Exercise 1 AD et HL (10 points)

This exercise studies the contact between a bobsleigh runner and ice. A general observation: the pressure is large enough to locally melt the ice and for sake of simplicity, we assume that a full water film is generated in the contact (with density ρ and viscosity η, assumed constant). The contact dimensions are (figure 1): length L, width $2b$, the runner form is given ($h(x)$ denotes the lubricant film thickness) and we assume the runner to be thin (infinitely thin), e.g. b is small with respect to L. The surface velocity with respect to the runner is U.

A. Dimensional analysis

To analyse the load carrying capacity F, we select a fixed film geometry $h(x)$, parameterized with the sole initial thickness $h_0 = h(x = 0)$.

1) Express F as a physical function of the problem parameters (geometry, kinematics, material…)
2) Choosing b, η and U as basic parameters, use dimensional analysis and simplify the obtained expression. What is this expression if one assumes $b << L$?
3) How does the load carrying capacity change if the thickness h_0 is divided by a factor 2?

B. Hydrodynamic lubrication

4) For a narrow contact, quickly show that the Reynolds equation, where $p(x,y)$ is the water film pressure, reduces to $\partial^2 p/\partial y^2 = f(x)$ where $f(x)$ is a function to be specified.
5) With boundary conditions at $y = \pm b$ to be recalled, give the solution of the pressure distribution of the form $p(x,y) = g(y)/h^3 \cdot dh/dx$ where $g(y)$ is to be specified.
6) Is it possible to obtain $p < 0$ with the previous expression? If the answer is yes, what would happen physically? In the following questions, one assumes that the pressure remains positive.
7) We now wish to obtain the load carrying capacity F. Give its general integral expression depending on p. The pressure goes to 0 when x increases due to the runner geometry; we therefore propose to simplify the expression of F by computing the integral from $x = 0$ up to $x = \infty$. Give the resulting expression of F depending on the given data, among which the back film thickness $h_0 = h(x = 0)$.

C. Comparison

8) Are the previous results for the load carrying capacity with the dimensional analysis and the lubrication solution consistent? Justify briefly.

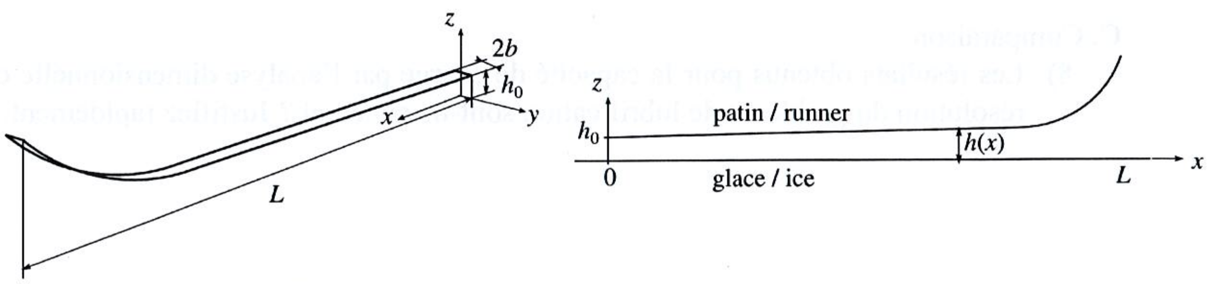


Figure 1. Géométrie au contact / contact geometry

Exercise 2 EHL and Stribeck (10 points)

This exercise studies a tribometer: a cylinder sliding on a flat surface, measuring the friction as a function of velocity.

1) For u=1 m/s, R=2 mm, L= 1cm, w=1000 N, η_0=0.1 Pa s, α=2e-8 Pa-1, E'=2e11 Pa, compute the Dowson and Higginson parameters, W1, U and G.

2) Compute the film thickness according to Ertel Grubin.

3) Calculate the Moes parameters M1 and L, indicate the lubrication regime and comment on the validity of the previous film thickness result.

The friction coefficient measurement is repeated at constant load and temperature, for different speeds:
u=0.10 m/s, f=0.08
u=0.20 m/s, f=0.10
u=0.50 m/s, f=0.12
u=1.00 m/s, f=0.14

4) Draw the film thickness profiles h(x) to scale for these 4 conditions, use the correct values in micrometers on the x and h-axis

5) Draw the Stribeck curve, using the correct dimensionless parameter on the horizontal axis, use a logarithmic horizontal axis.

6) Comment on the lubrication regime for 0.1 < u < 1.0 m/s

The friction coefficient measurement is extended at constant load and temperature, for lower speeds:
u=0.01 m/s, f=0.12
u=0.02 m/s, f=0.08
u=0.05 m/s, f=0.07

7) Physically explain the observed transition in friction coefficient

8) Estimate from these results the combined roughness of the two surfaces.

9) A refined analysis of the friction coefficient for u>0.1 m/s shows that f is proportional to u to the power 0.25, show the origin of this 1/4 power.

10) For classical "Stribeck curves", the minimum friction coefficient is around 0.001, explain the difference with the current result.

Reynolds equation:

$$\frac{d}{dx}\left(\frac{\rho h^3}{12\eta}\frac{dp}{dx}\right)+\frac{d}{dy}\left(\frac{\rho h^3}{12\eta}\frac{dp}{dy}\right)=\frac{u_1+u_2}{2}\frac{d\rho h}{dx}+\frac{d\rho h}{dt}$$

Velocity profile

$$u(y)=\frac{1}{2\eta}\frac{dp}{dx}(\mathrm{y}-h)\,\mathrm{y}+u_1\frac{h-y}{h}+u_2\frac{y}{h}$$

Barus equation

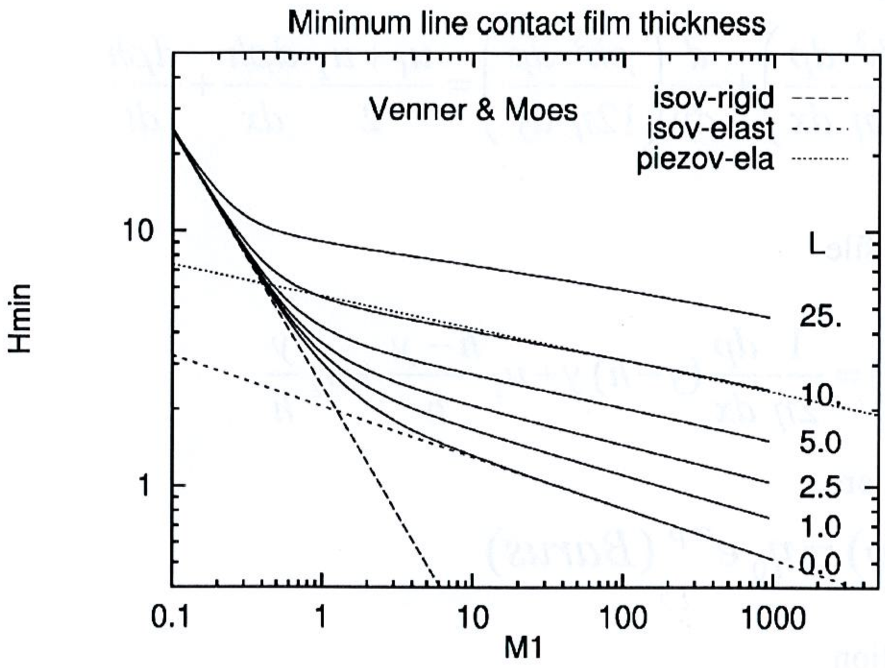
$$\eta(p)=\eta_0\, e^{\alpha p}\ (Barus)$$

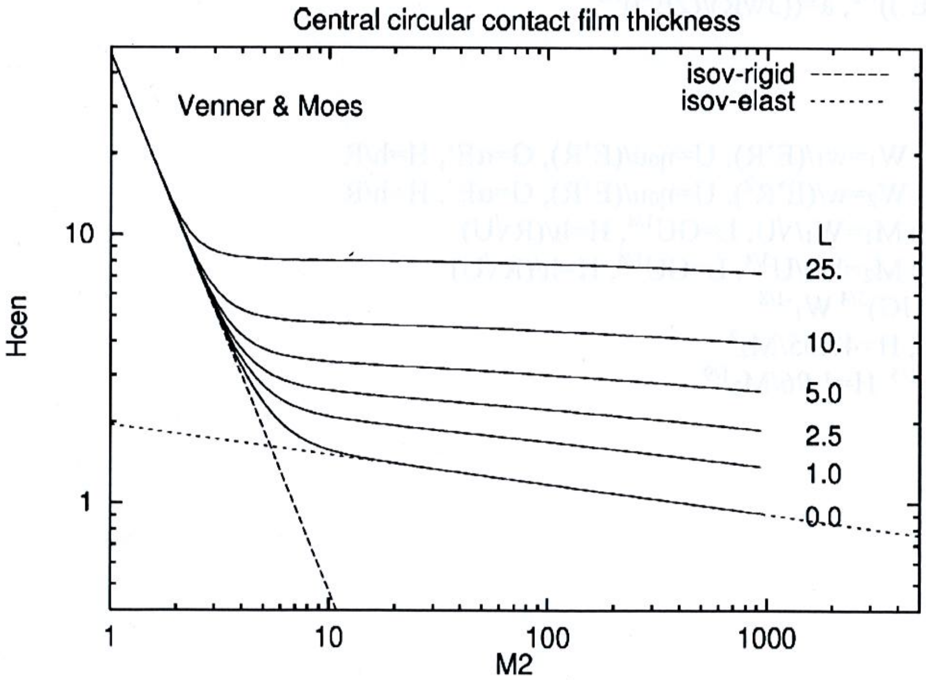
Vogel equation

$$\eta(T)=\eta_0\, e^{\frac{b}{T+\theta}}\ (Vogel)$$

Hertz:
$p_h=(2w_1)/(\pi b)$, $p_h=(3w)/(2\pi a^2)$
$b=((8w_1R)/(\pi E'))^{1/2}$, $a=((3wR)/(2E'))^{1/3}$

Lubricated:
DH parameters $W_1=w_1/(E'R)$, $U=\eta_0 u/(E'R)$, $G=\alpha E'$, $H=h/R$
HD parameters $W_2=w/(E'R^2)$, $U=\eta_0 u/(E'R)$, $G=\alpha E'$, $H=h/R$
MV parameters $M_1=W_1/\sqrt{U}$, $L=GU^{1/4}$, $H=h/(R\sqrt{U})$
MV parameters $M_2=W_2/U^{3/4}$, $L=GU^{1/4}$, $H=h/(R\sqrt{U})$
EG: $H^*=1.31\ (UG)^{3/4}\ W_1^{-1/8}$
IR: $H=2.45/M_1$, $H=47.55/M_2^2$
IE: $H=2.05/M_1^{1/5}$, $H=1.96/M_2^{1/9}$


Minimum line contact film thickness
Venner & Moes
isov-rigid
isov-elast
piezov-ela
L
25.
10.
5.0
2.5
1.0
0.0
Hmin
10
1
0.1
1
10
100
1000
M1


Central circular contact film thickness
Venner & Moes
isov-rigid
isov-elast
L
25.
10.
5.0
2.5
1.0
0.0
Hcen
10
1
1
10
100
1000
M2