	433 kN/m <sup>2</sup>	433 kN/m <sup>2</sup>	433kN/m <sup>2</sup>

### 13.5 SOFT SOIL CREEP MODEL: RESPONSE IN ONE-DIMENSIONAL COMPRESSION TEST

In this section the behaviour of the Soft Soil Creep model is illustrated on the basis of a one-dimensional compression test on clay. Two types of analysis are performed. First, the test is simulated assuming drained conditions in order to demonstrate the logarithmic stress-strain relationship and the logarithmic time-settlement behaviour on the long term (secondary compression). Second, the test is simulated more realistically by including undrained conditions and consolidation. Since the consolidation process depends on the drainage length, it is important to use actual dimensions of the test set-up. In this case an axisymmetric configuration with specimen height of 0.01 m, Figure 13.23, is used. The material parameters are shown in Table 13.7. The parameter values are selected arbitrarily, but they are realistic for normally consolidated clay. The vertical preconsolidation stress is fixed at 50 kPa (POP = 50 kPa).

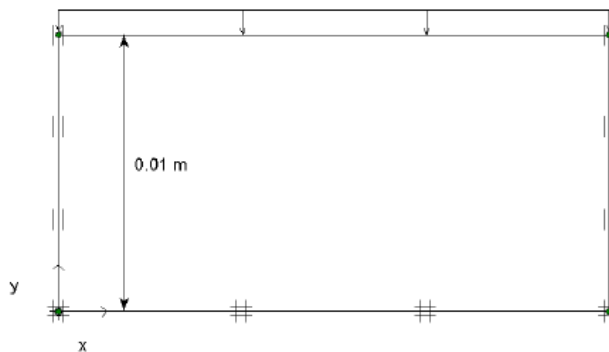


Figure 13.23 One-dimensional compression test

#### **Drained analysis**

In the first analysis successive plastic loading steps are applied using drained conditions. The load is doubled in every step using *Staged construction* with time increments of 1 day. After the last loading step an additional creep period of 100 days is applied. The calculation scheme is listed in Table 13.8. All calculations are performed with a tolerance

Table 13.7 Soft Soil Creep model parameters for one-dimensional compression test

Parameter	Symbol	Value	Unit
Unit weight	$\gamma$	19	kN/m <sup>3</sup>
Permeability	$k_x, k_y$	0.0001	m/day
Modified compression index	$\lambda^*$	0.10	-
Modified swelling index	$\kappa^*$	0.02	-
Secondary compression index	$\mu^*$	0.005	-
Poisson's ratio	$\nu_{ur}$	0.15	-
Cohesion	$c$	1.0	kN/m <sup>2</sup>
Friction angle	$\varphi$	30	°
Dilatancy angle	$\psi$	0.0	°
Coefficient of lateral stress	$K_0^{nc}$	0.5	-

of 1%.

### Undrained analysis

In the second analysis the loading steps are instantaneously applied using undrained conditions. After each loading step a consolidation of 1 day is applied to let the excess pore pressures fully dissipate. After the last loading step, an additional creep period of 100 days is again introduced. The calculation scheme for this analysis is listed in Table 13.9. All calculations are performed with a reduced tolerance of 1%.

Figure 13.24 shows the load-settlement curves of both analyses. It can be seen that, after consolidation, the results of the undrained test match those of the drained test. The influence of the preconsolidation stress can clearly be seen, although the transition between reloading and primary loading is not as sharp as when using the Soft Soil model. In fact, the results presented here are more realistic. The transition is indeed around 50 kPa.

From the slope of the primary loading line one can back-calculate the modified compression index  $\lambda^* = \Delta\varepsilon_1 / \ln(\sigma_1 + \Delta\sigma_1) / \sigma_1 \approx 0.10$ . Note that 1 mm settlement corresponds to  $\varepsilon_1 = 10\%$ . For an axial strain of 30% one would normally use an *Updated mesh* analysis, which has not been done in this simple analysis. If, however, the Soft Soil Creep model would have been used in an *Updated mesh* analysis with axial strains over 15% one would observe a stiffening effect as indicated by line *C* in Figure 13.24.

Figure 13.25 shows the time-settlement curves of the drained and the undrained analyses. From the last part of the curve one can back-calculate the secondary compression index  $\mu^* = \Delta\varepsilon_1 / \ln(\Delta t / t_0) \approx 0.005$  (with  $t_0 = 1$  day).

Another interesting phenomenon is the development of lateral stresses. During primary loading, the lateral stress is determined by  $K_0^{nc}$ , appropriate for normally consolidated soil. During unloading, the lateral stress decreases much less than the vertical stress, so that the ratio  $\sigma'_{xx} / \sigma'_{yy}$  increases.

To show these effects the calculation is continued after with a new drained unloading phase that starts from phase 7 (see Table 13.8) in which the vertical stress is reduced to -80 kPa.

Table 13.8 Calculation scheme for the drained case

Phase	Calculation type	Loading input	Load [kPa]	Time interval [day]	End time [day]
1	Plastic	Staged construction	10	1	1
2	Plastic	Staged construction	20	1	2
3	Plastic	Staged construction	40	1	3
4	Plastic	Staged construction	80	1	4
5	Plastic	Staged construction	160	1	5
6	Plastic	Staged construction	320	1	6
7	Plastic	Staged construction	640	1	7
8	Plastic	Staged construction	640	100	107

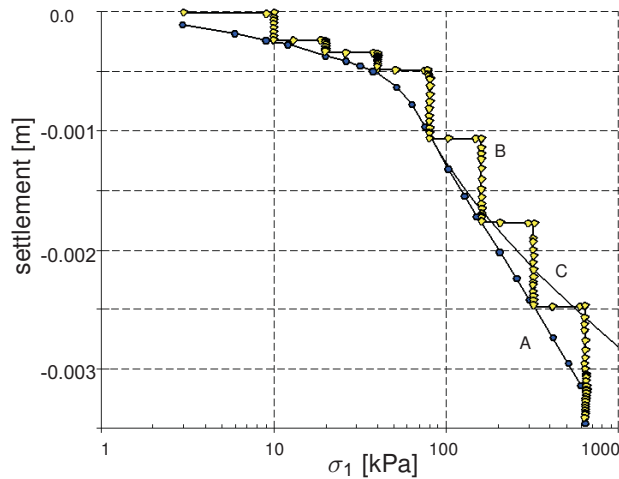


Figure 13.24 Load-settlement curve of oedometer test with Soft Soil Creep model. A) Transient loading with doubling of loading within one day. B) Instantaneous loading with doubling of load at the beginning of a new day. C) As 'A' using Updated Mesh calculation

Figure 13.26 shows the stress state for two different calculation phases, both at a vertical stress level of 80 kPa. The plot in the left hand side shows the stress state after primary loading. As expected the horizontal stress is found to be approximately  $-40$  kPa (corresponding to  $K_0^{nc} = 0.5$ ). The plot in the right hand side shows the final situation after unloading down to  $-80$  kPa. In this case the horizontal stress is decreased from

Table 13.9 Calculation scheme for second analysis

Phase	Calculation type	Loading input	Load [kPa]	Time interval [day]	End time [day]
1	Plastic	Staged construction	10	0	0
2	Consolidation	Staged construction	10	1	1
3	Plastic	Staged construction	20	0	1
4	Consolidation	Staged construction	20	1	2
5	Plastic	Staged construction	40	0	2
6	Consolidation	Staged construction	40	1	3
7	Plastic	Staged construction	80	0	3
8	Consolidation	Staged construction	80	1	4
9	Plastic	Staged construction	160	0	4
10	Consolidation	Staged construction	160	1	5
11	Plastic	Staged construction	320	0	5
12	Consolidation	Staged construction	320	1	6
13	Plastic	Staged construction	640	0	6
14	Consolidation	Staged construction	640	1	7
15	Consolidation	Staged construction	640	100	107

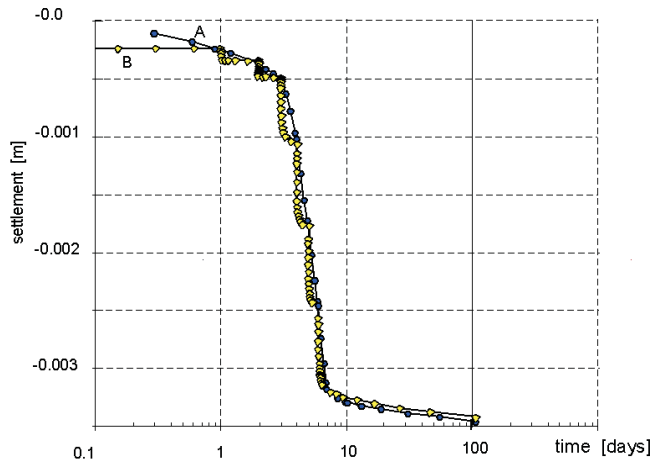


Figure 13.25 Time-settlement curve of oedometer test with Soft Soil Creep model. A) Transient loading with doubling of loading within one day. B) Instantaneous loading with doubling of load at the beginning of a new day

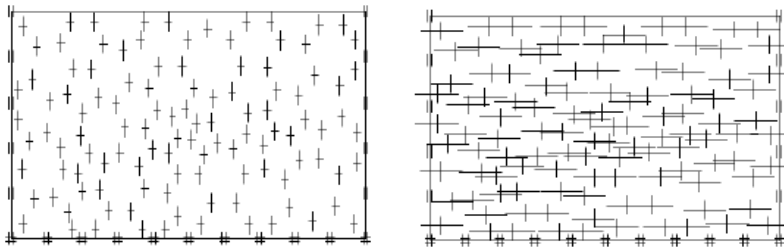


Figure 13.26 Stress states at a vertical stress level of  $-80$  kPa. Left, after primary loading  $\sigma'_{xx} \approx 40$  kPa. Right, after unloading from  $-640$  kPa  $\sigma'_{xx} \approx -220$  kPa

$-320$  kPa to approximately  $-220$  kPa, ( $\Delta\sigma'_{xx} = 100$  kPa), i.e, much less than the decrease of the vertical stress ( $\Delta\sigma'_{yy} = 560$  kPa). Thus, a situation where  $\sigma'_{xx}$  is larger than  $\sigma'_{yy}$  is obtained.

During sudden unloading in a one-dimensional compression test, the behaviour is purely elastic. Hence, the ratio of the horizontal and vertical stress increments can be determined as:

$$\frac{\Delta\sigma'_{xx}}{\Delta\sigma'_{yy}} = \frac{\nu_{ur}}{1 - \nu_{ur}} \tag{13.3}$$

It is easy to verify that the results correspond to Poisson's ratio  $\nu_{ur} = 0.15$  as listed in Table 13.3.

### 13.6 SOFT SOIL MODEL : RESPONSE IN ISOTROPIC COMPRESSION TEST

In this section it will be demonstrated that the Soft Soil model obeys a logarithmic relationship between the volumetric strain and the mean stress in isotropic compression. For this purpose the test set up is simulated as that presented in Figure 13.1. The vertical load (A) and the horizontal load (B) are simultaneously applied to the same level so that a fully isotropic stress state occurs. The parameters of the Soft Soil model are chosen arbitrarily, but the values are realistic for normally consolidated clay. The parameters are presented in Table 13.10.

From a stress-free state, the model is isotropically loaded to a mean stress of  $p' = 100$  kPa, after which the displacements are reset to zero. As a result, the material becomes 'normally consolidated', i.e., the preconsolidation stress is equivalent to the current state-of-stress. After that, the isotropic pressure is increased to  $p' = 1000$  kPa. This loading path is denoted as 'primary loading'. Then, the sample is isotropically 'unloaded' to  $p' = 100$  kPa. Finally, the sample is loaded up to  $p' = 10000$  kPa. In the last loading path, the maximum preload of 1000 kPa is exceeded. and hence, it consists of two parts: the part of the loading path for which  $p' < 1000$  kPa is referred to as 'reloading', and the part of the loading path for  $p' > 1000$  kPa consists of further primary loading. The calculation phases are indicated in Table 13.11.

The computational results are presented in Figure 13.27, which shows the relation between the vertical strain  $\varepsilon_{yy}$  and the vertical stress  $\sigma'_{yy}$ .

The latter quantity is plotted on a logarithmic scale. The plot shows two straight lines, which indicates that there is indeed a logarithmic relation for loading and unloading. The vertical strain is 1/3 of the volumetric strain,  $\varepsilon_v$ , and the vertical stress is equal to the mean stress,  $p'$ . The volumetric strains obtained from the calculation are given in Table 13.12.

From these strains and corresponding stresses, the parameters  $\lambda^*$  and  $\kappa^*$  can be back-calculated using Eqs. (9.1) and (9.2).

$$\begin{aligned} \text{Phase 1} \quad \lambda^* &= -\frac{\varepsilon_v^1 - \varepsilon_v^0}{\ln(p^1/p^0)} = \frac{0.235}{\ln(1000/100)} = 0.102 \\ \text{Phase 2} \quad \kappa^* &= -\frac{\varepsilon_v^2 - \varepsilon_v^1}{\ln(p^2/p^1)} = \frac{0.188 - 0.235}{\ln(100/1000)} = 0.020 \end{aligned}$$

Table 13.10 Soft Soil model parameters for isotropic compression test

Modified compression index	$\lambda^*$	0.10
Modified swelling index	$\kappa^*$	0.02
Poisson's ratio	$\nu_{ur}$	0.15
Friction angle	$\varphi$	30°
Cohesion	$c$	1.0 kPa
Normally consolidated $K_0$	$K_0^{nc}$	0.5

Table 13.11 Calculation phases for isotropic compression test on clay

Stage		Initial stress	Final stress
0	Initial situation		$p^0 = 100 \text{ kPa}$
1	Primary loading	$p^0 = 100 \text{ kPa}$	$p^1 = 1000 \text{ kPa}$
2	Unloading	$p^1 = 1000 \text{ kPa}$	$p^2 = 100 \text{ kPa}$
3	Reloading	$p^2 = 100 \text{ kPa}$	$p^3 = 1000 \text{ kPa}$
4	Primary loading	$p^3 = 1000 \text{ kPa}$	$p^4 = 10000 \text{ kPa}$

Table 13.12 Volumetric strains from various calculation phases

Phase	Initial strain	Final strain
0		$\varepsilon_v^0 = 0.000$
1	$\varepsilon_v^0 = 0.000$	$\varepsilon_v^1 = -0.235$
2	$\varepsilon_v^1 = -0.235$	$\varepsilon_v^2 = -0.188$
3	$\varepsilon_v^2 = -0.188$	$\varepsilon_v^3 = -0.235$
4	$\varepsilon_v^3 = -0.235$	$\varepsilon_v^4 = -0.471$

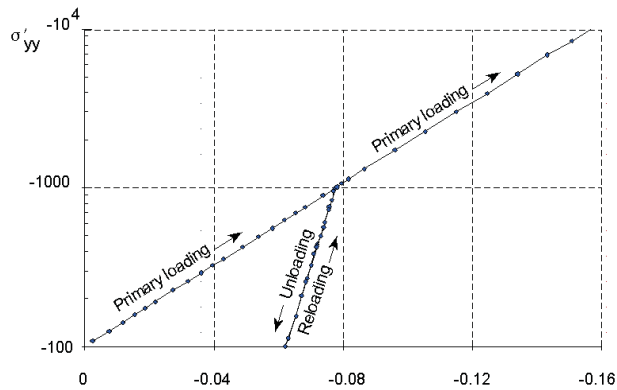


Figure 13.27 Results of isotropic compression test

$$\text{Phase 3} \quad \kappa^* = -\frac{\varepsilon_v^3 - \varepsilon_v^2}{\ln(p^3/p^2)} = \frac{0.235 - 0.188}{\ln(1000/100)} = 0.020$$

$$\text{Phase 4} \quad \lambda^* = -\frac{\varepsilon_v^4 - \varepsilon_v^3}{\ln(p^4/p^3)} = \frac{0.471 - 0.235}{\ln(10000/1000)} = 0.102$$

The back-calculated values correspond to the input values as given in Table 13.10.

Note that the Soft Soil model does not include time effects such as in the secondary compression. Such behaviour is included in the Soft Soil Creep model.



### 13.7 HARDENING SOIL MODEL AND HS SMALL MODEL: EXCAVATION IN BERLIN SAND

In the previous example, the advantage of the Hardening Soil model's distinct loading and unloading stiffness was highlighted. With those, the calculated excavation heave could be reduced to a more realistic, but in most cases still too high value. In the Berlin excavation example, now the further advantage of considering small-strain stiffness in the analysis is demonstrated.

The working group 1.6 Numerical methods in Geotechnics of the German Geotechnical Society (DGGT) has organized several comparative finite element studies (benchmarks). One of these benchmark examples is the installation of a triple anchored deep excavation wall in Berlin sand. The reference solution by Schweiger (2002) is used here as the starting point for the next validation example: Both, the mesh shown in Figure 13.28, and the soil parameters given in Table 13.13 are taken from this reference solution. However, the bottom soil layer 3 defined by Schweiger (2002) is assigned the parameters of layer 2 in the HSsmall analysis. In the reference solution this layer's only purpose is the simulation of small-strain stiffness due to a lack of small-strain stiffness constitutive models back then.

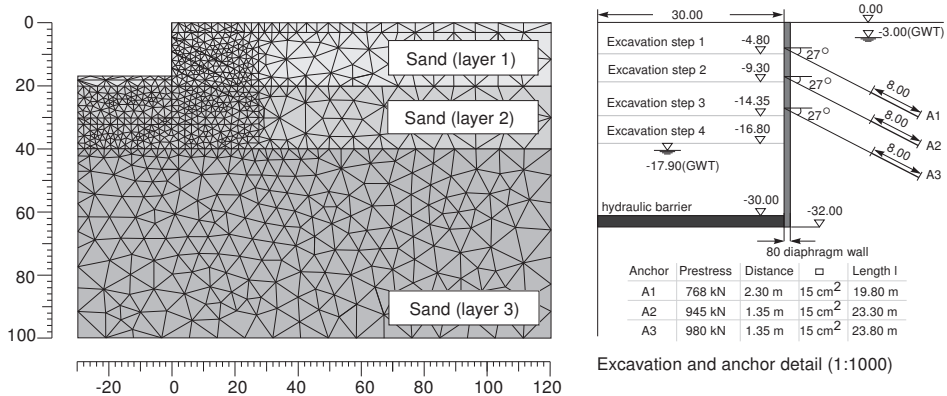


Figure 13.28 Excavation in Berlin sand: plane strain mesh (left) and geometry detail (right).

Figure 13.29 shows results from the finite element calculation using the original Hardening Soil model and the HS small model. The small-strain stiffness formulation in the HS small model accumulates more settlements right next to the wall, whereas the settlement trough is smaller. The triple anchored retaining wall is deflected less when using the HSsmall model, almost fitting the measured deflection. Calculated excavation heave at the end of excavation is shown in Figure 13.30. Compared to the HS results, the heave which is due to elastic unloading, is roughly halved when using the HS small model.

Table 13.13 Hardening Soil model and HS small model parameters for the three sand layers in the excavation project

Parameter	Layer 1	Layer 2	Layer 3	Unit
Unit weight above/below phreatic level	19 / 20	19 / 20	19 / 20	kN/m <sup>3</sup>
$E_{50}^{ref}$ ( $p_{ref} = 100$ kPa)	45000	75000	105000	kN/m <sup>2</sup>
$E_{ur}^{ref}$ ( $p_{ref} = 100$ kPa)	45000	75000	105000	kN/m <sup>2</sup>
$E_{oed}^{ref}$ ( $p_{ref} = 100$ kPa)	180000	300000	315000	kN/m <sup>2</sup>
$G_0^{ref}$ ( $p_{ref} = 100$ kPa)	168750	281250	NA	kN/m <sup>2</sup>
Shear strain $\gamma_{0.7}$	0.0002	0.0002	NA	-
Cohesion $c$	1.0	1.0	1.0	kN/m <sup>2</sup>
Friction angle $\varphi$	35	38	38	°
Dilatancy angle $\psi$	5.0	6.0	6.0	°
Poisson's ratio $\nu_{ur}$	0.2	0.2	0.2	-
Power $m$	0.55	0.55	0.55	-
$K_0^{nc}$	0.43	0.38	0.38	-
Tensile strength	0.0	0.0	0.0	kN/m <sup>2</sup>
Failure ratio	0.9	0.9	0.9	-

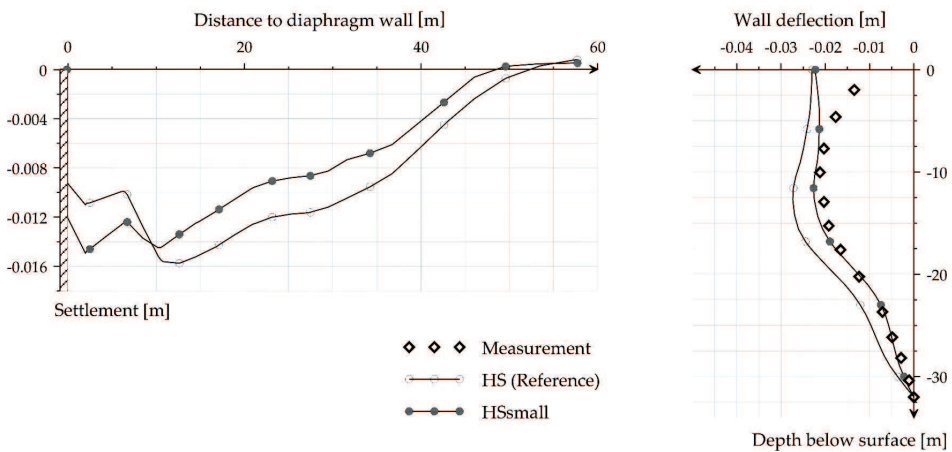


Figure 13.29 Hardening Soil model and HS small model predictions versus measured displacements after the final excavation step. Left: Surface settlement trough. Right: Lateral wall deflection.

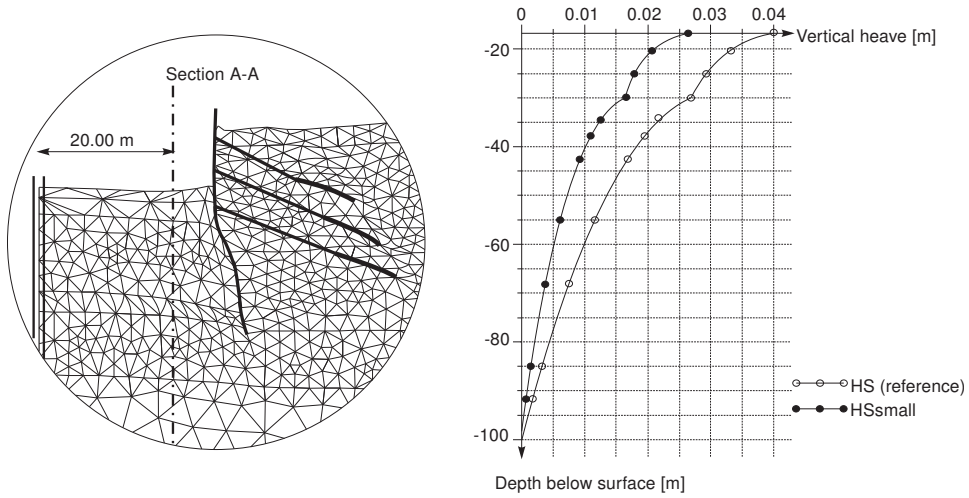


Figure 13.30 Vertical displacements in the excavation pit at a distance of 10 m from the retaining wall (Section A-A).





## 14 USER-DEFINED SOIL MODELS

### 14.1 INTRODUCTION

PLAXIS has a facility for user-defined (UD) soil models. This facility allows users to implement a wide range of constitutive soil models (stress-strain-time relationship) in PLAXIS. Such models must be programmed in FORTRAN (or another programming language), then compiled as a Dynamic Link Library (DLL) and then added to the UDSM sub-folder of the PLAXIS program directory.

In principle the user provides information about the current stresses and state variables and PLAXIS provides information about the previous ones and also the strain and time increments. In the material data base of the PLAXIS input program, the required model parameters can be entered in the material data sets.

$\sigma_{ij}^t + \Delta t, \kappa^{t+\Delta t}$	current stresses and state variables
$\sigma_{ij}^t, \kappa^t$	previous stresses and state variables
$\Delta \varepsilon_{ij}, \Delta t$	strain and time increments

As an example, a UD subroutine based on the Drucker-Prager material model is provided in the user-defined soil models directory, which is included in the program installation package. In this section, a step-by-step description on how a user-defined soil model can be formed and utilised in PLAXIS is presented.

**Hint:** Please note that the PLAXIS organization cannot be held responsible for any malfunctioning or wrong results due to the implementation and/or use of user-defined soil models.

### 14.2 IMPLEMENTATION OF UD MODELS IN CALCULATIONS PROGRAM

The PLAXIS calculations program has been designed to allow for User-defined soil models. There are mainly four tasks (functionalities) to be performed in the calculations program:

- Initialisation of state variables
- Calculation of constitutive stresses (stresses computed from the material model at certain step)
- Creation of effective material stiffness matrix
- Creation of elastic material stiffness matrix

These main tasks (and other tasks) have to be defined by the user in a subroutine called 'User\_Mod'. In this subroutine more than one user-defined soil model can be defined. If a UD soil model is used in an application, the calculation program calls the corresponding task from the subroutine User\_Mod. To create a UD soil model, the User\_Mod subroutine must have the following structure:

Subroutine User\_Mod (IDTask, iMod, IsUndr, iStep, iTer, Iel, Int, X, Y, Z, Time0, dTime, Props, Sig0, Swp0, StVar0, dEps, D, Bulk\_W, Sig, Swp, StVar, ipl, nStat, NonSym, iStrsDep, iTimeDep, iTang, iPrjDir, iPrjLen, iAbort)

where:

- IDTask = Identification of the task (1 = Initialise state variables; 2 = Calculate constitutive stresses; 3 = Create effective material stiffness matrix; 4 = Return the number of state variables; 5 = Return matrix attributes (NonSym, iStrsDep, iTimeDep, iTang); 6 = Create elastic material stiffness matrix)
- iMod = User-defined soil model number (This option allows for more than one UD model, up to 10.)
- IsUndr = Drained condition (IsUndr = 0) or undrained condition (IsUndr = 1). In the latter case, PLAXIS will add a large bulk stiffness for water.
- iStep = Current calculation step number
- iter = Current iteration number
- Iel = Current element number
- Int = Current local stress point number (1..3 for 6-noded elements, or 1..12 for 15-noded elements)
- X, Y, Z = Global coordinates of current stress point
- Time0 = Time at the start of the current step
- dTime = Time increment of current step
- Props = Array(1..50) with User-defined model parameters for the current stress point
- Sig0 = Array(1..20) with previous (= at the start of the current step) effective stress components of the current stress point and some other variables ( $\sigma_{xx}^0$ ,  $\sigma_{yy}^0$ ,  $\sigma_{zz}^0$ ,  $\sigma_{xy}^0$ ,  $\sigma_{yz}^0$ ,  $\sigma_{zx}^0$ ,  $p_{steady}$ ,  $\Sigma Mstage^0$ ,  $\Sigma Mstage$ ,  $Sat$ ,  $Sat^0$ ,  $Suc$ ,  $Suc^0$ ,  $\Sigma Msf^0$ ,  $\Sigma Msf$ ,  $X_j$ ,  $SatRes$ ,  $Temp$ ,  $UnfrozenW$ , 0). In 2D calculations  $\sigma_{yz}$  and  $\sigma_{zx}$  should be zero.
- Swp0 = Previous excess pore pressure of the current stress point
- StVar0 = Array(1..nStat) with previous values of state variables of the current stress point

dEps	=	Array(1..12) with strain increments of the current stress point in the current step ( $\Delta\varepsilon_{xx}$ , $\Delta\varepsilon_{yy}$ , $\Delta\varepsilon_{zz}$ , $\Delta\gamma_{xy}$ , $\Delta\gamma_{yz}$ , $\Delta\gamma_{zx}$ , $\varepsilon_{xx}^0$ , $\varepsilon_{yy}^0$ , $\varepsilon_{zz}^0$ , $\gamma_{xy}^0$ , $\gamma_{yz}^0$ , $\gamma_{zx}^0$ ). In 2D calculations $\Delta\gamma_{yz}$ , $\Delta\gamma_{zx}$ , $\gamma_{yz}^0$ and $\gamma_{zx}^0$ should be zero. In PLAXIS 2D this array may also contain non-local strains. Contact Plaxis for more details.
D	=	Effective material stiffness matrix of the current stress point (1..6, 1..6)
Bulk_W	=	Bulk modulus of water for the current stress point (for undrained calculations and consolidation)
Sig	=	Array (1..6) with resulting constitutive stresses of the current stress point ( $\sigma'_{xx}$ , $\sigma'_{yy}$ , $\sigma'_{zz}$ , $\sigma'_{xy}$ , $\sigma'_{yz}$ , $\sigma'_{zx}$ )
Swp	=	Resulting excess pore pressure of the current stress point
StVar	=	Array(1..nStat) with resulting values of state variables for the current stress point
ipl	=	Plasticity indicator: 0 = no plasticity, 1 = Mohr-Coulomb (failure) point; 2 = Tension cut-off point, 3 = Cap hardening point, 4 = Cap friction point, 5 = Friction hardening point.
nStat	=	Number of state variables (unlimited)
NonSym	=	Parameter indicating whether the material stiffness matrix is non-symmetric (NonSym = 1) or not (NonSym = 0) (required for matrix storage and solution).
iStrsDep	=	Parameter indicating whether the material stiffness matrix is stress-dependent (iStrsDep = 1) or not (iStrsDep = 0).
iTimeDep	=	Parameter indicating whether the material stiffness matrix is time-dependent (iTimeDep = 1) or not (iTimeDep = 0).
iTang	=	Parameter indicating whether the material stiffness matrix is a tangent stiffness matrix, to be used in a full Newton-Raphson iteration process (iTang = 1) or not (iTang = 0).
iPrjDir	=	Project directory (for debugging purposes)
iPrjLen	=	Length of project directory name (for debugging purposes)
iAbort	=	Parameter forcing the calculation to stop (iAbort = 1).

In the above, 'increment' means 'the total contribution within the current step' and not per iteration. 'Previous' means 'at the start of the current step', which is equal to the value at the end of the previous step.

In the terminology of the above parameters it is assumed that the standard type of parameters is used, i.e. parameters beginning with the characters A-H and O-Z are double (8-byte) floating point values and the remaining parameters are 4-byte integer values.

The parameters `IDTask` to `dEps` and `iPrjDir` and `iPrjLen` are input parameters; The values of these parameters are provided by PLAXIS and can be used within the subroutine. These input parameters should not be modified (except for `StVar0` in case `IDTask = 1`). The parameters `D` to `iTang` and `iAbort` are output parameters. The values of these parameters are to be determined by the user. In case `IDTask = 1`, `StVar0` becomes output parameter.

The user subroutine should contain program code for listing the tasks and output parameters (`IDTask = 1` to 6). After the declaration of variables, the `User_Mod` subroutine must have the following structure (here specified in pseudo code):

```
Case IDTask of
  1 Begin
      { Initialise state variables StVar0 }
  End
  2 Begin
      { Calculate constitutive stresses Sig (and Swp) }
  End
  3 Begin
      { Create effective material stiffness matrix D }
  End
  4 Begin
      { Return the number of state variables nStat }
  End
  5 Begin
      { Return matrix attributes NonSym, iStrsDep,
        iTimeDep }
  End
  6 Begin
      { Create elastic material stiffness matrix De }
  End
End Case
```

If more than one UD model is considered, distinction should be made between different models, indicated by the UD model number `iMod`.

### ***Initialise state variables (`IDTask = 1`)***

State variables (also called the hardening parameters) are, for example, used in hardening models to indicate the current position of the yield loci. The update of state variables is considered in the calculation of constitutive stresses based on the previous value of the state variables and the new stress state. Hence, it is necessary to know about the initial value of the state variables, i.e. the value at the beginning of the calculation step. Within a continuous calculation phase, state variables are automatically transferred from one calculation step to another. The resulting value of the state variable in the previous step, `StVar`, is stored in the output files and automatically used as the initial value in the current step, `StVar0`. When starting a new calculation phase, the initial



value of the state variables is read from the output file of the previous calculation step and put in the StVar0 array. In this case it is not necessary to modify the StVar0 array.

However, if the previous calculation step does not contain information on the state variables (for example in the very first calculation step), the StVar0 array would contain zeros. For this case the initial value has to be calculated based on the actual conditions (actual stress state) at the beginning of the step. Consider, for example, the situation where the first state variable is the minimum mean effective stress,  $p'$  (considering that compression is negative). If the initial stresses have been generated using the  $K_0$ -procedure, then the initial effective stresses are non-zero, but the initial value of the state variable is zero, because the initialization of this user-defined variable is not considered in the  $K_0$ -procedure. In this case, part 1 of the user subroutine may look like:

```
1 Begin
  { Initialise state variables StVar0}
  p = (Sig0[1] + Sig0[2] + Sig0[3] ) / 3.0
  StVar0[1] = Min(StVar0[1] ,p)
End
```

### **Calculate constitutive stresses ( $IDTask = 2$ )**

This task constitutes the main part of the user subroutine in which the stress integration and correction are performed according to the user-defined soil model formulation. Let us consider a simple example using a linear elastic  $D$ -matrix as created under  $IDTask = 3$ .

In this case the stress components, Sig, can directly be calculated from the initial stresses, Sig0, the material stiffness matrix,  $D$ , and the strain increments, dEps:  $Sig[i]=Sig0[i] + \sum (D[i, j]*dEps[j])$ . In this case, part 2 of the user subroutine may look like:

```
2 Begin
  { Calculate constitutive stresses Sig (and Swp) }
  For i=1 to 6 do
    Sig[i] = Sig0[i]
    For j=1 to 6 do
      Sig[i] = Sig[i] + D[i,j]*dEps[j]
    End for {j}
  End for {i}
End
```

### **Create effective material stiffness matrix ( $IDTask = 3$ )**

The material stiffness matrix,  $D$ , may be a matrix containing only the elastic components of the stress-strain relationship (as it is the case for the existing soil models in PLAXIS), or the full elastoplastic material stiffness matrix (tangent stiffness matrix). Let us consider the very simple example of Hooke's law of isotropic linear elasticity. There are only two model parameters involved: Young's modulus,  $E$ , and Poisson's ratio,  $\nu$ . These parameters are stored, respectively, in position 1 and 2 of the model parameters array, Props(1..50). In this case, part 3 of the user subroutine may look like:

```
3 Begin
  { Create effective material stiffness matrix D}
  E = Props[1]
  v = Props[2]
```

```

G = 0.5*E/(1.0+v)
Fac = 2*G/(1.0-2*v)  { make sure that v < 0.5 !! }
Term1 = Fac*(1-v)
Term2 = Fac*v
D[1,1] = Term1
D[1,2] = Term2
D[1,3] = Term2
D[2,1] = Term2
D[2,2] = Term1
D[2,3] = Term2
D[3,1] = Term2
D[3,2] = Term2
D[3,3] = Term1
D[4,4] = G
D[5,5] = G
D[6,6] = G
End

```

(By default, D will be initialized to zero, so the remaining terms are still zero; however, it is a good habit to explicitly define zero terms as well.)

If undrained behaviour is considered (`IsUndr = 1`), then a bulk stiffness for water (`Bulk_W`) must be specified at the end of part 3. After calling the user subroutine with `IDTask = 3` and `IsUndr = 1`, PLAXIS will automatically add the stiffness of the water to the material stiffness matrix D such that:  $D[i=1..3, j=1..3] = D[i,j] + \text{Bulk\_W}$ . If `Bulk_W` is not specified, PLAXIS will give it a default value of  $100 * \text{Avg}(D[i=1..3, j=1..3])$ .

### **Return the number of state variables (`IDTask = 4`)**

This part of the user subroutine returns the parameter `nStat`, i.e. the number of state variables. In the case of just a single state parameter, the user subroutine should look like:

```

4  Begin
    { Return the number of state variables nStat }
    nStat = 1
End

```

### **Return matrix attributes (`IDTask = 5`)**

The material stiffness matrix may be stress-dependent (such as in the Hardening Soil model) or time-dependent (such as in the Soft Soil Creep model). When using a tangent stiffness matrix, the matrix may even be non-symmetric, for example in the case of non-associated plasticity. The last part of the user subroutine is used to initialize the matrix attributes in order to update and store the global stiffness matrix properly during the calculation process. For the simple example of Hooke's law, as described earlier, the matrix is symmetric and neither stress- nor time-dependent. In this case the user subroutine may be written as:

```

5  Begin
    { Return matrix attributes NonSym, iStrsDep, }
    { iTimeDep, iTang }
    NonSym = 0

```

```

iStrsDep = 0
iTimeDep = 0
iTang = 0
End

```

For NonSym = 0 only half of the global stiffness matrix is stored using a profile structure, whereas for Nonsym = 1 the full matrix profile is stored.

For iStrsDep = 1 the global stiffness matrix is created and decomposed at the beginning of each calculation step based on the actual stress state (modified Newton-Raphson procedure).

For iTimeDep = 1 the global stiffness matrix is created and decomposed every time when the time step changes.

For iTang = 1 the global stiffness matrix is created and decomposed at the beginning of each iteration based on the actual stress state (full Newton-Raphson procedure; to be used in combination with iStrsDep=1).

### **Create elastic material stiffness matrix (*IDTask* = 6)**

The elastic material stiffness matrix,  $D^e$ , is the elastic part of the effective material stiffness matrix as described earlier.

In the case that the effective material stiffness matrix was taken to be the elastic stiffness matrix, this matrix may just be adopted here. However in the case that an elastoplastic or tangent matrix was used for the effective stiffness matrix, then the matrix to be created here should only contain the elastic components.

The reason that an elastic material stiffness matrix is required is because PLAXIS calculates the current relative global stiffness of the finite element model as a whole (CSP = Current Stiffness Parameter). The CSP parameter is defined as:

$$CSP = \frac{\text{Total work}}{\text{Total elastic work}}$$

The elastic material stiffness matrix is required to calculate the total elastic work in the definition of the CSP. The CSP equals unity if all the material is elastic whereas it gradually reduces to zero when failure is approached.

The CSP parameter is used in the calculation of the global error. The global error is defined as:

$$\text{Global error} = \frac{|\text{unbalance force}|}{|\text{currently activated load}| + CSP \cdot |\text{previously activated load}|}$$

The unbalance force is the difference between the external forces and the internal reactions. The currently activated load is the load that is being activated in the current calculation phase, whereas the previously activated load is the load that has been activated in previous calculation phases and that is still active in the current phase.

Using the above definition for the global error in combination with a fixed tolerated error results in an improved equilibrium situation when plasticity increases or failure is approached. The idea is that a small out-of-balance is not a problem when a situation is mostly elastic, but in order to accurately calculate failure state, safety factor or bearing capacity, a stricter equilibrium condition must be adopted.

Part 6 of the user subroutine looks very similar to part 3, except that only elastic components are considered here. It should be noted that the same variable D is used to store the elastic material stiffness matrix, whereas in Part 3 this variable is used to store the effective material stiffness matrix.

```
6  Begin
    { Create elastic material stiffness matrix D }
    D[1,1] =
    D[1,2] =
    D[1,3] =
    . . . . .
    D[6,6] =
End
```

### ***Using predefined subroutines from the source code***

In order to simplify the creation of user subroutines, a number of FORTRAN subroutines and functions for vector and matrix operations are available in the source code (to be included in the file with the user subroutine). The available subroutines may be called by User\_Mod subroutine to shorten the code. An overview of the available subroutines is given in Appendix C.

### ***Definition of user-interface functions***

In addition to the user-defined model itself it is possible to define functions that will facilitate its use within the Plaxis user-interface. If available, Plaxis Input will retrieve information about the model and its parameters using the procedures described hereafter.

```
procedure GetModelCount(var C:longint) ;
```

C = number of models (return parameter)

This procedure retrieves the number of models that have been defined in the DLL. PLAXIS assumes that model IDs are successive starting at model ID = 1.

```
procedure GetModelName(var iModel : longint;
                       var Name : shortstring) ;
```

iModel = User-defined soil model number to retrieve the name for (input parameter)

Name = model name (return parameter)

This procedure retrieves the names of the models defined in the DLL.

```
procedure GetParamCount(var iModel : longint; var C :longint) ;
```

iModel = User-defined soil model number (input parameter)

C = number of parameters for the specified model (return parameter)

This procedure retrieves the number of parameters of a specific model.

```
procedure GetParamName(var iModel,iParam : longint;
                       var Name : shortstring);
```

iModel = User-defined soil modelnumber (input parameter)  
 iParam = Parameter number (input parameter)  
 Name = parameter name (return parameter)

This procedure retrieves the parameter name of a specific parameter.

```
Procedure GetParamUnit(var iModel,iParam : longint;
                      var Units : shortstring) ;
```

iModel = User-defined soil model number (input parameter)  
 iParam = Parameter number (input parameter)  
 Units = Parameter units (return parameter)

This procedure retrieves the parameter units of a specific parameter. Since the chosen units are dependent on the units of length, force and time chosen by the user the following characters should be used for defining parameter units:

'L' or 'l' for units of length 'F' or 'f' for units of force 'T' or 't' for units of time.

For model names, model parameter names and model parameter units special characters can be used for indicating subscript, superscript or symbol font (for instance for Greek characters).

^ : From here characters will be superscript  
 \_ : From here characters will be subscript  
 @ : From here characters will be in symbol font  
 # : Ends the current superscript or subscript.

Pairs of '^.#', '\_...#' and '@...#' can be nested.

For example:

A UD model parameter uses the oedometer stiffness as parameter. The parameter name can be defined as 'E\_oed#' and its units as 'F/L^2#'.

When defining a unit containing one of the letters 'l', 'f' or 't', like 'cal/mol', these letters will be replaced by the unit of length, the unit of force or the unit of time respectively. To avoid this, these letters should be preceded by a backslash. For example 'cal/mol' should be defined as 'ca\l/mo\l' to avoid getting 'cam/mom'.

The state variables to be displayed in the Output program can be defined.

```
procedure GetStateVarCount(var iModel : longint; var C :longint) ;
```

iModel = User-defined soil model number (input parameter)  
 C = number of state variables for the specified model (return parameter)

This procedure retrieves the number of state variables of a specific model.

```
procedure GetStateVarName(var iModel,iParam : longint;
                          var Name : shortstring);
```

*iModel* = Used-defined soil model number (input parameter)  
*iParam* = Parameter number (input parameter)  
*Name* = parameter name (return parameter)

This procedure retrieves the state parameter name of a specific parameter.

```

Procedure GetStateVArUnit(var iModel,iParam : longint;
                          var Units : shortstring) ;
  
```

*iModel* = User-defined soil model number (input parameter)  
*iParam* = Parameter number (input parameter)  
*Units* = Parameter units (return parameter)

This procedure retrieves the state parameter units of a specific parameter.

All procedures are defined in Pascal but equivalent procedures can be created, for instance in a Fortran programming language. Please make sure that the data format of the parameters in the subroutine headers is identical to those formulated before. For instance, the procedures mentioned above use a "shortstring" type; a "shortstring" is an array of 256 characters where the first character contains the actual length of the shortstring contents. Some programming languages only have null-terminated strings; in this case it may be necessary to use an array of 256 bytes representing the ASCII values of the characters to return names and units. An example of Fortran subroutines is included in the software package.

### ***Compiling the user subroutine***

The user subroutine `User_Mod` has to be compiled into a DLL file using an appropriate compiler. Note that the compiler must have the option for compiling DLL files. Below are examples for two different FORTRAN compilers. It is supposed that the user subroutine `User_Mod` is contained in the file `USRMOD.FOR`.

After creating the user subroutine `User_Mod`, a command must be included to export data to the DLL.

The following statement has to be inserted in the subroutine just after the declaration of variables:

- Using Lahey Fortran (LF95, ...): `DLL_Export User_Mod`
- Using Intel Visual Fortran: `!DEC$ ATTRIBUTES DLLEExport,StdCall,Reference :: User_Mod`

In order to compile the `USRMOD.FOR` into a DLL file, the following command must be executed:

- Using Lahey Fortran 90: `LF90 -win -dll USRMOD.FOR -lib LFUsrLib -ml bd`
- Using Lahey Fortran 95: `LF95 -win -dll USRMOD.FOR -lib LFUsrLib -ml bd`
- Using Intel Visual Fortran: `ifort /winapp USRMOD.FOR DFUsrLib.lib /dll`
- Using GCC compiler: `g95 USRMOD.FOR -o usermod.dll -shared -fcase -upper -fno-underscoring -mrtcd`

- For GCC compiler, the following statement has to be included in the subroutine just after the declaration of variables: NONE

In all cases USRMOD.DLL file will be created. It can be renamed to 'any' .dll. This file should be placed in the **usdm** folder under the PLAXIS program directory, thereafter it can be used together with the existing PLAXIS calculations program (PLASW.EXE in PLAXIS 2D or PLASW3DF.EXE in PLAXIS 3D). Once the UD model is used, PLAXIS will execute the commands as listed in the USRMOD.DLL file.

In order to compile as 64-bit, you need both a 32-bit compiled 'USRMOD.DLL' and a 64-bit compiled ' USRMOD64.DLL' file in the **usdm** folder under the PLAXIS program directory. The last one only needs to contain the 'User\_Mod' subroutine.

### ***Debugging possibilities***

When making computer programs, usually some time is spent to 'debug' earlier written source code. In order to be able to effectively debug the user subroutine, there should be a possibility for the user to write any kind of data to a file. Such a 'debug-file' is not automatically available and has to be created in the user subroutine.

After the debug-file is created, data can be written to this file from within the user subroutine. This can be done by using, for example, the available written subroutines (Section C).

## **14.3 INPUT OF UD MODEL PARAMETERS VIA USER-INTERFACE**

Input of the model parameters for user-defined soil models can be done using the PLAXIS material data base. In fact, the procedure is very similar to the input of parameters for the existing PLAXIS models.

When creating a new material data set for soil and interfaces in the material data base, a window appears with six tabsheets: *General, Parameters, Groundwater, Thermal, Interfaces, Initial* Figure 14.1. A user-defined model can be selected from the *Material model* combo box in the *General* tabsheet.

After inputting general properties, the appropriate UD model can be chosen from the available models that have been found by PLAXIS Input.

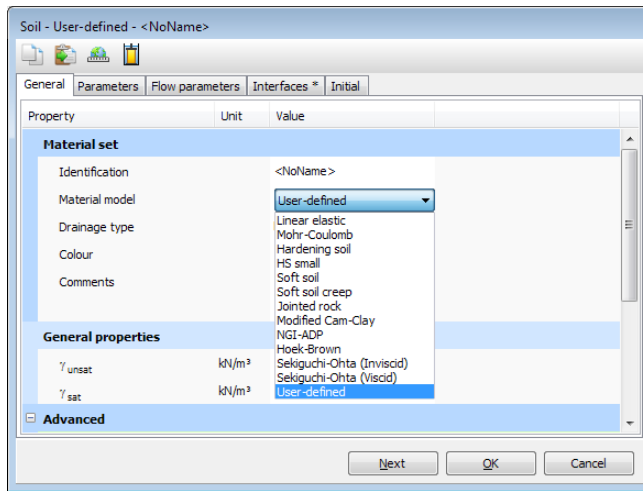
The *Parameters* tabsheet shows two combo boxes; the top combo box lists all the DLLs that contain valid UD models and the next combo box shows the models defined in the selected DLL. Each UD model has its own set of model parameters, defined in the same DLL that contains the model definition.

When an available model is chosen PLAXIS will automatically read its parameter names and units from the DLL and fill the parameter table below.

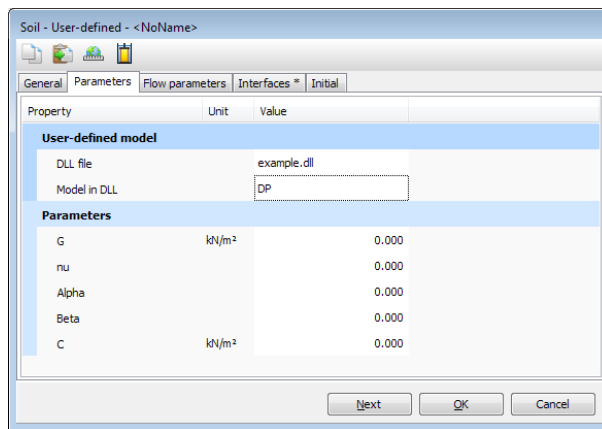
### ***Interfaces***

The *Interfaces* tabsheet, Figure 14.2, contains the material data for interfaces.

Normally, this tabsheet contains the  $R_{inter}$  parameter. For user-defined soil models the interface tabsheet is slightly different and contains the interface oedometer modulus,  $E_{oed}^{ref}$ , and the interface strength parameters  $c_{inter}$ ,  $\varphi_{inter}$  and  $\psi_{inter}$ . Hence, the interface shear strength is directly given in strength parameters instead of using a factor relating



a. Selection of user-defined soil models



b. Input of parameters

Figure 14.1 Selection window

the interface shear strength to the soil shear strength, as it is the case in PLAXIS models.

In addition, two parameters are included to enable stress-dependency of the interface stiffness according to a power law formulation:

$$E_{oed}(\sigma'_n) = E_{oed}^{ref} \left( \frac{\sigma'_n}{UD-P^{ref}} \right)^{UD-Power} \quad (14.1)$$

where UD-Power, is the rate of stress dependency of the interface stiffness, UD-P<sup>ref</sup> is the reference stress level (usually 100 kN/m<sup>2</sup>) and  $\sigma'_n$  is the effective normal stress in the interface stress point.

After having entered values for all parameters, the data sets can be assigned to the corresponding soil clusters, in a similar way as for the existing material models in PLAXIS. The user-defined parameters are transmitted to the calculation program and appear for the appropriate stress points as Props(1..50) in the User\_Mod subroutine.



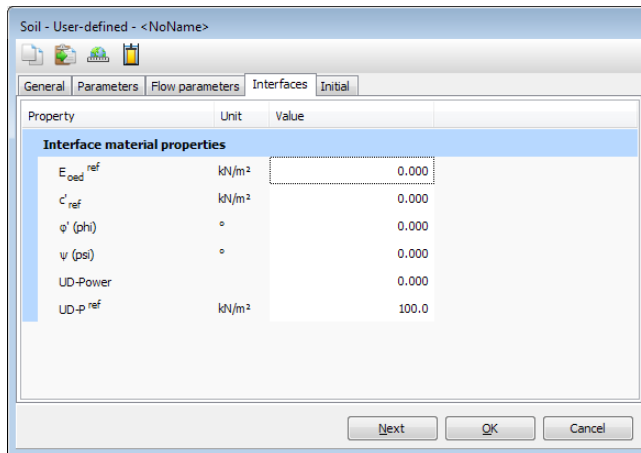


Figure 14.2 Interface tabsheet



## 15 STRUCTURAL BEHAVIOUR

### 15.1 ANCHORS

The elastic behaviour of an anchor involves only a relationship between axial force  $N$  and displacement (elongation)  $u$  of the form:

$$N = \frac{EA}{L}u \quad (15.1)$$

The anchor stiffness  $EA$  is defined by the user based on the material stiffness  $E$  and cross section  $A$ .

In case of elastoplastic behaviour of the anchor the maximum tension force is bound by  $F_{max,tens}$  and the maximum compression force is bound by  $F_{max,comp}$ .



### 15.2 BEAMS

The local system of axes of a beam element is such that the first axis corresponds with the axial beam direction. The second and third axis are always perpendicular to the beam axis.

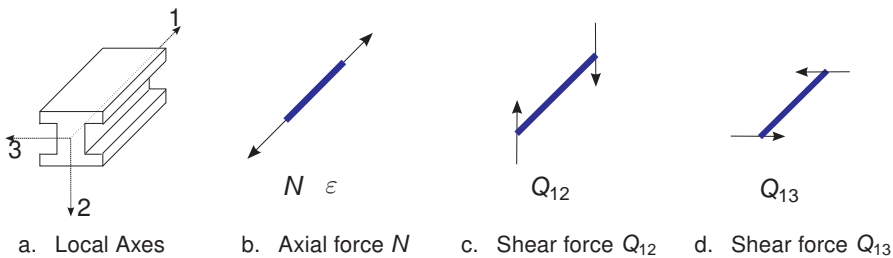


Figure 15.1 Axial force and shear forces

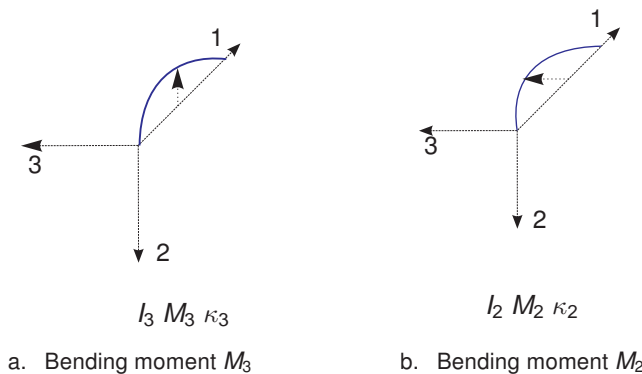


Figure 15.2 Bending moments

Elastic behaviour of beam elements is defined by the following parameters:

- A: Beam cross section area
- E: Young's modulus in axial direction
- $I_2$ : Moment of inertia against bending around the second axis
- $I_3$ : Moment of inertia against bending around the third axis

The relationships between the different force components and the strain / gradient / curvature components are defined as:

$$\begin{aligned}
 N &= EA\varepsilon & M_2 &= EI_2\kappa_2 \\
 Q_{12} &= kGA\gamma_{12} & M_3 &= EI_3\kappa_3 \\
 Q_{13} &= kGA\gamma_{13}
 \end{aligned}
 \tag{15.2}$$

In which  $k$  is the shear correction factor, which is taken as  $\frac{5}{6}$ . The shear modulus is taken as  $G = \frac{1}{2}E$ .



### 15.3 2D GEOGRIDS

The PLAXIS 2D program allows for orthotropic behaviour of geogrid elements, which is defined by the parameters  $EA_1$  and  $EA_2$ . The stiffnesses are defined by the user and are based on the material tension stiffnesses ( $E_1, E_2$ ) and the cross section areas ( $A_1, A_2$ ) corresponding to the local axes of the geogrid. Geogrid elements cannot sustain compression forces.

The relationship between the force and the strain in axisymmetric models is defined as:

$$\begin{bmatrix} N \\ H \end{bmatrix} = \begin{bmatrix} EA_1 & 0 \\ 0 & EA_2 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \varepsilon_H \end{bmatrix}
 \tag{15.3}$$

where  $H$  is the hoop force and  $\varepsilon_H$  is the hoop strain. For plane strain model  $H = 0$ .

In case of elastoplastic behaviour the maximum tension force in any direction is bound by  $N_{p,1}$  and  $N_{p,2}$ .



### 15.4 3D GEOGRIDS

The PLAXIS 3D program allows for orthotropic as well as anisotropic material behaviour in geogrid elements, which is defined by the following parameters:

$$\begin{aligned}
 N_1 &= EA_1\varepsilon_1 \\
 N_2 &= EA_2\varepsilon_2 \\
 Q_{12} &= GA\gamma_{12}
 \end{aligned}
 \tag{15.4}$$

In the case of orthotropic behaviour  $EA_1 = EA_2$  and  $GA = \frac{1}{2}EA_1$  in the general three dimensional case.

When plasticity is considered, the maximum tensile forces can be defined:

- $N_{p,1}$ : Maximum tensile force in 1-direction

$N_{p,2}$ : Maximum tensile force in 2-direction

Axial forces are calculated at the stress points of the geogrid elements. If  $N_p$  is exceeded, stresses are redistributed according to the theory of plasticity, so that the maximum forces are complied with. This will result in irreversible deformations. Output of axial forces is given in the nodes, which requires extrapolation of the values at the stress points. Nodal forces are not checked against the maximum forces. Hence, it is possible that the nodal values of the axial force may slightly exceed  $N_p$ .



## 15.5 2D PLATES

The PLAXIS 2D program allows for orthotropic elasto-plastic material behaviour in plate elements. The elastic behaviour is defined by the following parameters:

$EA_1$ : Normal stiffness

$EA_2$ : Stiffness in the out of plane direction

$EI$ : Bending stiffness

$\nu$ : Poisson's ratio

The material behaviour in plate elements is defined by the following relationship between strains and stresses.

$$\begin{bmatrix} \sigma_N \\ \sigma_2 \\ \tau \end{bmatrix} = \begin{bmatrix} \frac{2G}{1-\nu} & \frac{2G\nu}{1-\nu} & 0 \\ \frac{2G\nu}{1-\nu} & \frac{2G}{1-\nu} & 0 \\ 0 & 0 & kG \end{bmatrix} \begin{bmatrix} \varepsilon_N \\ \varepsilon_2 \\ \gamma \end{bmatrix} \quad (15.5)$$

In which  $k$  is the shear correction factor, which is taken as  $\frac{5}{6}$ . For isotropic material:

$$G = \frac{E_1}{2(1+\nu)} \quad (15.6)$$

For anisotropic plates, the following relationship between strains and stresses is used:

$$\begin{bmatrix} \sigma_N \\ \sigma_2 \\ \tau \end{bmatrix} = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & kG \end{bmatrix} \begin{bmatrix} \varepsilon_N \\ \varepsilon_2 \\ \gamma \end{bmatrix} \quad (15.7)$$

where  $E_1 = EA_1/d$  and  $E_2 = EA_2/d$ . Note that the Poisson's ratio ( $\nu$ ) is assumed to be zero in anisotropic case.

The material behaviour in plate elements is defined by the following relationship between structural forces and strains:

$$N = EA_1 \varepsilon \quad (15.8)$$

$$H = EA_2 \varepsilon_H \quad (15.9)$$

$$Q = \frac{kEA}{2(1 + \nu)} \gamma^* \quad (15.10)$$

$$M = EI \kappa \quad (15.11)$$

The modified shear strain  $\gamma^*$  takes into account the shear strain  $\gamma$  and some additional terms in order to give a more accurate approximation of the problem.  $k$  is the shear correction factor, which is taken as  $\frac{5}{6}$ . This implies that the shear stiffness is determined from the assumption that the plate has a rectangular cross section. In the case of modelling a solid wall, this will give the correct shear deformation. However, in the case of steel profile elements, like sheet-pile walls, the computed shear deformation may be too large. You can check this by judging the value of  $d_{eq}$ , which can be computed as  $\sqrt{12EI/EA}$ . For steel profile elements,  $d_{eq}$  should be at least of the order of a factor 10 times smaller than the length of the plate to ensure negligible shear deformations.

When plasticity is considered, the maximum bending moment and maximum normal force can be defined:

$M_p$ : Maximum bending moment

$N_p$ : Maximum normal force

The maximum bending moment is given in units of force times length per unit width. The maximum axial force,  $N_p$ , is specified in units of force per unit width. When the combination of a bending moment and an axial force occur in a plate, then the actual bending moment or axial force at which plasticity occurs is lower than respectively  $M_p$  or  $N_p$ . The relationship between  $M_p$  and  $N_p$  is visualised in (Figure 15.3). The diamond shape represents the ultimate combination of forces for which plasticity will occur. Force combinations inside the diamond will result in elastic deformations only.

By default the maximum moment is set to  $1 \cdot 10^{15}$  units if the material type is set to elastic (the default setting).

Bending moments and axial forces are calculated at the stress points of the beam elements. When yield function is violated, stresses are redistributed according to the theory of plasticity, so that the maxima are complied with. This will result in irreversible deformations. Output of bending moments and axial forces is given in the nodes, which requires extrapolation of the values at the stress points. Nodal forces are not checked against the maximum forces. Hence, it is possible that the nodal values of the axial force may slightly exceed  $M_p$ .

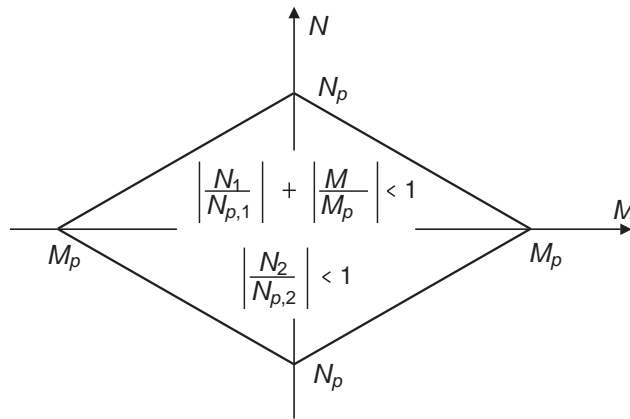


Figure 15.3 Combinations of maximum bending moment and axial force



## 15.6 3D PLATES

The PLAXIS 3D program allows for orthotropic material behaviour in plate elements, which is defined by the following parameters:

- $E_1$ : Young's modulus in first axial direction
- $E_2$ : Young's modulus in second axial direction
- $G_{12}$ : In-plane shear modulus
- $G_{13}$ : Out-of-plane shear modulus related to shear deformation over first direction
- $G_{23}$ : Out-of-plane shear modulus related to shear deformation over second direction
- $\nu_{12}$ : Poisson's ratio ( $\nu_{12} < \sqrt{E_1/E_2}$ )

The material behaviour in plate elements is defined by the following relationship between strains and stresses, which is based on the general three-dimensional continuum mechanics theory and the assumption that  $\sigma_{33} = 0$ .

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 & 0 & 0 \\ 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 1/kG_{13} & 0 \\ 0 & 0 & 0 & 0 & 1/kG_{23} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} \quad (15.12)$$

In which  $k$  is the shear correction factor, which is taken as  $\frac{5}{6}$ . Inverting this relationship and ignoring the higher order terms in  $\nu$  gives the following stress-strain relationship:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} E_1 & \nu_{12}E_2 & 0 & 0 & 0 \\ \nu_{12}E_2 & E_2 & 0 & 0 & 0 \\ 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & kG_{13} & 0 \\ 0 & 0 & 0 & 0 & kG_{23} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} \quad (15.13)$$

This approximation holds as long as the Poisson's ratio is small. These stress-strain relationships can be transformed into relationships for structural forces:

$$\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} E_1 d & \nu_{12}E_2 d \\ \nu_{12}E_2 d & E_2 d \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \quad (15.14a)$$

$$\begin{bmatrix} Q_{12} \\ Q_{13} \\ Q_{23} \end{bmatrix} = \begin{bmatrix} G_{12} d & 0 & 0 \\ 0 & kG_{13} d & 0 \\ 0 & 0 & kG_{23} d \end{bmatrix} \begin{bmatrix} \gamma_{12} \\ \gamma_{13}^* \\ \gamma_{23}^* \end{bmatrix} \quad (15.14b)$$

$$\begin{bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1 d^3}{12} & \frac{\nu_{12} E_2 d^3}{12} & 0 \\ \frac{\nu_{12} E_2 d^3}{12} & \frac{E_2 d^3}{12} & 0 \\ 0 & 0 & \frac{G_{12} d^3}{12} \end{bmatrix} \begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ \kappa_{12} \end{bmatrix} \quad (15.14c)$$

In which  $d$  is the thickness of the plate, which also determines the distributed weight of the plate together with the unit weight of the plate material:  $\gamma \cdot d$ . The modified shear strain  $\gamma^*$  takes into account the shear strain  $\gamma$  and some additional terms in order to give a more accurate approximation of the problem.

The local system of axes in a plate element is such that the first and the second local axis lie in the plane of the plate whereas the third axis is perpendicular to the plane of the plate (Figure 15.4).

When geometric orthotropy is considered rather than material orthotropy, the following relationships for structural forces apply:

$$\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} \frac{EA_1}{1-\nu^2} & \frac{\nu EA_1}{1-\nu^2} \\ \frac{\nu EA_1}{1-\nu^2} & \frac{EA_2}{1-\nu^2} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \approx \begin{bmatrix} EA_1 & \nu EA_1 \\ \nu EA_1 & EA_2 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \quad (15.15a)$$

$$\begin{bmatrix} Q_{12} \\ Q_{13} \\ Q_{23} \end{bmatrix} = \begin{bmatrix} \frac{EA_{12}}{2(1+\nu)} & 0 & 0 \\ 0 & \frac{EA_{13}}{2(1+\nu)} & 0 \\ 0 & 0 & \frac{EA_{23}}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \gamma_{12} \\ \gamma_{13}^* \\ \gamma_{23}^* \end{bmatrix} \quad (15.15b)$$



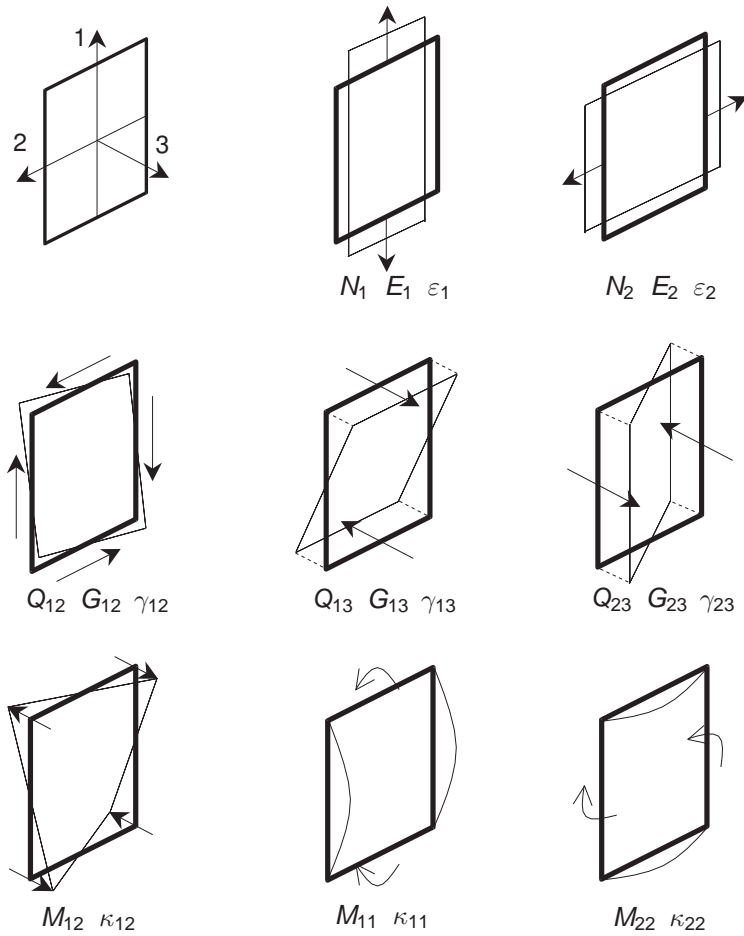


Figure 15.4 Definition of positive normal forces (N), shear forces (Q) and bending moments (M) for a plate based on local system of axes

$$\begin{bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu^2} & \frac{\nu E_1}{1-\nu^2} & 0 \\ \frac{\nu E_1}{1-\nu^2} & \frac{E_2}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E_{12}}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ \kappa_{12} \end{bmatrix} \approx \begin{bmatrix} E_1 & \nu E_1 & 0 \\ \nu E_1 & E_2 & 0 \\ 0 & 0 & \frac{E_{12}}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ \kappa_{12} \end{bmatrix} \quad (15.15c)$$

where the approximations hold for a small Poisson's ratio. In these relations the following alternative parameters are used:

- $A_1$ : Effective material cross section area for axial forces in the first direction
- $A_2$ : Effective material cross section area for axial forces in the second direction
- $A_{12}$ : Effective material cross section area for shear forces  $Q_{12}$
- $A_{13}$ : Effective material cross section area for shear forces  $Q_{13}$

- $A_{23}$ : Effective material cross section area for shear forces  $Q_{23}$
- $I_1$ : Moment of inertia against bending over the first axis
- $I_2$ : Moment of inertia against bending over the second axis
- $I_{12}$ : Moment of inertia against torsion

In order to use the available plate elements for geometric orthotropy, the basic material parameters should be chosen in such a way that the resulting normal stiffness  $E_1 A$  is equal to the normal stiffness  $EA_1$  of the plate. Here  $E_1$  is the input value for the Young's modulus in the first direction,  $A$  is the internally calculated area of the plate,  $E$  is the actual Young's modulus of the material and  $A_1$  is the cross sectional area of the element to be modelled. Similar parameter choices should be made for the other normal stiffness, flexural rigidities and shear stiffnesses. However, for a given choice of the equivalent plate thickness  $d$  it will not be possible to match all stiffness components exactly. The thickness  $d$  is the equivalent plate thickness such that the average distributed weight of the plate corresponds to  $\gamma \cdot d$ .

When bending is considered as the most important type of deformation, the following choices are recommended:

$$\begin{aligned}
 E_1 &= 12EI_1/d^3 & E_2 &= 12EI_2/d^3 \\
 G_{12} &= \frac{6EI_{12}}{(1+\nu)d^3} & G_{13} &= \frac{EA_{13}}{2(1+\nu)d} & G_{23} &= \frac{EA_{23}}{2(1+\nu)d} & (15.16) \\
 \nu_{12} &= \nu
 \end{aligned}$$

In this case the resulting flexural rigidities  $E_1 d^3/12$  and  $E_2 d^3/12$  and shear stiffnesses  $G_{13}d$  and  $G_{23}d$  prove to be independent of the chosen value for equivalent plate thickness. Only the normal stiffnesses  $E_1 A$  and  $E_2 A$  and shear stiffness  $G_{12}d$  are not independent of the chosen value of the equivalent plate thickness, and a suitable selection for  $d$  has to be made. What is the most suitable selection for  $d$  depends on the construction that is to be modelled. Two examples are given below.

*Engineering examples:*

In the following, two types of applications are given that frequently occur in the engineering practice. The first example is a sheet-pile wall, as depicted in Figure 15.5. From the sheet-pile manufacturer, the following properties are known:  $t$  (wall thickness),  $h$  (total height),  $A$  (per m wall width),  $I_1$ ,  $E_{steel}$  and  $\gamma_{steel}$ .

The structure is geometrically orthotropic with significant different stiffnesses in horizontal and vertical direction. It is known that the axial stiffness in vertical direction is larger than the effective stiffness in horizontal direction ( $E_1 > E_2$ ). Moreover, the flexural rigidity against bending over the vertical direction,  $I_1$ , is much larger than the stiffness against bending over the horizontal direction,  $I_2$ , ( $I_1 \gg I_2$  say  $I_1 \approx 20I_2$  and  $I_1 \gg I_{12}$  say  $I_1 \approx 10I_{12}$ ).<sup>‡</sup>

Furthermore, it is assumed that the cross section area that is effective against shear deformation over the vertical direction is about 1/3 of the total cross section area,

<sup>‡</sup> A factor of 20 is used here to move the bending stiffness over the first direction sufficiently small compared to the bending stiffness over the second direction, whilst the matrix condition is still OK. Note that in reality bending stiffness differences in order of 1000 may exist.

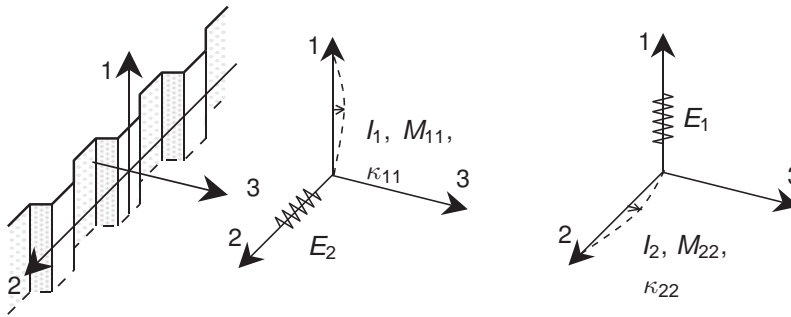


Figure 15.5 Example of sheet-pile wall with its major quantities

whereas the area that is effective against shear deformation over the horizontal direction is about 1/10 of the total cross section area. Finally, the Poisson's ratio's for sheet pile walls can be assumed zero. With these assumptions, the situation could be modelled by selecting the model parameters in the following way:

$$d = h \quad (\text{which is considerably larger than } t)$$

$$E_1 = 12E_{steel}I_1/d^3$$

$$E_2 = 12E_{steel}I_2/d^3 \approx I_2/I_1 E_1 \approx E_1/20$$

$$G_{12} = \frac{6E_{steel}I_2}{(1 + \nu_{steel})d^3} \approx 6E_{steel}I_1/10d^3$$

$$G_{13} = \frac{E_{steel}A_{13}}{2(1 + \nu_{steel})d} \approx E_{steel}(A/3)/2d \approx E_{steel}A/6d$$

$$G_{23} = \frac{E_{steel}A_{23}}{2(1 + \nu_{steel})d} \approx E_{steel}(A/10)/2d \approx E_{steel}A/20d$$

$$\nu_{12} \approx 0$$

$$\gamma = A\gamma_{steel}/d$$

The second example is a concrete T-shaped floor profile, as depicted in Figure 15.6. In addition to the precise geometry dimensions, the following properties are known:

$$E_{concrete}, \nu_{concrete}$$

The structure is again geometrically orthotropic with significant different stiffnesses in the two major floor directions. The flexural rigidity against bending over the second direction,  $I_2$ , is larger than the stiffness against bending over the first direction,  $I_1$ , ( $I_2 > I_1$ ), since  $I_2$

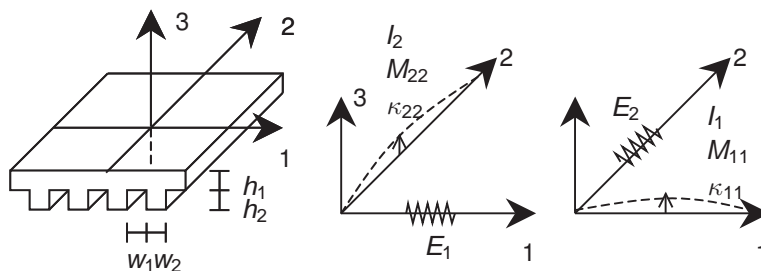


Figure 15.6 Example of concrete floor profile with its major quantities

is dominated by  $h_1 + h_2$  whereas  $I_1$  only depends on  $h_1$ . Furthermore, the cross section area (per unit of width) that is effective against shear deformation over the second direction ( $G_{23}$ ) is equal to the total cross section area  $A$ , whereas the area that is effective against shear deformation over the first direction ( $G_{13}$ ) is equal to  $h_1$ . With these assumptions, the situation could be modelled by selecting the model parameters in the following way:

$$d = \frac{h_1 w_1 + (h_1 + h_2) w_2}{w_1 + w_2}$$

$$E_1 = 12 E_{\text{concrete}} I_1 / d^3 = 12 E_{\text{concrete}} \frac{1}{12} h_1^3 / d^3 = E_{\text{concrete}} h_1^3 / d^3$$

$$E_2 = 12 E_{\text{concrete}} I_2 / d^3 \quad \text{where } I_2 = \frac{1}{12} h_1^3 w_1 + \frac{1}{12} (h_1 + h_2)^3 w_2$$

$$G_{12} = \frac{6 E_{\text{concrete}} I_{12}}{(1 + \nu_{\text{concrete}}) d^3} \quad \text{where } I_{12} \approx I_1 = \frac{1}{12} h_1^3$$

$$G_{13} = \frac{E_{\text{concrete}} A_{13}}{2(1 + \nu_{\text{concrete}}) d} = \frac{E_{\text{concrete}} h_1}{2(1 + \nu_{\text{concrete}}) d}$$

$$G_{23} = \frac{E_{\text{concrete}} A_{23}}{2(1 + \nu_{\text{concrete}}) d} = \frac{E_{\text{concrete}} d}{2(1 + \nu_{\text{concrete}}) d} = \frac{E_{\text{concrete}}}{2(1 + \nu_{\text{concrete}})}$$

$$\nu_{12} = \nu_{\text{concrete}}$$



## 15.7 2D EMBEDDED PILE ROW

An embedded pile in PLAXIS 2D consists of plate elements with embedded interface elements to describe the interaction with the soil at the pile skin and at the pile foot (bearing capacity). The material parameters of the embedded pile distinguish between

the parameters of the pile and the parameters of the skin resistance and foot resistance. The plate elements are considered to be linear elastic. The elastic behaviour is defined by the following parameters:

$E$ : Young's modulus in axial direction

$A$ : Pile cross section area

$I$ : Moment of inertia against bending around the pile axis

The material behaviour in plate elements is defined by the following relationship between strains and stresses.

$$\begin{bmatrix} \sigma_N \\ \tau \end{bmatrix} = \begin{bmatrix} E & 0 \\ 0 & kG \end{bmatrix} \begin{bmatrix} \varepsilon_N \\ \gamma \end{bmatrix} \quad (15.17)$$

Where

$\sigma_N$ : Normal stress

$\tau$ : Shear stress

$G$ : Shear modulus

$$G = \frac{E}{2(1 + \nu)}$$

$\kappa$ : Shear correction factor taken as  $\frac{5}{6}$

$\varepsilon_N$ : Normal strain

The material behaviour in plate elements is defined by the following relationship between structural forces and strains:

$$N = EA_1 \varepsilon_N \quad (15.18)$$

$$Q = \frac{kEA}{2(1 + \nu)} \gamma^* \quad (15.19)$$

$$M = EI\kappa \quad (15.20)$$

The modified shear strain  $\gamma^*$  takes into account the shear strain  $\gamma$  and some additional terms in order to give a more accurate approximation of the problem.

The interaction of the pile with the soil at the skin of the pile is described by linear elastic behaviour with a finite strength and is defined by the following parameter:

$T_{max}$ : Maximum traction allowed at the skin of the embedded pile (can vary along the pile)

The constitutive equation at the skin of the pile is defined by (see Figure 15.7):

$$\begin{bmatrix} t_s \\ t_n \end{bmatrix} = \begin{bmatrix} K_s & 0 \\ 0 & K_n \end{bmatrix} \begin{bmatrix} u_s^p - u_s^s \\ u_n^p - u_n^s \end{bmatrix} \quad (15.21)$$

where  $u^p$  denotes the displacement of the pile and  $u^s$  denotes the displacement of the soil.  $K_s$  denotes the elastic shear stiffness (against longitudinal (axial) displacement differences) of the embedded interface elements and  $K_n$  denotes the elastic normal

stiffness (against transverse (lateral) displacement differences) of the embedded interface elements. These values are calculated using the interface stiffness factors and the pile spacing defined by the user:

$$\begin{aligned}
 K_s &= ISF_{RS} \frac{G_{soil}}{L_{spacing}} \\
 K_n &= ISF_{RN} \frac{G_{soil}}{L_{spacing}}
 \end{aligned}
 \tag{15.22}$$

Default interface stiffness factors are calculated based on the pile spacing ( $L_{spacing}$ ) in relation to the pile diameter ( $D_{eq}$ ):

$$ISF_{RS} = 2.5 \left( \frac{L_{spacing}}{D_{eq}} \right)^{-0.75}
 \tag{15.23}$$

$$ISF_{RN} = 2.5 \left( \frac{L_{spacing}}{D_{eq}} \right)^{-0.75}
 \tag{15.24}$$

$$ISF_{KF} = 25 \left( \frac{L_{spacing}}{D_{eq}} \right)^{-0.75}
 \tag{15.25}$$

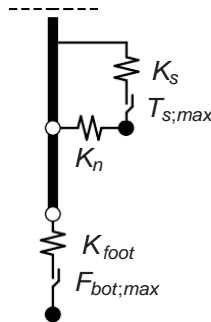


Figure 15.7 Stiffness of the embedded interface elements for piles

The normal stress  $t_n$  will always remain elastic. For the shear stress in axial direction  $t_s$  to remain elastic it is given by:

$$|t_s| < T_{max}
 \tag{15.26}$$

For plastic behaviour the shear force  $t_s$  is given by:

$$|t_s| = T_{max}
 \tag{15.27}$$

The interaction of the pile with the soil at the foot of the pile is described by a linear elastic perfectly plastic interface element. The strength of the base is described by the following parameter:

$F_{max}$ : Maximum force allowed at the foot of the embedded pile

In addition, no tension forces are allowed. In order to ensure that a realistic pile bearing capacity as specified can actually be reached, a zone in the soil surrounding the pile foot

is identified where any kind of soil plasticity is excluded (elastic zone; Figure 15.8). The size of this zone is determined by the embedded pile's equivalent radius  $R_{eq}$  (Reference Manual).

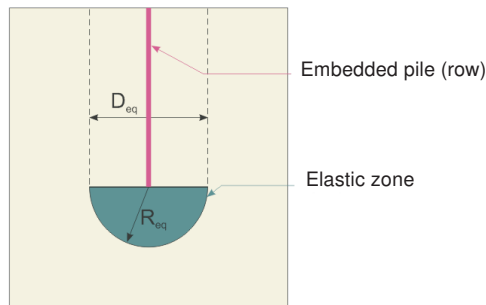


Figure 15.8 Elastic zone surrounding the bottom of the pile (after Sluis (2012))

The constitutive relationship at the foot of the pile and its failure criterion are defined by:

$$F_{foot} = K_{foot}(u_{foot}^p - u_{foot}^s) < F_{max} \quad (15.28)$$

where  $K_{foot}$  denotes the stiffness of the spring which is defined in the same way as the stiffness of the embedded interface elements:

$$K_{foot} = ISF_{KF} \frac{G_{soil} R_{eq}}{L_s} \quad (15.29)$$

In case of plastic behaviour, the foot force  $F_{foot}$  is given by:

$$F_{foot} = F_{max} \quad (15.30)$$



## 15.8 3D EMBEDDED PILE

An embedded pile in PLAXIS 3D consists of beam elements with embedded interface elements to describe the interaction with the soil at the pile skin and at the pile foot (bearing capacity). The material parameters of the embedded pile distinguish between the parameters of the beam and the parameters of the skin resistance and foot resistance. The beam elements are considered to be linear elastic and are defined by the same material parameters as a regular beam element (Section 15.2).

The interaction of the pile with the soil at the skin of the pile is described by linear elastic behaviour with a finite strength and is defined by the following parameter:

$T_{max}$ : Maximum traction allowed at the skin of the embedded pile (can vary along the pile)

The constitutive equation at the skin of the pile is defined by (see Figure 15.9):

$$\begin{bmatrix} t_s \\ t_n \\ t_t \end{bmatrix} = \begin{bmatrix} K_s & 0 & 0 \\ 0 & K_n & 0 \\ 0 & 0 & K_t \end{bmatrix} \begin{bmatrix} u_s^p - u_s^s \\ u_n^p - u_n^s \\ u_t^p - u_t^s \end{bmatrix} \quad (15.31)$$

where  $u^p$  denotes the displacement of the pile and  $u^s$  denotes the displacement of the soil.  $K_s$  denotes the elastic shear stiffness (against parallel displacement differences) of the embedded interface elements and  $K_n$  and  $K_t$  denote the elastic normal stiffness (against perpendicular displacement differences) of the embedded interface elements. By default these values are defined such that the stiffness of the embedded interface elements does not influence the total elastic stiffness of the pile-soil structure:

$$\begin{aligned} K_s &\gg G_{soil} \\ K_n = K_t &= \frac{2(1-\nu)}{1-2\nu} K_s \end{aligned} \quad (15.32)$$

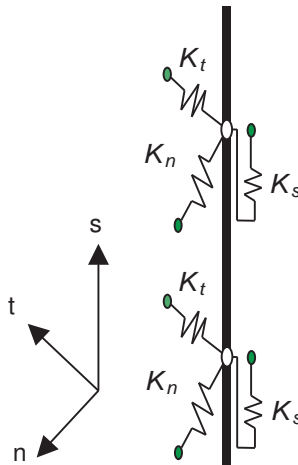


Figure 15.9 Stiffness of the embedded interface elements at the skin of the pile

The normal stresses  $t_n$  and  $t_t$  will always remain elastic. For the shear stress in axial direction  $t_s$  to remain elastic it is given by:

$$|t_s| < T_{\max} \quad (15.33)$$

For plastic behaviour the shear force  $t_s$  is given by:

$$|t_s| = T_{\max} \quad (15.34)$$

In case of defining a layer dependent skin resistance the shear force  $t_s$  will remain elastic as long as:

$$|t_s| < (\sigma_n^{avg} \tan \varphi_i + c_i) \pi D_{eq} \quad \text{and} \quad |t_s| < T_{\max} \quad (15.35)$$



where  $D_{eq}$  denotes the diameter or the equivalent diameter (in the case alternative beam properties have been specified) of the embedded pile  
 $(D_{eq} = 2 \max \left\{ \sqrt{(A/\pi)}, \sqrt{(2I_{avg}/A)} \right\})$  where  $I_{avg} = (I_2 + I_3)/2$  and  $\sigma_n^{avg}$  is the average lateral (perpendicular) stress of the soil around the pile:

$$\sigma_n^{avg} = \frac{1}{2}(\sigma_{xx} + \sigma_{zz}) \quad (\text{in the case of a vertical pile}) \quad (15.36)$$

The parameters  $\varphi_i$  and  $c_i$  are the friction angle and cohesion of the embedded interface. The strength properties of embedded interfaces with layer dependent skin resistance are linked to the strength properties of a soil layer. Each data set has an associated strength reduction factor for interfaces  $R_{inter}$ . The embedded interface properties are calculated from the soil properties in the associated data set and the strength reduction factor by applying the following rules:

$$\begin{aligned} \tan \varphi_i &= R_{inter} \tan \varphi_{soil} \\ c_i &= R_{inter} c_{soil} \end{aligned} \quad (15.37)$$

For plastic behaviour  $t_s$  is given by:

$$|t_s| = (\sigma_n^{avg} \tan \varphi_i + c_i) \pi D \quad \text{provided that} \quad (\sigma_n^{avg} \tan \varphi_i + c_i) \pi D \leq T_{max} \quad (15.38)$$

In the case of a layer dependent skin resistance where the actual bearing capacity is not known,  $T_{max}$  can be used as an ultimate cut-off value. The interaction of the pile with the soil at the foot of the pile is described by a linear elastic perfectly plastic interface element. The strength of the base is described by the following parameter:

$F_{max}$ : Maximum force allowed at the foot of the embedded pile

In addition, no tension forces are allowed. The constitutive relationship at the foot of the pile and its failure criterion are defined by (see Figure 15.10):

$$F_{foot} = K_{foot}(u_{foot}^p - u_{foot}^s) < F_{max} \quad (15.39)$$

where  $K_{foot}$  denotes the stiffness of the spring which is defined in the same way as the stiffness of the embedded interface elements:

$$K_{foot} \gg G_{soil} \quad (15.40)$$

In case of plastic behaviour, the foot force  $F_{foot}$  is given by:

$$F_{foot} = F_{max} \quad (15.41)$$

In order to ensure that a realistic pile bearing capacity as specified can actually be reached, a zone in the soil volume elements surrounding the beam is identified where any kind of soil plasticity is excluded (elastic zone Figure 15.8). The size of this zone is determined by the embedded pile's equivalent radius  $R_{eq} = \max \left\{ \sqrt{(A/\pi)}, \sqrt{(2I_{avg}/A)} \right\}$  where  $I_{avg} = (I_2 + I_3)/2$ . The elastic zone makes the embedded pile almost behave like a volume pile. However, installation effects of piles are not taken into account and the pile-soil interaction is modelled at the centre rather than at the circumference.

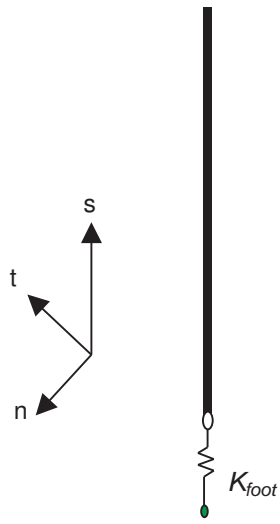


Figure 15.10 Stiffness of the embedded interface element at the foot of the pile



## 16 HYDRAULIC MODELS

### 16.1 VAN GENUCHTEN MODEL

A Soil Water Characteristic Curve (SWCC) is introduced to describe hydraulic parameters of the groundwater flow in unsaturated zones (usually above the phreatic surface). The SWCC describes the capacity of the soil to keep water at different stresses. There are many models which describe the hydraulic behaviour of unsaturated soils. The most common in the groundwater literature is the model proposed by Van Genuchten (1980), which is used in PLAXIS. The Van Genuchten function is a three-parameter equation and relates the saturation to the pressure head  $\phi_p$ :

$$S(\phi_p) = S_{res} + (S_{sat} - S_{res})[1 + (g_a|\phi_p|)^{g_n}]^{g_c} \quad (16.1)$$

Where

$$\phi_p = -\frac{p_w}{\gamma_w} \quad (16.2)$$

$p_w$	Suction pore stress.
$\gamma_w$	Unit weight of the pore fluid.
$S_{res}$	A residual saturation which describes a part of the fluid that remains in the pores even at high suction heads.
$S_{sat}$	In general at saturated conditions the pores will not be completely filled with water as air can get trapped and the saturation in this situation, $S_{sat}$ , will be less than one. However, the default is $S_{sat} = 1.0$
$g_a$	A fitting parameter which is related to the air entry value of the soil and has to be measured for a specific material. It is in the unit of $1/L$ and is a positive value.
$g_n$	A fitting parameter which is a function of the rate of water extraction from the soil once the air entry value has been exceeded. This parameter has to be measured for a specific material.
$g_c$	A fitting parameter which is used in the general Van Genuchten equation. In PLAXIS the following assumption is made to convert the Van Genuchten to a two-parameter equation (Eq. (16.3)).

$$g_c = \left( \frac{1 - g_n}{g_n} \right) \quad (16.3)$$

The Van Genuchten relationship provides reasonable results for low and intermediate suctions. For very high suction values, saturation remains at the residual saturation.

Figure 16.1 and Figure 16.2 show the effect of the parameters  $g_a$  and  $g_n$  on the shape of the SWCC.

Relative permeability is related to the saturation via the effective saturation. The effective

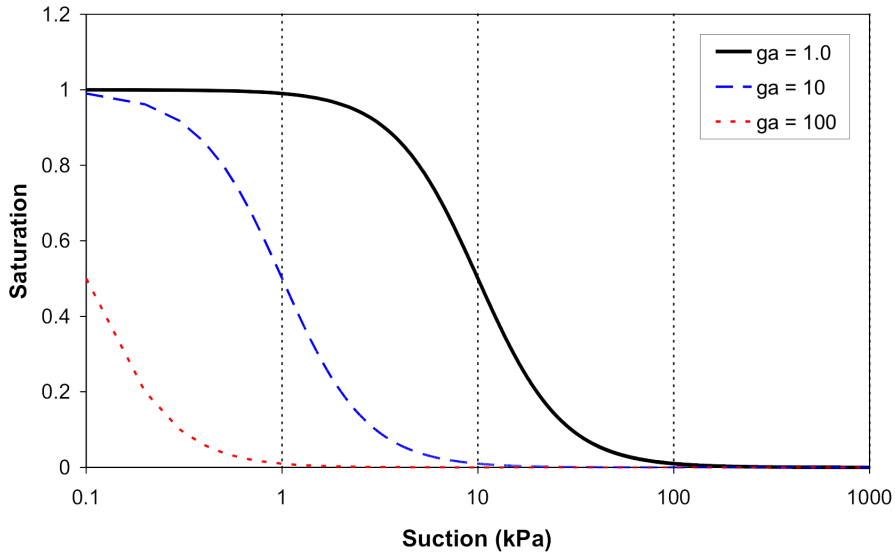


Figure 16.1 Effect of the parameter  $g_a$  on the SWCC

saturation  $S_e$  is expressed as:

$$S_e = \frac{S - S_{res}}{S_{sat} - S_{res}} \quad (16.4)$$

The relative permeability according to Van Genuchten now reads:

$$k_{rel}(S) = \max \left[ (S_e)^{g_l} \left( 1 - \left[ 1 - S_e \left( \frac{g_n}{g_n - 1} \right) \right] \left( \frac{g_n - 1}{g_n} \right)^2 \right), 10^{-4} \right] \quad (16.5)$$

$g_l$  is a fitting parameter and has to be measured for a specific material. Note that using the above expressions, the relative permeability can be related to the suction pore pressure directly.

The derivative of the degree of saturation with respect to the suction pore pressure reads:

$$\frac{\partial S(p_w)}{\partial p_w} = (S_{sat} - S_{res}) \left[ \frac{1 - g_n}{g_n} \right] \left[ g_n \left( \frac{g_a}{\gamma_w} \right)^{g_n} \cdot p_w^{(g_n-1)} \right] \left[ 1 + \left( g_a \cdot \frac{p_w}{\gamma_w} \right)^{g_n} \right] \left( \frac{1 - 2g_n}{g_n} \right) \quad (16.6)$$

Figure 16.3 and Figure 16.4 present the Van Genuchten relations for a sandy material with parameters  $S_{sat} = 1.0$ ,  $S_{res} = 0.027$ ,  $g_a = 2.24 \text{ m}^{-1}$ ,  $g_l = 0.0$  and  $g_n = 2.286$  graphically.

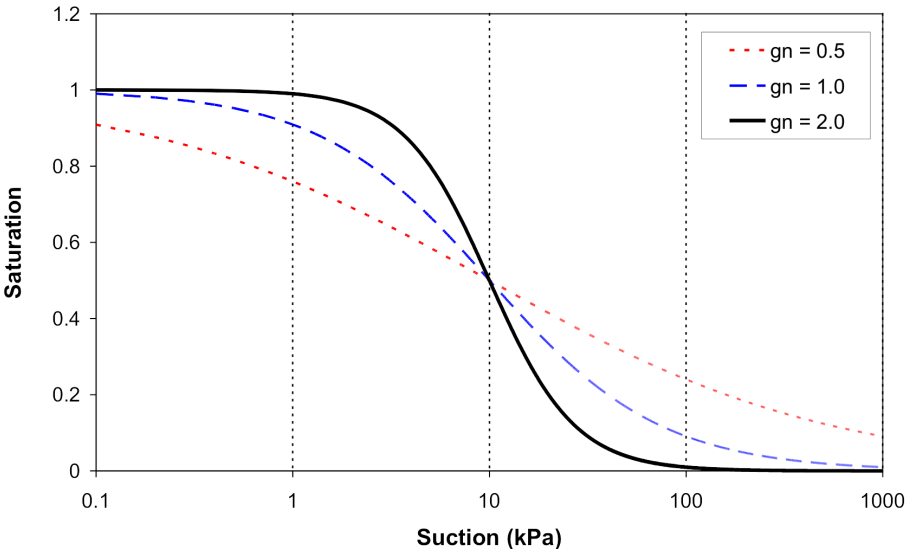


Figure 16.2 Effect of the parameter  $g_n$  on the SWCC

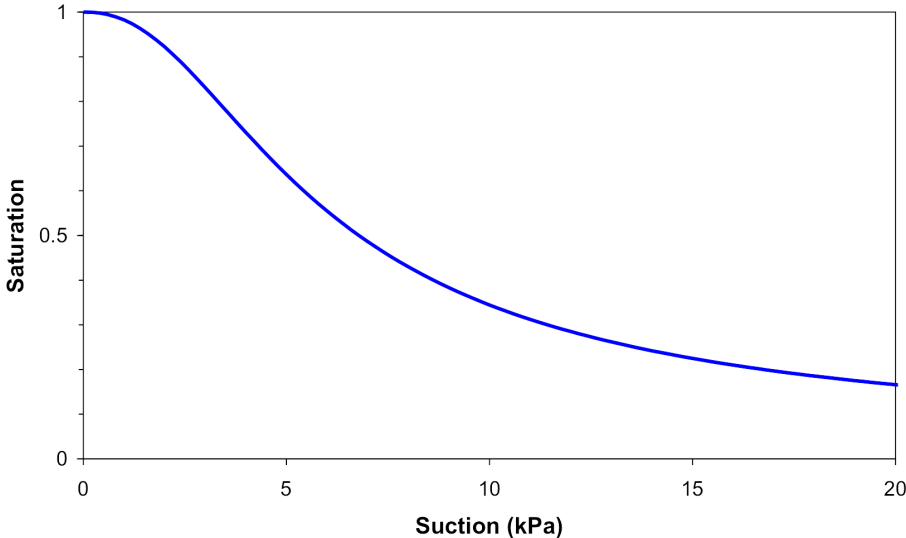


Figure 16.3 Van Genuchten pressure-saturation

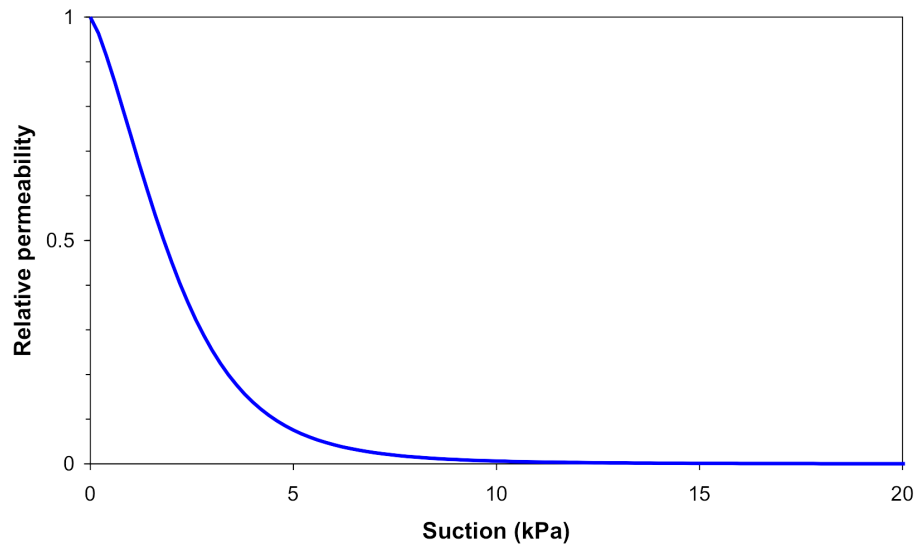


Figure 16.4 Van Genuchten pressure-relative permeability

## 16.2 APPROXIMATE VAN GENUCHTEN MODEL

As an alternative, the PlaxFlow program supports a linearized Van Genuchten model for which the approximate Van Genuchten parameters can be derived. According to this concept saturation relates to the pore pressure head as:

$$S(\phi_p) = \begin{cases} 1 & \text{if } \phi_p \geq 0 \\ 1 + \frac{\phi_p}{|\phi_{ps}|} & \text{if } \phi_{ps} < \phi_p < 0 \\ 0 & \text{if } \phi_p \leq \phi_{ps} \end{cases} \quad (16.7)$$

The variable  $\phi_{ps}$  is a material dependent pressure head which specifies the extent of the unsaturated zone under hydrostatic conditions. Below this threshold value the saturation is assumed to be zero. For saturated conditions the degree of saturation equals one. The relation between relative permeability and pressure head is written as:

$$k_{rel}(\phi_p) = \begin{cases} 1 & \text{if } \phi_p \geq 0 \\ 10 \frac{4\phi_p}{|\phi_{pk}|} & \text{if } \phi_{pk} < \phi_p < 0 \\ 10^{-4} & \text{if } \phi_p \leq \phi_{pk} \end{cases} \quad (16.8)$$

According to this formulation the permeability in the transition zone is described as a log-linear relation of pressure head where  $\phi_{pk}$  is the pressure head at which the relative permeability is reduced to  $10^{-4}$ . The permeability remains constant for higher values of the pressure head. Under saturated conditions the relative permeability equals one and the effective permeability is equal to the saturated permeability which is assumed to be constant.

The input parameters of the “approximate Van Genuchten model” are derived from the classical Van Genuchten model. These parameters are translated into approximately equivalent process parameters for the numerically more robust linearized model. For  $\phi_{ps}$  the translation is as follows:

$$\phi_{ps} = \frac{1}{S_{\phi_p = -1.0 \text{ m}} - S_{sat}} \quad (16.9)$$

The parameter  $\phi_{pk}$  is set equal to the pressure head at which the relative permeability according to Van Genuchten is  $10^{-2}$ , with a lower limit of -0.5 m. Figure 16.5 presents the functional relation between pressure and saturation according to the approximate Van Genuchten model using  $\phi_{ps} = 1.48$  m. The corresponding pressure-relative saturation relation  $\phi_{pk} = 1.15$  m is given in Figure 16.6.

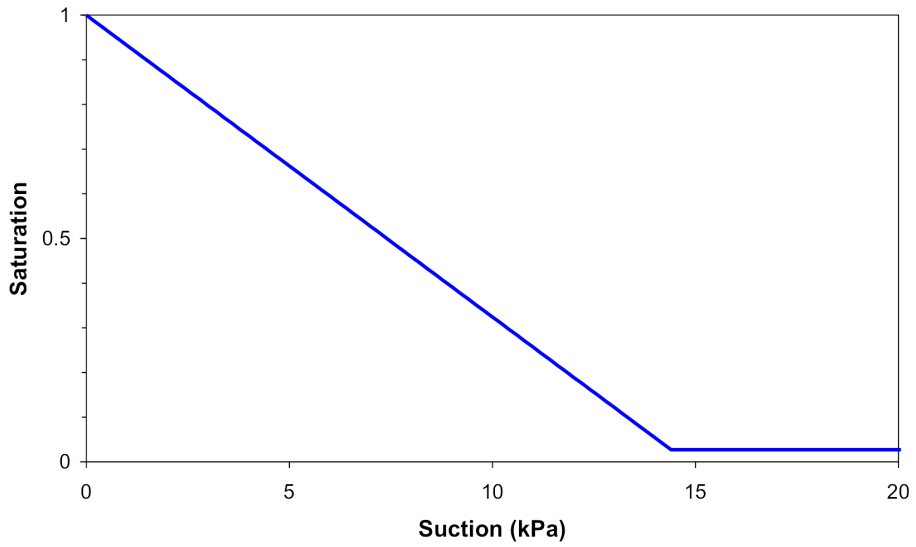


Figure 16.5 Approximate Van Genuchten pressure-saturation

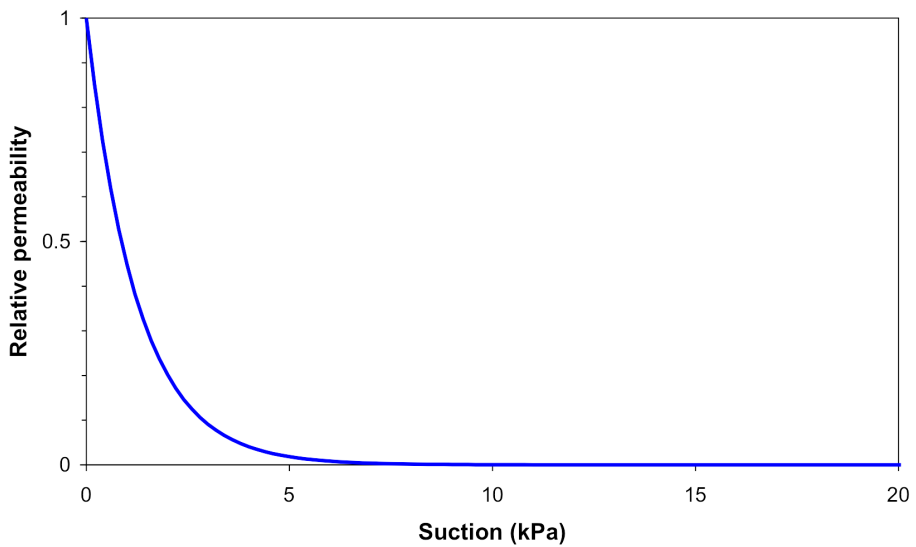


Figure 16.6 Approximate Van Genuchten pressure-relative permeability



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## APPENDIX A - SYMBOLS

$A$	:	Cross section area
$c$	:	Cohesion
$csp$	:	Current stiffness parameter
$C_u, S_u$	:	Undrained shear-strength
$d$	:	Thickness
$\underline{D}^e$	:	Elastic material matrix representing Hooke's law
$e$	:	Void ratio
$E$	:	Young's modulus
$E_{oed}$	:	Oedometer modulus
$f$	:	Yield function
$g$	:	Plastic potential function
$G$	:	Shear modulus
$I$	:	Moment of inertia
$K$	:	Bulk modulus
$K_0$	:	Coefficient of lateral earth pressure (initial stress state)
$K_0^{nc}$	:	Coefficient of lateral earth pressure for a normally consolidated stress state
$m$	:	Power in stress-dependent stiffness relation
$M$	:	Slope of critical state line in $p' - q$ space
$M$	:	Bending moment
$n$	:	Porosity
$N$	:	Normal force
OCR	:	Over-consolidation ratio
$p$	:	Isotropic stress or mean stress, negative for pressure; positive for tension
$p_p$	:	Isotropic pre-consolidation stress, negative for pressure
POP	:	Pre overburden pressure, positive for (over)pressure
$q$	:	Equivalent shear stress or deviatoric stress
$Q$	:	Shear force
$R_f$	:	Failure ratio
$t$	:	Time
$\underline{u}$	:	Vector with displacement components
$\gamma$	:	Unit weight
$\gamma$	:	Various types of shear strain
$\Delta$	:	Increment
$\underline{\varepsilon}$	:	Vector with Cartesian strain components, normal components positive for extension; negative for compression
$\varepsilon_q$	:	Deviatoric strain (invariant)
$\varepsilon_v$	:	Volumetric strain, negative for compression; positive for extension
$\kappa$	:	Cam-Clay swelling index
$\kappa^*$	:	Modified swelling index
$\lambda$	:	Plastic multiplier
$\lambda$	:	Cam-Clay compression index
$\lambda^*$	:	Modified compression index
$\mu^*$	:	Modified creep index

$\nu$	:	Poisson's ratio
$\underline{\sigma}$	:	Vector with Cartesian stress components, normal components positive for tension; negative for pressure
$\sigma_p$	:	Vertical pre-consolidation stress, negative for pressure
$\varphi$	:	Friction angle
$\psi$	:	Dilatancy angle
$\times^c$	:	denotes creep component
$\times^e$	:	denotes elastic component
$\times^p$	:	denotes plastic component
$\times^{ref}$	:	denotes reference value (related to a reference stress)
$\times_u$	:	denotes undrained
$\times_{ur}$	:	denotes unloading and reloading
$\times_m$	:	denotes mobilised



## APPENDIX B - APPLICABILITY OF THE MATERIAL MODELS

In this appendix, an overview of the applicability of the material models is given.

Considering different types of soils		Concrete	Rock	Gravel	Sand	Silt	OC clay	NC clay	Peat (org)
Model									
Linear Elastic model	C								
Mohr-Coulomb model	A	B	C	C	C	C	C	C	C
Hardening Soil model			B	B	B	B	B	B	
HS small model			A	A	A	A	A	B	
Soft Soil Creep model								A*	A*
Soft Soil model								A*	A*
Jointed Rock model			A**						
Modified Cam-Clay model								C	C
NGI-ADP model								A*	A*
Hoek-Brown model			A**						

A : The best standard model in PLAXIS for this application

B : Reasonable modelling

C : First order (crude) approximation

\* : Soft Soil Creep model in case time-dependent behaviour is important; NGI-ADP model for short-term analysis, in case only undrained strength is known

\*\* : Jointed Rock model in case of anisotropy and stratification; Hoek-Brown model for rock in general

Considering different types of applications (consider also type of soil!)

Model	Foundation	Excavation	Tunnel	Embankment	Slope	Dam	Offshore	Other
Linear Elastic model			C					
Mohr-Coulomb model	C	C	C	C	C	C	C	C
Hardening Soil model	B	B	B	B	B	B	B	B
HS small model	A	A	A	A	A	A	A	A
Soft Soil Creep model	B	B	B	A	A	B	B	B
Soft Soil model	B	B	B	A	A	B	B	B
Jointed Rock model	B	B	B	B	B	B	B	B
Modified Cam-Clay model	C	C	C	C	C	C	C	C
NGI-ADP model	B	B	B	A	A	B	A	B
Hoek-Brown model	B	B	B	B	B	B	B	B

A : The best standard model in PLAXIS for this application

B : Reasonable modelling

C : First order (crude) approximation

Considering different types of loading and soils (consider also type of soil!)

Model	Primary compression	Unloading / Reloading	Shear / Deviatoric loading	Undrained loading	Cyclic	Compression + Shear	Extension + Shear
Linear Elastic model	C	C					
Mohr-Coulomb model	C	B	C	C		C	C
Hardening Soil model	A	B	B	B	C	A	A
HS small model	A	A	A	B	B	A	A
Soft Soil Creep model	A	B	B	B	C	A	B
Soft Soil model	A	B	B	B	C	A	B
Jointed Rock model	B	B	B			B	B
Modified Cam-Clay model	C	C	C	C	C	C	C
NGI-ADP model	B	B	B	A	C	B	B
Hoek-Brown model	B	B	B			B	B

A : The best standard model in PLAXIS for this application

B : Reasonable modelling

C : First order (crude) approximation



## APPENDIX C - FORTRAN SUBROUTINES FOR USER-DEFINED SOIL MODELS

In this appendix, a listing is given of the subroutines and functions which are provided by PLAXIS in libraries and source code in the User-defined soil models directory. These can be called by the User\_Mod subroutine:

### Subroutines

MZeroR( R, K ):

To initialize  $K$  terms of double array  $R$  to zero

MZeroI( I, K ):

To initialize  $K$  terms of integer array  $I$  to zero

SetRVal( R, K, V ):

To initialize  $K$  terms of double array  $R$  to  $V$

SetIVal( I, K, IV ):

To initialize  $K$  terms of integer array  $I$  to  $IV$

CopyIVec( I1, I2, K ):

To copy  $K$  values from integer array  $I1$  to  $I2$

CopyRVec( R1, R2, K ):

To copy  $K$  values from double array  $R1$  to  $R2$

MulVec( V, F, n ):

To multiply a vector  $V$  by a factor  $F$ ,  $n$  values

MatVec( xMat, im, Vec, n, VecR ):

Matrix (xMat)-vector(Vec) operation.

First dimension of matrix is  $im$ ; resulting vector is  $VecR$

AddVec( Vec1, Vec2, R1, R2, n, VecR ):

To add  $n$  terms of two vectors; result in  $VecR$

$$VecR_i = R1 \cdot Vec1_i + R2 \cdot Vec2_i$$

MatMat( xMat1, id1, xMat2, id2, nR1, nC2, nC1, xMatR, idR ):

Matrix multiplication  $xMatR_{ij} = xMat1_{ik} \cdot xMat2_{kj}$

$id1, id2, idR$ : first dimension of matrices

$nR1$  number of rows in  $xMat1$  and resulting  $xMatR$

$nC2$  number of column in  $xMat2$  and resulting  $xMatR$

$nC1$  number of columns in  $xMat2$  =rows in  $xMat2$

MatMatSq( n, xMat1, xMat2, xMatR ):

Matrix multiplication  $xMatR_{ij} = xMat1_{ik} \cdot xMat2_{kj}$

Fully filled square matrices with dimensions  $n$

MatInvPiv( AOrig, B, n ):

Matrix inversion of square matrices  $AOrig$  and  $B$  with dimensions  $n$ .

$AOrig$  is NOT destroyed,  $B$  contains inverse matrix of  $AOrig$ .

Row-pivoting is used.

WriVal( io, C, V ):

To write a double value  $V$  to file unit  $io$  (when  $io > 0$ )

The value is preceded by the character string  $C$ .

`WriIVl( io, C, I ):`

As *WriVal* but for integer value  $I$

`WriVec( io, C, V, n ):`

As *WriVal* but for  $n$  values of double array  $V$

`WriIVc( io, C, iV, n ):`

As *WriVal* but for  $n$  values of integer array  $iV$

`WriMat( io, C, V, nd, nr, nc ):`

As *WriVal* but for double matrix  $V$ .  $nd$  is first dimension of  $V$ ,  $nr$  and  $nc$  are the number of rows and columns to print respectively.

`PrnSig( iOpt, S, xN1, xN2, xN3, S1, S2, S3, P, Q ):`

To determine principal stresses and (for  $iOpt=1$ ) principal directions.

$iOpt = 0$  to obtain principal stresses without directions

$iOpt = 1$  to obtain principal stresses and directions

$S$  array containing 6 stress components (XX, YY, ZZ, XY, YZ, ZX)

$xN1, xN2, xN3$  array containing 3 values of principal normalized directions only when  $iOpt=1$ .

$S1, S2, S3$  sorted principal stresses ( $S \leq S2 \leq S3$ )

$P$  isotropic stress (negative for compression)

$Q$  deviatoric stress

`CarSig( S1, S2, S3, xN1, xN2, xN3, SNew ):`

To calculate Cartesian stresses from principal stresses and principal directions.

$S1, S2, S3$  principal stresses

$xN1, xN2, xN3$  arrays containing principal directions (from `PrnSig`)

$SNew$  contains 6 stress components (XX, YY, ZZ, XY, YZ, ZX)

`CrossProd( xN1, xN2, xN3 ):`

Cross product of vectors  $xN1$  and  $xN2$

`SetVecLen( xN, n, xL ):`

To multiply the  $n$  components of vector  $xN$  such that the length of  $xN$  becomes  $xL$  (for example to normalize vector  $xN$  to unit length).

## Functions

Logical Function `LEqual( A, B, Eps ):`

Returns *TRUE* when two values  $A$  and  $B$  are almost equal, *FALSE* otherwise.

$LEqual = |A-B| < Eps * (|A| + |B| + Eps) / 2$

Logical Function `Is0Arr( A, n ):`

Returns *TRUE* when all  $n$  values of real (double) array  $A$  are zero, *FALSE* otherwise

Logical Function `Is0IArr( IArr, n ):`

Returns *TRUE* when all *n* values of integer array *IArr* are zero, *FALSE* otherwise  
Double Precision Function *DInProd( A, B, n )*:  
Returns the dot product of two vectors with length *n*

