C.3.3 Contact forces:

The aim of this paragraph is the study of direct mechanical actions between solids in contact.

a) Generality:

Consider two solids \( (S_1) \) and \( (S_2) \) in contact in \( I \) (point contact).

Point contact kinematics is defined by the wrench:

\[
\{K_{1/2}\} = \begin{cases} 
\vec{\Omega}^1_2 = \vec{n} \cdot (\vec{\Omega}^1_2 \cdot \vec{n}) + \vec{n} \wedge (\vec{\Omega}^1_2 \wedge \vec{n}) & = (\vec{P}^1_2)_I + (\vec{R}^1_2)_I \\
\vec{V}^1_2(I) & 
\end{cases}
\]
b) *Contact force wrench:*

Basic problem: To avoid infinite pressures, local elastic deformations have to be considered and a finite contact area (instead of a point of contact) has to be introduced (Hertz).

The contact force wrench can be derived by:

\[
\begin{align*}
\{F_{1/2}\} &= \left\{ \begin{array}{l}
F_{1/2} = \int_{M \in (A)} dF_{1/2} \\
M_{1/2}(I) = \int_{M \in (A)} IM \wedge dF_{1/2}
\end{array} \right. \\
\{F_{1/2}\} &= \left\{ \begin{array}{l}
F_{1/2} = T_{1/2} + N_{1/2} \\
M_{1/2}(I) = C_{p1/2} + C_{r1/2}
\end{array} \right.
\]

\( I, \text{ nominal point of contact} \)

\( (A) : \text{contact area} \)
C. GENERAL THEOREMS OF DYNAMICS

C.3.3.1 Constitutive equations:

Some experimental evidences led to constitutive equations relating kinematic parameters to forces and moments. Caution: Their validity range is narrow.

C.3.3.1.1 DRY contacts

a) for slipping (Coulomb friction law):

<table>
<thead>
<tr>
<th></th>
<th>Slipping</th>
<th>No slip</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kinematics</strong></td>
<td>( \ddot{V}^i_2(I) \neq 0 )</td>
<td>( \ddot{V}^i_2(I) = 0 )</td>
</tr>
<tr>
<td><strong>Forces</strong></td>
<td>( \dddot{T}_{i/2} ) is opposed to ( \dddot{V}^i_2(I) )</td>
<td>( \dddot{T}_{i/2} )</td>
</tr>
<tr>
<td></td>
<td>( | \dddot{T}<em>{i/2} | = f | \dddot{N}</em>{i/2} | )</td>
<td>( | \dddot{T}<em>{i/2} | &lt; f | \dddot{N}</em>{i/2} | )</td>
</tr>
<tr>
<td>( f ) : friction coefficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>which is synthesised as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \dddot{T}<em>{i/2} = -f | \dddot{N}</em>{i/2} | \dddot{V}^i_2(I) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Rightarrow ) 2 equations for 3D problem</td>
<td></td>
<td>1 equation for 2D problem</td>
</tr>
</tbody>
</table>
some orders of magnitude for $f$:

metal against metal: between 0.5 and 2

it can be reduced by some coatings on the contact surfaces.

**Cautions:**

- some variations with the speed have been established,
- in vacuum, these values have to be multiplied by nearly 100,
- also depends on geometry (closed / open contacts), frequencies, …

**b) for rolling:**

<table>
<thead>
<tr>
<th>Rolling</th>
<th>No rolling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kinematics</strong></td>
<td></td>
</tr>
<tr>
<td>$(\vec{R}_1')_i \neq \bar{\theta}$</td>
<td>$(\vec{R}_i')_i = \bar{\theta}$</td>
</tr>
<tr>
<td><strong>Forces</strong></td>
<td></td>
</tr>
<tr>
<td>$\vec{C}r_{1/2} = -h | \vec{N}_{1/2} | \frac{(\vec{R}_1')_i}{| (\vec{R}_2')_i |}$</td>
<td>$| \vec{C}r_{1/2} | &lt; h | \vec{N}_{1/2} |$</td>
</tr>
<tr>
<td>$h$ : rolling parameter</td>
<td></td>
</tr>
<tr>
<td>(2 equations in 3D, 1 in 2D)</td>
<td></td>
</tr>
</tbody>
</table>

_Numerical values, $h = 0.5$ to $1$ mm for a steel wheel on a rail_
C. GENERAL THEOREMS OF DYNAMICS

c) for pitching:

<table>
<thead>
<tr>
<th>Kinematics</th>
<th>Pitching</th>
<th>No pitching</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left( \vec{P}_2' \right) \neq \hat{0}$</td>
<td>$\left( \vec{P}_2' \right) = \hat{0}$</td>
<td></td>
</tr>
<tr>
<td>Forces</td>
<td>$\vec{C}p_{1/2} = -k.\left| \vec{N}_{1/2} \right| \left( \vec{P}_2' \right)$</td>
<td>$\left| \vec{C}p_{1/2} \right| &lt; k.\left| \vec{N}_{1/2} \right|$</td>
</tr>
<tr>
<td></td>
<td>$k :$ pitching parameter (1 equation)</td>
<td></td>
</tr>
</tbody>
</table>

the pitching parameter can be calculated from the theory of Elasticity, it comes:

$$k = \frac{3}{32} \cdot f \cdot a$$ where $a$ is the semi-major axis of the ellipse of contact.
C. GENERAL THEOREMS OF DYNAMICS

d) Remarks:

1 – about the number of equations: parity

2 – Tangential forces exist EVEN if there is NO SLIP (3 components for the contact force) ****!!!!

3 - \( \vec{F}_{1/2} \) reduces to \( \vec{N}_{1/2} \) if there is NO FRICTION \( (f = 0) \) => perfect contact

4 – when slipping, \( f = \tan \varphi = \frac{\|T_{1/2}\|}{\|N_{1/2}\|} \)

\[ \begin{align*}
\text{a) if there is slipping, } \vec{F}_{1/2} \text{ is on the friction cone surface } f &= \tan \varphi = \frac{\|T_{1/2}\|}{\|N_{1/2}\|}.
\end{align*} \]

\[ \begin{align*}
\text{b) if there is no slip in } I, \vec{F}_{1/2} \text{ remains inside the friction cone,}
\end{align*} \]

\[ \begin{align*}
f > \tan \varphi &= \frac{\|T_{1/2}\|}{\|N_{1/2}\|}.
\end{align*} \]
5 – In case of slipping, pitching and rolling can be reasonably neglected. For no slip conditions, this leads to unrealistic situations (perpetual motions).

6 - $f$ decreases with an increasing sliding velocity amplitude:

$$f_a = \text{coefficient of adhesion}$$

C.3.3.1.2 **Laws for lubricated contacts:**

friction coefficients fall down to $0.01 - 0.05$ for fully lubricated contacts

higher values are obtained for mixed lubrication

analytical or numerical solutions are available (hydrodynamic and elastohydrodynamics).
C.4 Application:

Oscillations of a cylinder on a circular track.

The friction coefficient in I is $f$. Upon assuming permanent contact and no slip in I, give the equations of motion.
C. GENERAL THEOREMS OF DYNAMICS

C.5 Dynamic analysis of the usual links:

All joints are considered perfect (friction forces can be neglected).

C.5.1 Revolute joint:

being \( M \) any point on the surface of contact \( (\Sigma_2) \), the elemental force acting on solid 2 reads:

\[
d\vec{F}_{1/2} = d\vec{N}_{1/2}
\]

and the complete force wrench is:

\[
\{F_{1/2}\} = \begin{cases} 
F_{1/2} = \int_{M \in \Sigma_2} d\vec{F}_{1/2} \\
\int_{M \in \Sigma_2} \overrightarrow{O_2M} \wedge d\vec{F}_{1/2}
\end{cases}
\]
C. GENERAL THEOREMS OF DYNAMICS

Conclusion:

\[ \overrightarrow{R}_{1/2} = \begin{bmatrix} X_{12} \\ Y_{12} \\ Z_{12} \end{bmatrix}_{1 \text{ or } 2} \quad \text{and} \quad \overrightarrow{M}_{1/2}(O_2) = \begin{bmatrix} L_{12} \\ M_{12} \end{bmatrix}_{1 \text{ or } 2} \]

\[ \{K_{1/2}\} = \begin{cases} \Omega_{1/2} = \dot{\psi} \cdot \overrightarrow{z}_{1,2} \\ \overrightarrow{V}_{1/2}(O_2) = \overrightarrow{0} \end{cases} \]

\[ \text{Power} = C_{12} = \overrightarrow{R}_{1/2} \cdot \overrightarrow{V}_{1/2}(O_2) + \overrightarrow{M}_{1/2}(O_2) \cdot \overrightarrow{\Omega}_{1/2} = 0 \rightarrow \text{perfect joint} \]

!! in case of friction, power is no more nil.

Balance of the formulation:

<table>
<thead>
<tr>
<th></th>
<th>unknowns</th>
<th>equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>kinematic</strong></td>
<td>( \psi )</td>
<td>1</td>
</tr>
<tr>
<td><strong>dynamic</strong></td>
<td>( x_{12}, y_{12}, z_{12} ) ( L_{12}, M_{12} )</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Note that it is an isodynamic problem.
C. GENERAL THEOREMS OF DYNAMICS

C.5.2 Prismatic joint:

The complete force wrench is:

\[ \{F_{1/2}\} = \left\{ \begin{array}{l} \vec{F}_{1/2} = \int_{M \in \Sigma_2} d\vec{F}_{1/2} \\ \int_{M \in \Sigma_2} \overrightarrow{O_2 M} \wedge d\vec{F}_{1/2} \end{array} \right\} \]

\[ \vec{R}_{1/2} = \begin{bmatrix} X_{12} \\ Y_{12} \\ 0 \end{bmatrix}_{1 or 2} \]

and

\[ \vec{M}_{1/2}(O_2) = \begin{bmatrix} L_{12} \\ M_{12} \\ N_{12} \end{bmatrix}_{1 or 2} \]
\[
\{K_{1/2}\} = \begin{cases}
  \overrightarrow{\Omega}_{1/2} = \overrightarrow{0} \\
  \overrightarrow{V}_{1/2}(O_2) = \dot{z} \cdot \overrightarrow{z}_{1/2}
\end{cases}
\]

\[
\text{Power} = C_{12} = R_{1/2} \cdot \overrightarrow{V}_{1/2}(O_2) + M_{1/2}(O_2) \cdot \overrightarrow{\Omega}_{1/2} = 0 \rightarrow \text{perfect joint}
\]

!! in case of friction, power is no more nil.

Balance of the formulation:

<table>
<thead>
<tr>
<th></th>
<th>unknowns</th>
<th>equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>kinematic</strong></td>
<td>(Z)</td>
<td>1</td>
</tr>
<tr>
<td><strong>dynamic</strong></td>
<td>(x_{12}, y_{12})</td>
<td>5</td>
</tr>
<tr>
<td>(L_{12}, M_{12}, N_{12})</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>
C. GENERAL THEOREMS OF DYNAMICS

C.5.3 Cylindrical joint:

The complete force wrench is:

\[
\{F_{1/2}\} = \left\{ \begin{array}{c}
  \overrightarrow{F_{1/2}} = \int_{M \in \Sigma_2} \overrightarrow{dF_{1/2}} \\
  \int_{M \in \Sigma_2} \overrightarrow{O_2M} \wedge \overrightarrow{dF_{1/2}}
\end{array} \right\} \rightarrow \overrightarrow{R_{1/2}} = \begin{bmatrix} X_{12} \\ Y_{12} \\ 0 \end{bmatrix}_{1 \text{ or } 2} \quad \text{and} \quad \overrightarrow{M_{1/2}(O_2)} = \begin{bmatrix} L_{12} \\ M_{12} \\ 0 \end{bmatrix}_{1 \text{ or } 2}
\]
C. GENERAL THEOREMS OF DYNAMICS

Conclusion:

\[
\{K_{1/2}\} = \begin{cases} 
\Omega_{1/2} = \dot{\psi}.z_{1,2} \\
V_{1/2}(O_2) = \dot{z}.z_{1,2}
\end{cases}
\]

\[Power = C_{12} = R_{1/2} \cdot V_{1/2}(O_2) + M_{1/2}(O_2) \cdot \Omega_{1/2} = 0 \rightarrow \text{perfect joint}\]

Balance of the formulation:

<table>
<thead>
<tr>
<th></th>
<th>unknowns</th>
<th></th>
<th>equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>kinematic</strong></td>
<td>(Z, \psi)</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td><strong>dynamic</strong></td>
<td>(X_{i2}, Y_{i2})</td>
<td>4</td>
<td>general theorems: 6</td>
</tr>
<tr>
<td></td>
<td>(L_{j2}, M_{j2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
C.5.4 *Spherical joint:*

The complete force wrench is:

\[
\{F_{1/2}\} = \left\{ \begin{array}{c}
\bar{F}_{1/2} = \int_{M \in \Sigma_2} \bar{dF}_{1/2} \\
\int_{M \in \Sigma_2} \bar{O}_2 M \wedge d\bar{F}_{1/2}
\end{array} \right\} \Rightarrow \bar{R}_{1/2} = \begin{bmatrix} X_{12} \\
Y_{12} \\
Z_{12} \end{bmatrix}_{1 \text{ or } 2} \quad \text{and} \quad \bar{M}_{1/2}(O_2) = \begin{bmatrix} 0 \\
0 \\
0 \end{bmatrix}_{1 \text{ or } 2}
\]
C. GENERAL THEOREMS OF DYNAMICS

\[
\{K_{1/2}\} = \begin{cases}
\Omega_{1/2} = \dot{\psi} \cdot x_{1,2} + \dot{\theta} \cdot y_{1,2} + \dot{\phi} \cdot z_{1,2} \\
V_{1/2}(O_2) = 0
\end{cases}
\]

\[\text{Power} = C_{12} = R_{1/2} \cdot V_{1/2}(O_2) + M_{1/2}(O_2) \cdot \Omega_{1/2} = 0 \rightarrow \text{perfect joint}\]

Balance of the formulation:

<table>
<thead>
<tr>
<th></th>
<th>unknowns</th>
<th>equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>kinematic</strong></td>
<td>(\psi, \theta, \phi)</td>
<td>3</td>
</tr>
<tr>
<td><strong>dynamic</strong></td>
<td>(x_{12}, y_{12}, z_{12})</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>
C.6 Springs and dampers

C.6.1 Purposes:

a) simulate the behaviour of real springs and dampers
b) simulate some properties of deformable parts which, strictly speaking, are beyond the scope of Rigid-Solid Mechanics.

For some applications, deflections cannot be ignored. They can be taken into account by linear springs, for example:

![Diagram of linear spring](image1)

![Diagram of spiral spring](image2)
C. GENERAL THEOREMS OF DYNAMICS

C.6.2 - Springs:

C.6.2.1 for translations: traction/compression springs

force wrench:

\[
\begin{aligned}
\{ F_{s/2} \} & = \vec{M}_{s/2}(A_2) = \vec{0} \\
\vec{F}_{s/2} &= -k(\ell - \ell_0). \left( \frac{A_1\vec{A}_2}{\parallel A_1\vec{A}_2 \parallel} \right)
\end{aligned}
\]

with \( k \) : spring stiffness

\( \ell_0 \) : length at rest (initial length of the spring)

\( \ell = \parallel A_1\vec{A}_2 \parallel \), actual spring length

Remarks:

1- the spring is massless, direct force transmission

\( \vec{F}_{1/s} + \vec{F}_{2/s} = \vec{0} \)

2- the spring is connected to solids by spherical joints since \( \vec{M}_{s/1}(A_1) = \vec{M}_{s/2}(A_2) = \vec{0} \)
C.6.2.2 for rotations: torsion spring

\[ \{ F_{s/2} \} \begin{cases} \vec{C}_{s/2} = -k_\theta \cdot (\theta_2 - \theta_1) \cdot \vec{u} \\ \vec{F}_{s/2} = \vec{0} \end{cases} \]

\[ \theta_1 \text{ and } \theta_2 \text{ are positively defined with respect to } \vec{u}. \]

\[ k_\theta = \text{torsional stiffness} \]

C.6.2.3 Limitations:

- massless springs cannot account for wave propagation,
- linearity is valid mainly for small deflections
C. GENERAL THEOREMS OF DYNAMICS

C.6.3 Dampers:

C.6.3.1 for translations:

\[ \begin{align*}
\{ F_{d/2} \} &= \{ M_{d/2} (A_2) = 0 \} \\
\vec{F}_{d/2} &= -c_\ell \cdot \frac{A_1 \vec{A}_2}{\| A_1 \vec{A}_2 \|}
\end{align*} \]

C.6.3.2 for rotations

one gets:

\[ \begin{align*}
\{ F_{d/2} \} &= \{ \vec{C}_{d/2} = -c_\theta \cdot (\dot{\theta}_2 - \dot{\theta}_1) \cdot \vec{u} \} \\
\vec{F}_{s/2} &= 0
\end{align*} \]

Important remark:

A real joint is often modelled as a perfect joint + damper(s) in order to account for energy dissipations.
C. GENERAL THEOREMS OF DYNAMICS

C.7 How to solve a problem of Dynamics:

C.7.1 Purposes:

Two different complementary aspects can be considered in Dynamics:

- **a)** define the motions (kinematic parameters) of the different system components produced by external forces and moments,

- **b)** knowing these motions, determine mechanical actions (forces and moments) in links (joints and contacts) (useful in Strength of Materials).

Order: a) then b)

C.7.2 Study of the motions:

**Aim:** Calculate the kinematic parameters (functions of time) by writing and solving the equations of motion (2\textsuperscript{nd} order differential equations) given by:

- the General Theorems equations
- the constraint equations
- the constitutive equations

**Procedure:**

Problem: Large amount of equations!! For N solids, 3D problem, General Theorems lead to 6*N equations.

- What are the necessary equations to be selected?
C. GENERAL THEOREMS OF DYNAMICS

C.7.2.1 Selection of the equations:

The selection is based upon the specific nature of the joints in order to eliminate the undesired unknown forces and moments in the joints, one will use the projections of the general theorems on to the axis of the links (nil component for the wrench of the forces).

<table>
<thead>
<tr>
<th>nature of the joint</th>
<th>kinematic parameter</th>
<th>equation(s) to be selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>revolute joint</td>
<td>( \psi / (O, \vec{z}) )</td>
<td>Moment theorem in ( O ) in the direction of ( \vec{z} ): ( \bar{\delta}^g (O).\vec{z} = \bar{M}_{ext} (O).\vec{z} )</td>
</tr>
<tr>
<td>prismatic joint</td>
<td>( \vec{z} / (O, \vec{z}) )</td>
<td>Sum theorem in the direction of ( \vec{z} ): ( \bar{\Sigma}^g .\vec{z} = \bar{F}_{ext} .\vec{z} )</td>
</tr>
<tr>
<td>cylindrical joint</td>
<td>( \psi, \vec{z} / (O, \vec{z}) )</td>
<td>( \bar{\delta}^g (O).\vec{z} = \bar{M}<em>{ext} (O).\vec{z} ) and ( \bar{\Sigma}^g .\vec{z} = \bar{F}</em>{ext} .\vec{z} )</td>
</tr>
<tr>
<td>spherical joint</td>
<td>( \psi, \theta, \phi ) with respect to ( (O, \vec{x}), (O, \vec{y}), (O, \vec{z}) ) respectively</td>
<td>( \bar{\delta}^g (O).\vec{x} = \bar{M}<em>{ext} (O).\vec{x} ) ( \bar{\delta}^g (O).\vec{y} = \bar{M}</em>{ext} (O).\vec{y} ) ( \bar{\delta}^g (O).\vec{z} = \bar{M}<em>{ext} (O).\vec{z} ) ( \Rightarrow ) ( \bar{\delta}^g (O) = \bar{M}</em>{ext} (O) )</td>
</tr>
</tbody>
</table>

**Generalisation:**

- for each translational parameter, dynamic sum theorem in the direction of the joint axis,
- for each rotational parameter, dynamic moment theorem in the direction of the joint axis (*note that the point of reference should be on the joint axis*).
C. GENERAL THEOREMS OF DYNAMICS

C.7.2.2 Selection of the isolated (sub)-systems:
   It is based upon the graph of links as shown below:

Conventionally, \( q_i \) represent the rotational parameters
\( p_i \) represent the translational parameters.

Balance of the formulation (general):

<table>
<thead>
<tr>
<th>Nature</th>
<th>unknowns</th>
<th>equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>kinematics</td>
<td>( q_1, q_2, q_3, p_3, q_4 )</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>contact in 1 (( \rightarrow ) the degree of mobility is ( \delta = 4 ))</td>
<td>1</td>
</tr>
<tr>
<td>dynamics</td>
<td>( \ell_{01} ) ( , \ell_{12}, \ell_{23}, \ell_{24}, \ell_{30}(point \ contact) )</td>
<td>5 ( , ), 4 ( , ), 5 ( , ), 3 ( , )</td>
</tr>
<tr>
<td></td>
<td>General theorems:</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>4 solids * 6 eq.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Constitutive equations:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coulomb's law</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>total</td>
<td>27</td>
</tr>
</tbody>
</table>

\( \rightarrow \) Isodynamic problem

But the complete solution of 27 equations is not necessarily simple and … some of the equations are useless in the context of developing the equations of motion.
C. GENERAL THEOREMS OF DYNAMICS

**Balance of the formulation (minimum system):**

<table>
<thead>
<tr>
<th>Nature</th>
<th>unknowns</th>
<th>equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>kinematics</td>
<td>$q_1, q_2, q_3, p_3, q_4$</td>
<td>contact in $I$ 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>=&gt; the degree of mobility is</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\delta = 4$</td>
</tr>
<tr>
<td>dynamics</td>
<td>* $l_{30}$ (ideal point contact with</td>
<td>D.M.T. / axis of $p_3, q_3$ for</td>
</tr>
<tr>
<td></td>
<td>negligible rolling and pitching</td>
<td>$(S_3)$ 1</td>
</tr>
<tr>
<td></td>
<td>torques)</td>
<td>D.S.T. / axis of $p_3, q_3$ for</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(S_3)$ 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D.M.T. / axis of $q_4$ for</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(S_4)$ 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D.M.T. / axis of $q_2$ for</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(S_2US_3US_4)$ 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D.M.T. / axis of $q_1$ for</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(S_1US_2US_3US_4)$ 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constitutive equations:</td>
</tr>
<tr>
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