Chapter 2
Hydrodynamic lubrication (HL)

Lubrification hydrodynamique

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(with some contributions by D. Dureisseix)

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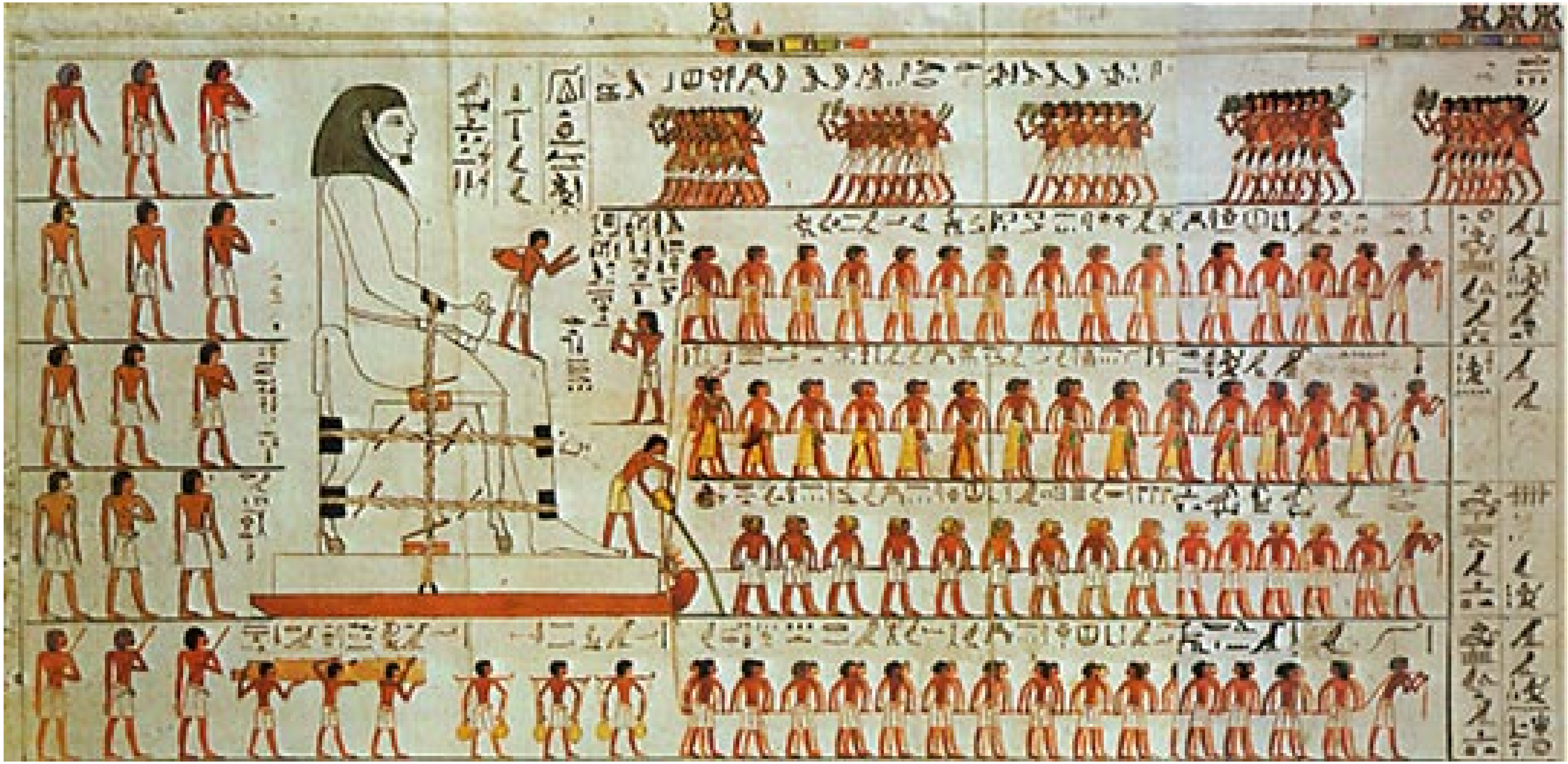
Lubricated friction

Frottement en contact lubrifié



Tomb of Djehutihotep (al berscheh)
172 slaves, 60 t stone statue
approx. 4000 years old

Lubricated friction

Frottement en contact lubrifié

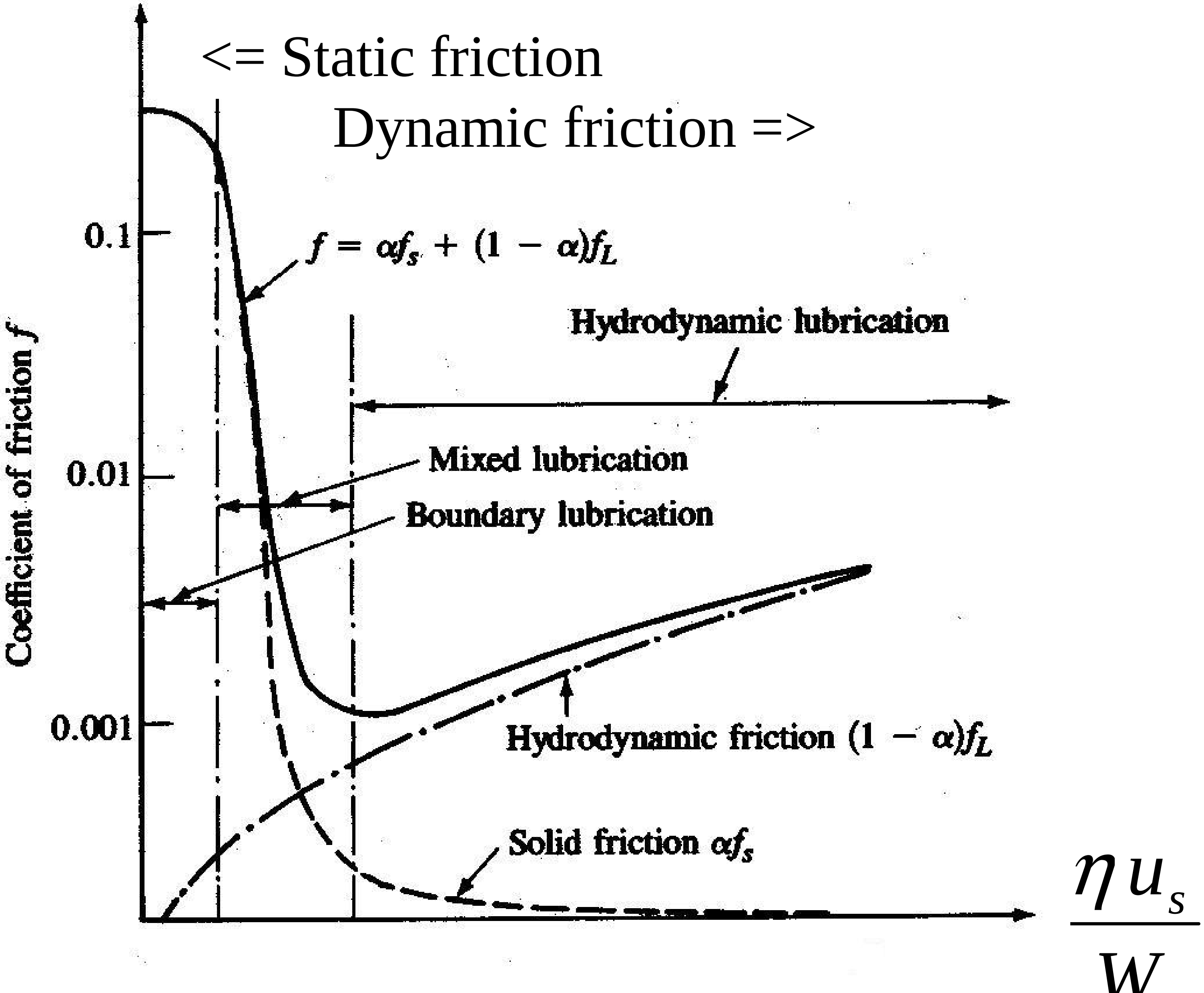
Lubricant functions

Fonctions des lubrifiants

- Separate rubbing surfaces by a low shear strength film
  $\Rightarrow$ low friction/wear
- Evacuate heat
  $\Rightarrow$ avoid scuffing
- Evacuate wear particles (cleaning)
  $\Rightarrow$ low friction/wear
- Transport additives into contact (high pressure zone)
  $\Rightarrow$ low friction
- Additives as anti-rust, anti-oxidant

Lubricated friction – Stribeck curve

Frottement en contact lubrifié – courbe de Stribeck



Lubricated friction – Stribeck curve

Frottement en contact lubrifié – courbe de Stribeck

Macroscopic explanation

Low speed

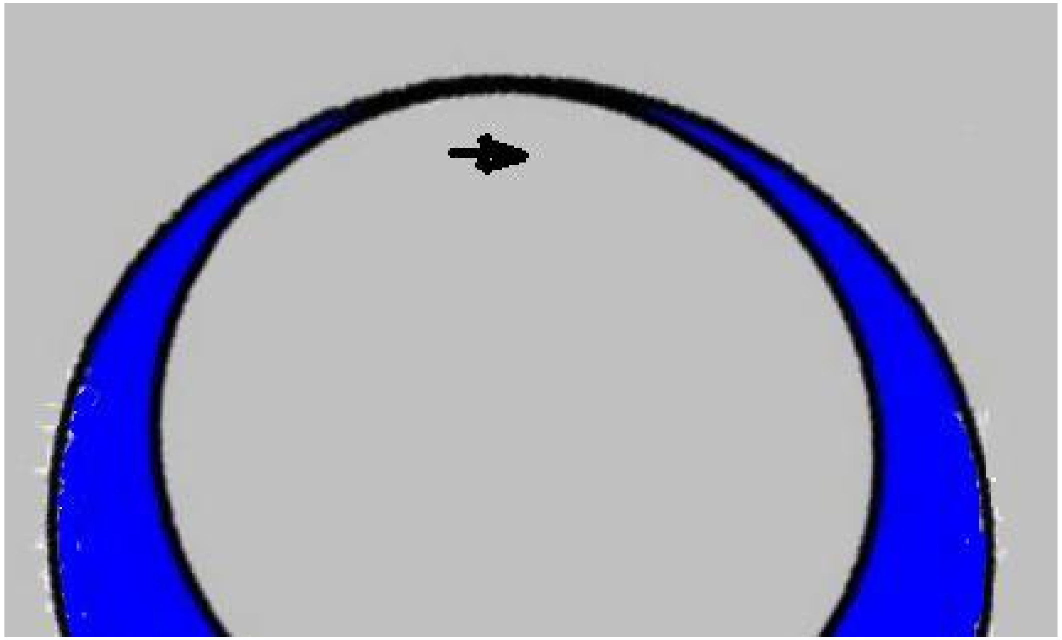
Contact

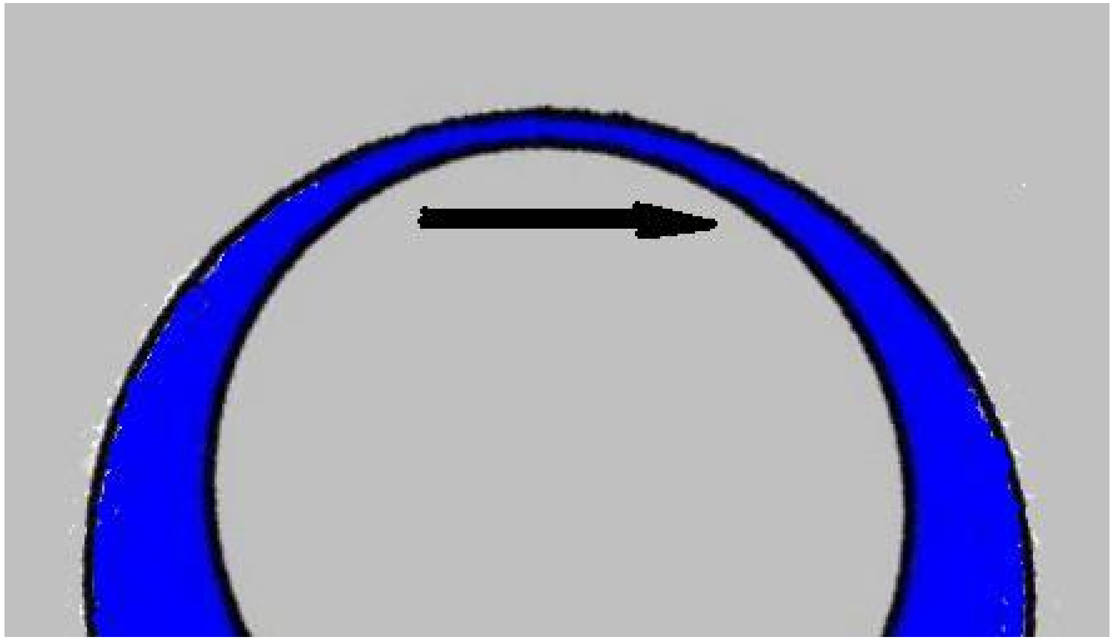
Static “dry” friction (high)

High speed

Separation

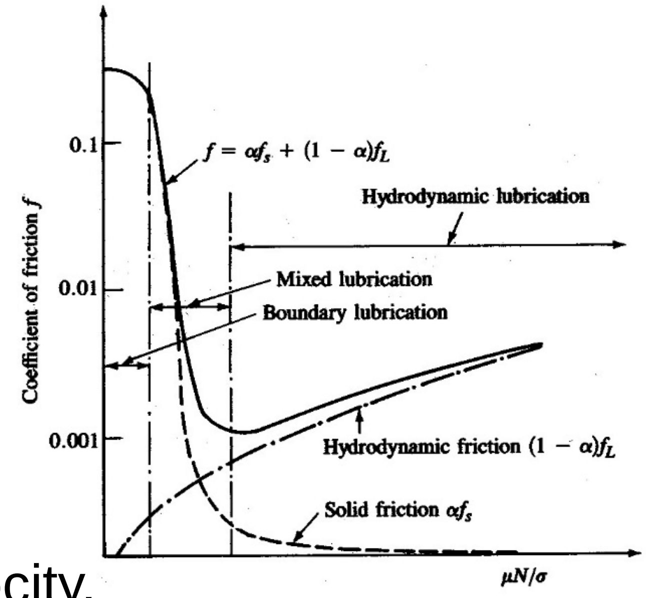
Dynamic friction (low)





Lubricated friction – Stribeck curve

Frottement en contact lubrifié – courbe de Stribeck



- Very low speed: boundary lubrication
  μ = const and high (“dry” friction).
- Low speed: mixed lubrication
  μ = medium to low and decreases with increasing velocity.
- High speed: full film regime
  μ = very low and slowly increases with increasing velocity.

$$\mu = f\left(\frac{\eta u_s}{W}\right)$$

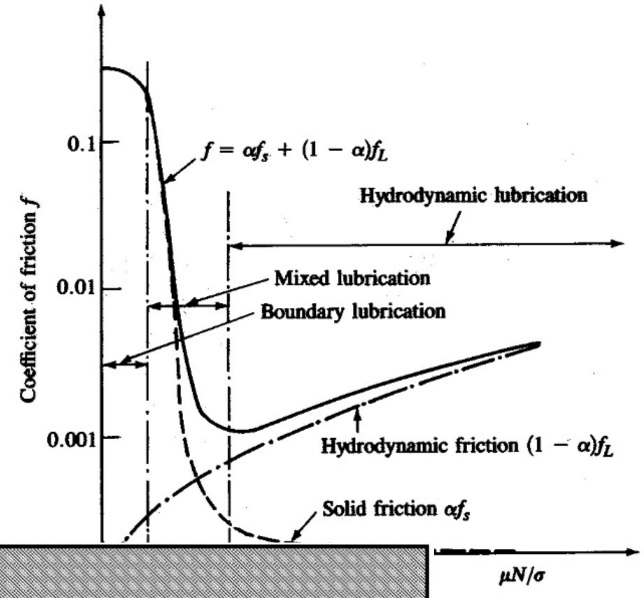
High speed = high viscosity = low load

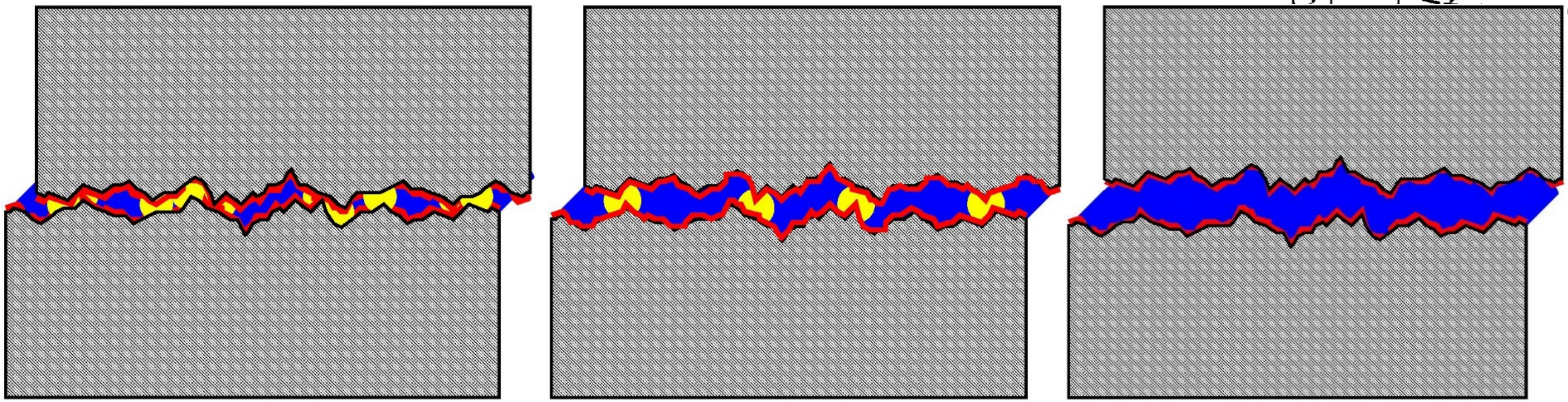
Low speed = low viscosity = high load

Lubricated friction – Stribeck curve

Frottement en contact lubrifié – courbe de Stribeck

Microscopic explanation



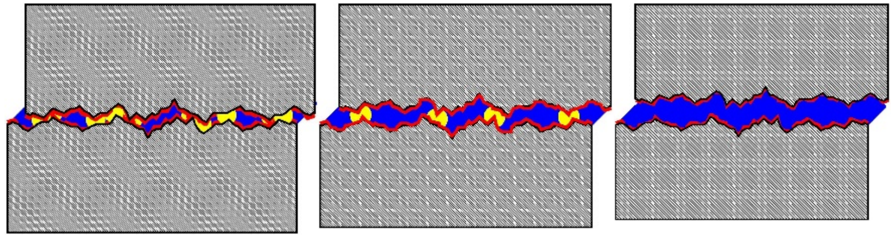


Left to right: Boundary lubrication, mixed lubrication, full film lubrication each regime determined by the distance separating the solid bodies: the **lubricant film thickness** *h*.

Lubricated friction – Stribeck curve

Frottement en contact lubrifié – courbe de Stribeck

Recap



Boundary lubrication:
friction coefficient is constant and high,
load is carried by direct metal to metal contact,
wear can be high depending on additives present in the oil
$h < R_a$

Mixed lubrication
friction is relatively high, decreasing with velocity,
load is partially carried by the fluid film, partially by metal-metal contact,
wear can be substantial
$h \approx R_a$

Full film lubrication
friction coefficient is very low and slowly increasing with speed,
friction is not modified by additives, wear is (virtually) zero
$h > R_a$

Lubricated friction – Stribeck curve

Frottement en contact lubrifié – courbe de Stribeck

Conclusions

For most engineering applications low friction and zero wear are required
$\Rightarrow$ full film lubrication

In the mixed lubrication regime, friction and wear can be made acceptable by choosing appropriate additives in the lubricant.

Reynolds equation

The Reynolds equation is derived from the Navier-Stokes equation for a slow viscous flow, using the simplification $h << R$ (small film thickness)

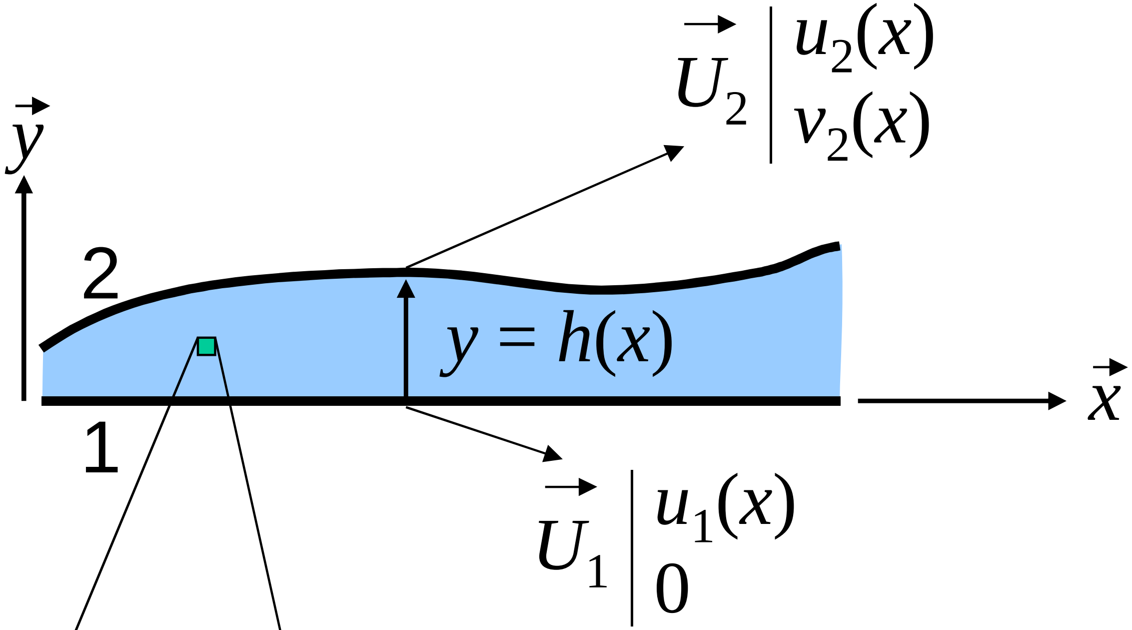
(for mathematicians: asymptotic expansion,

for fluid physicists: dimensional analysis,

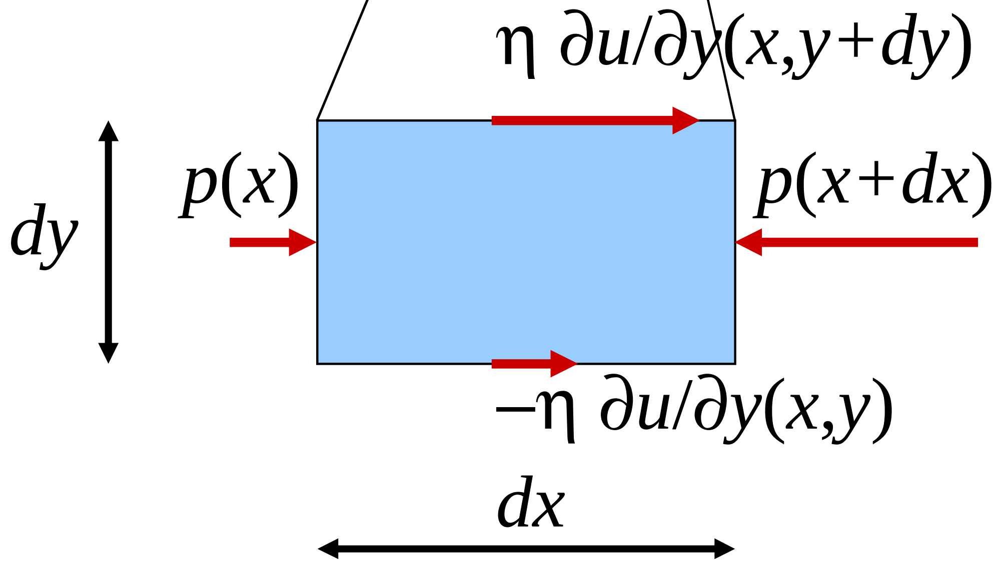
or: with sensible assumptions on the solution)

1D Reynolds equation

Simplified geometry

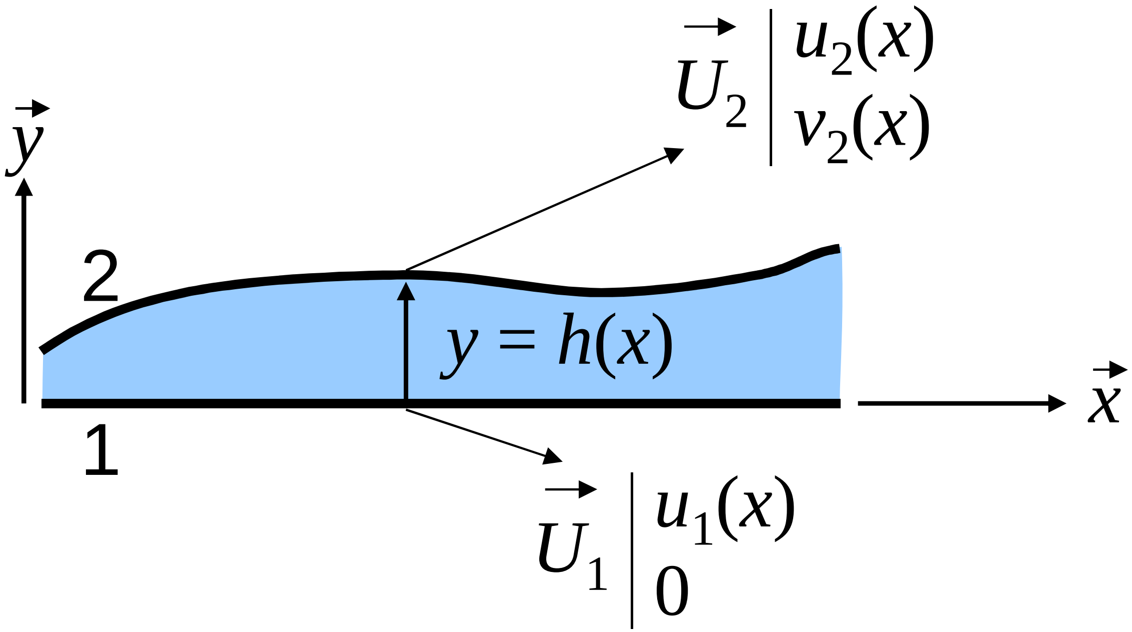


Dynamical equation (no inertial force, no body force)

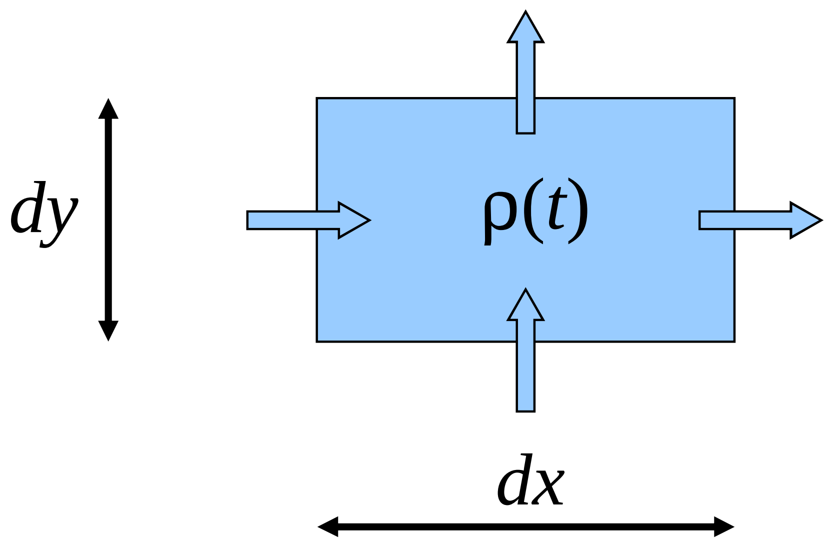


1D Reynolds equation

Simplified geometry

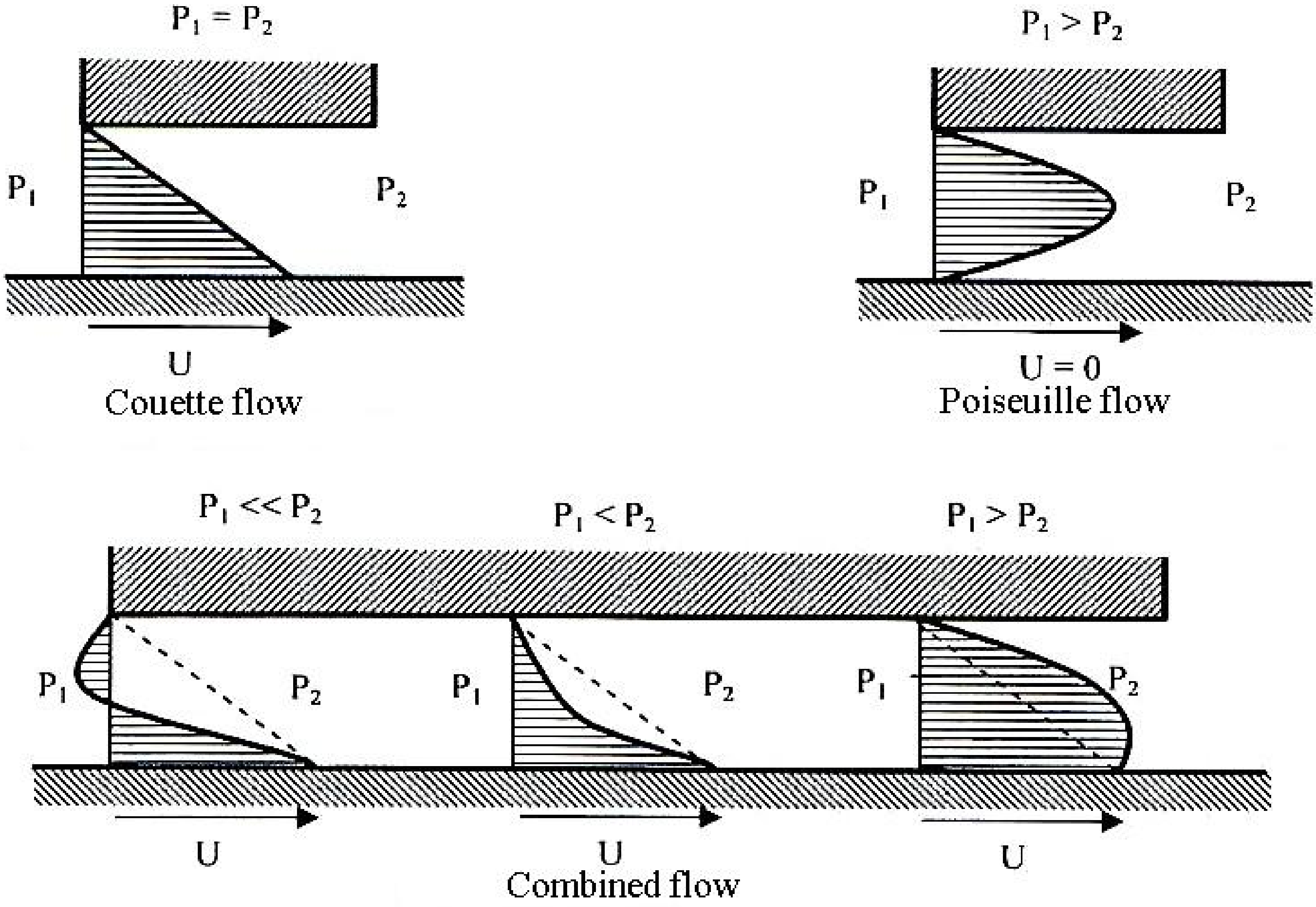


Mass conservation



Different flows

Différents types d'écoulement



1D Reynolds equation

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)=\rho\frac{u_1-u_2}{2}\frac{\partial h}{\partial x}+h\frac{\partial}{\partial x}\left(\rho\frac{u_1+u_2}{2}\right)+\rho v_2+h\frac{\partial \rho}{\partial t}$$

or

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)=\frac{\partial}{\partial x}\left(\rho h\frac{u_1+u_2}{2}\right)+\frac{\partial}{\partial t}(\rho h)$$

Note that this is non linear mechanics!
large influence of film thickness with h^3

1D Reynolds equation

Simpler case: stationary conditions ($\partial/\partial t = 0$),
1D form reads

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right) = \frac{\partial}{\partial x}\left(\rho h \frac{u_1 + u_2}{2}\right) + \frac{\partial}{\partial t}(\rho h)$$

integrating once:

$$\frac{dp}{dx} = 6\eta(u_1 + u_2)\frac{h - h^*}{h^3}$$

Example: Thrust bearing

butée

Fig. 4.3 Tilting pad thrust bearing
(NEYRPIC)

(Slimline)

Example: Thrust bearing
butée

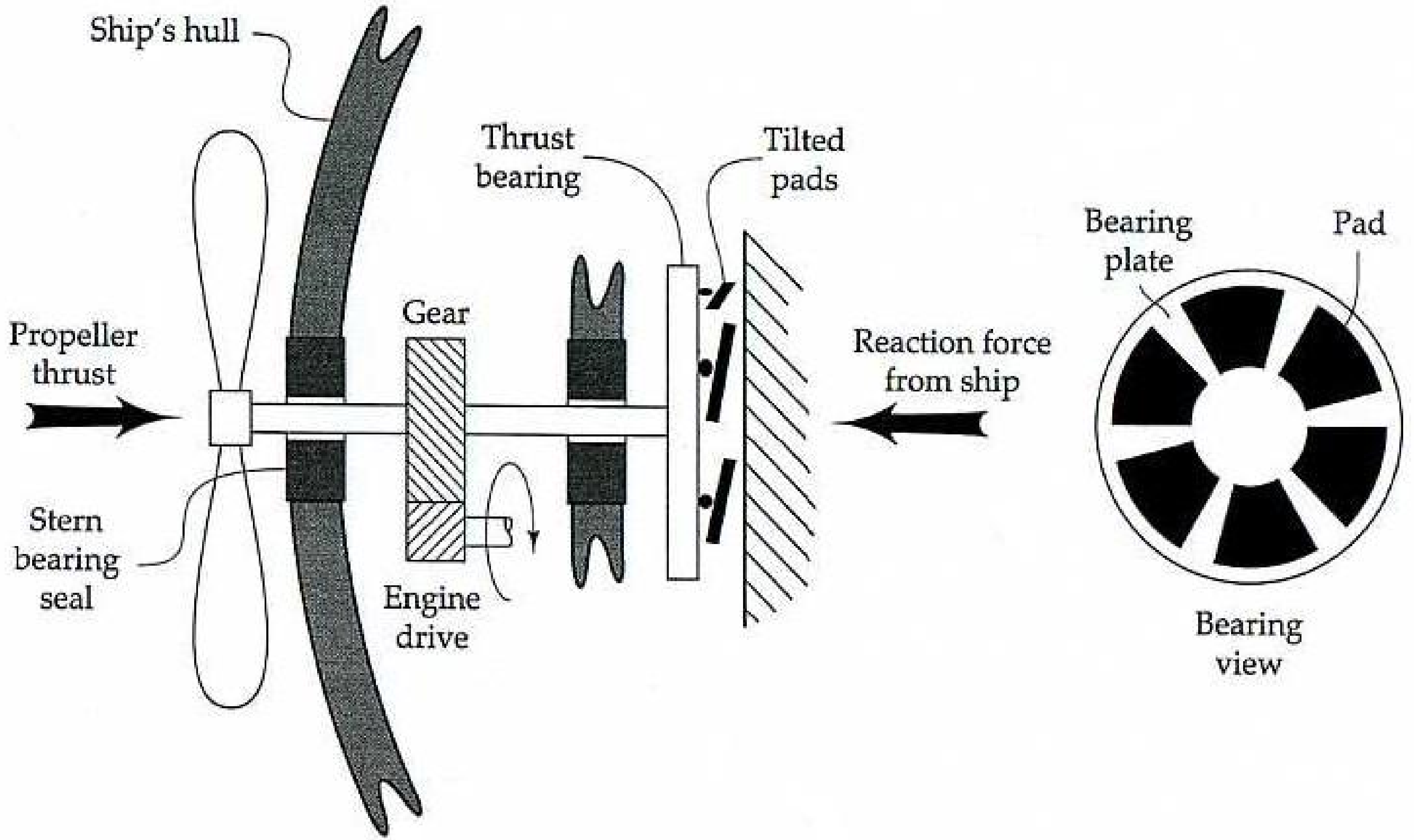


FIGURE 4.8 Example of a pad bearing application to sustain the thrust loads from the ship propeller shaft.

Example: Thrust bearing
butée

Simplified geometry

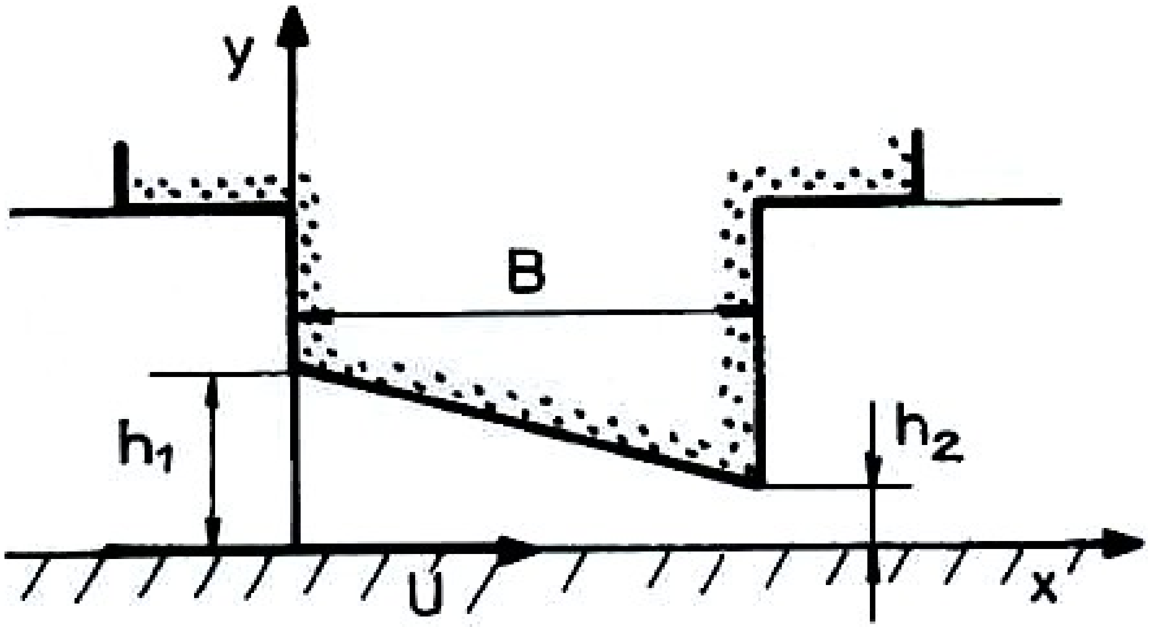


Fig. 4.1 Plane slider scheme

1D problem

Stationary problem ($\partial/\partial t = 0$)

Inside gap, $\partial h / \partial x = \text{constant}$

Density and viscosity are assumed constant

$u_1 = U, u_2 = 0$

Example: Thrust bearing
butée

Simplified geometry

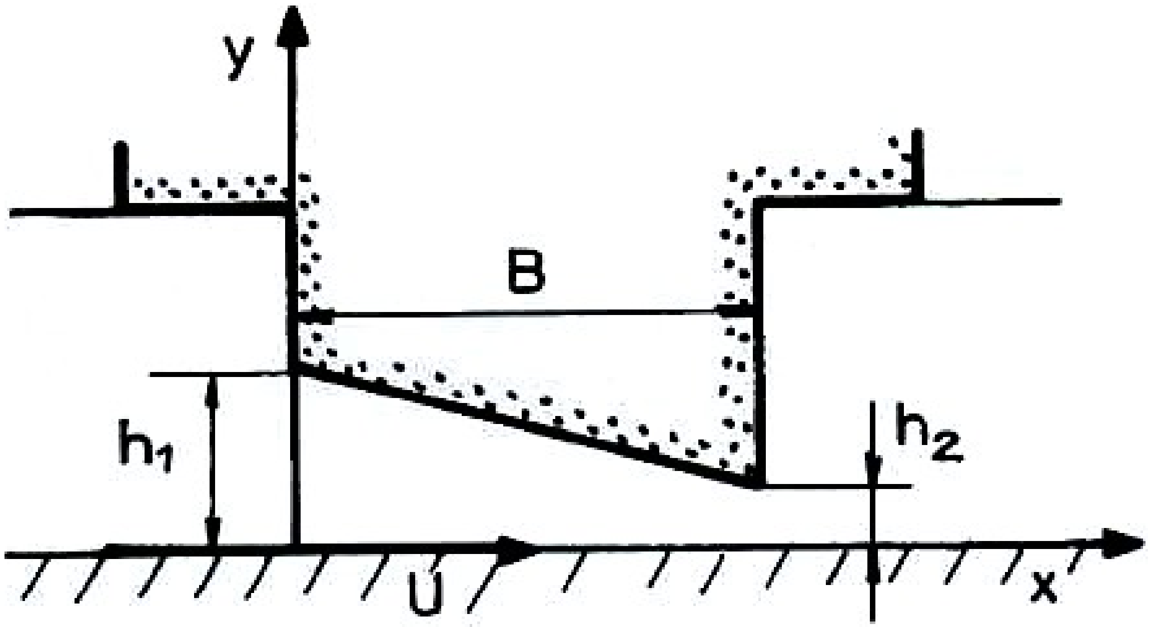


Fig. 4.1 Plane slider scheme

$$\frac{dp}{dx} = 6\eta U \frac{h - h^*}{h^3}$$

$h = h_1 - (h_1 - h_2)\, x/B$

Boundary conditions: $p(x = 0) = p(x = B) = 0$

Example: Thrust bearing
butée

Results

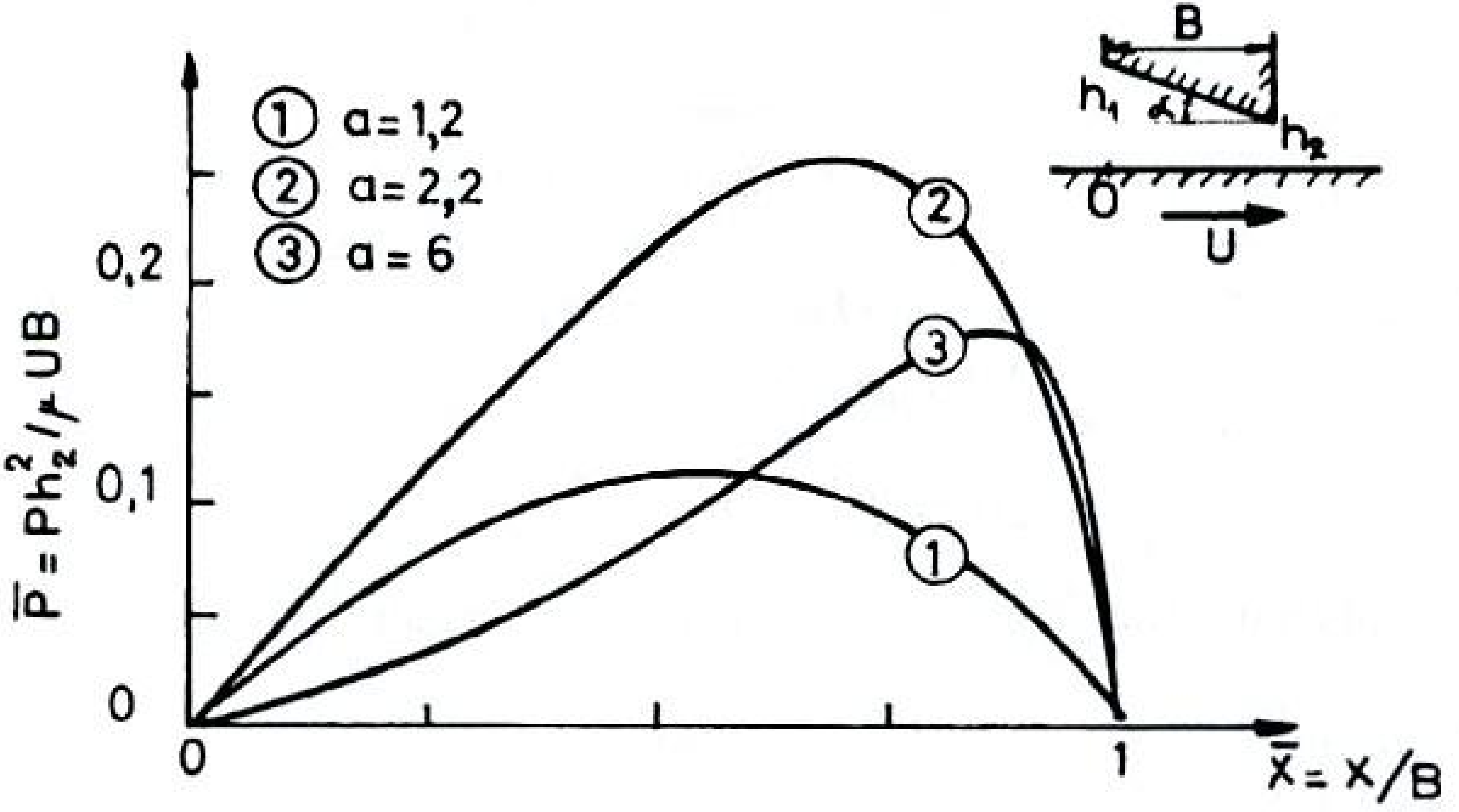
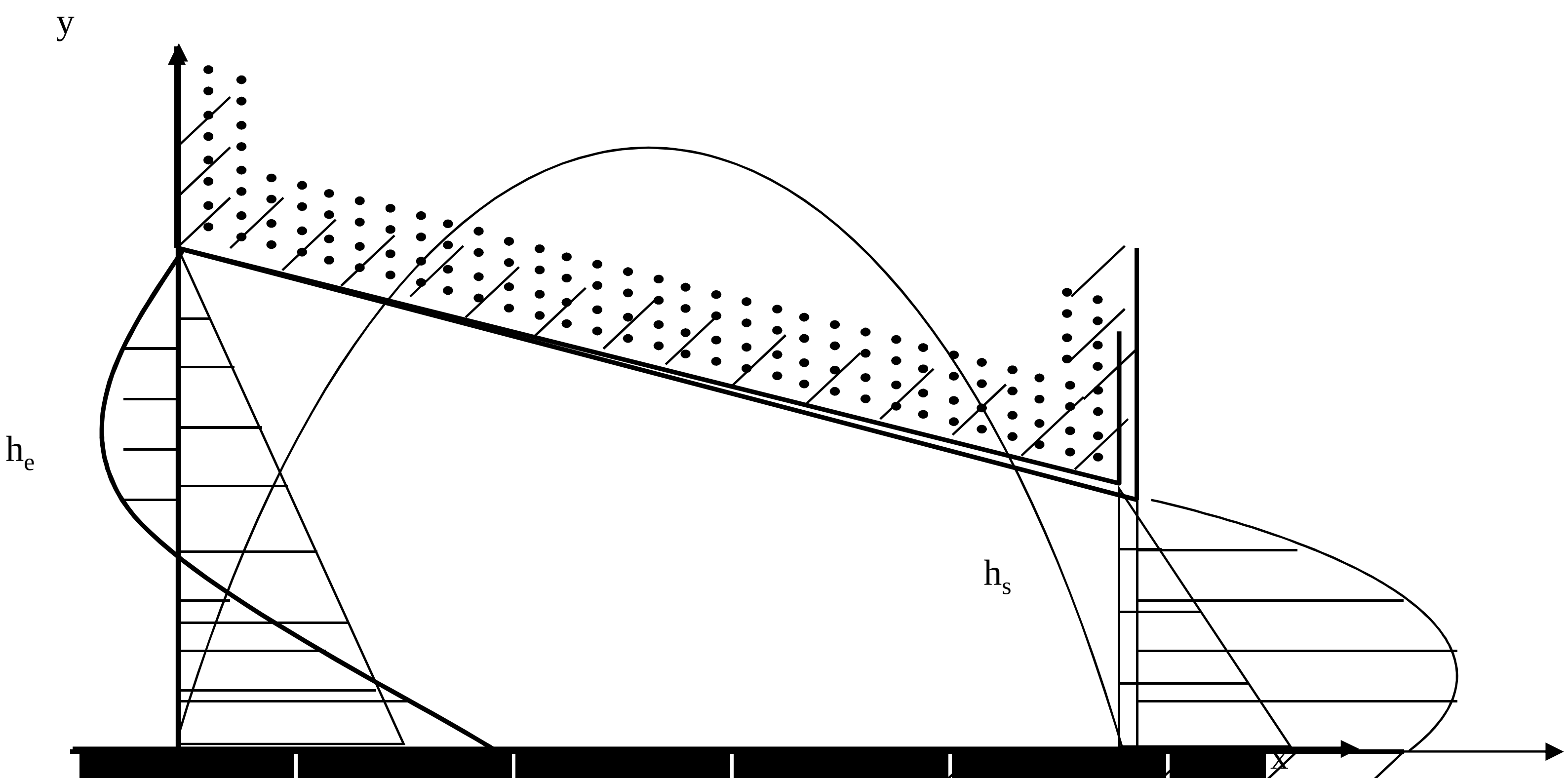


Fig. 3.13. Pressure distribution for various ratios $a = h_1 / h_2$

Physical analysis
Analyse physique

Analyzing the Couette flow: much more oil flows into the wedge than what flows out.

This leads to pressure generation, to restore the flow balance.



Reynolds equation (2D)

Trivial generalization:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)=$$

$$\rho\frac{u_1-u_2}{2}\frac{\partial h}{\partial x}+\rho\frac{v_1-v_2}{2}\frac{\partial h}{\partial y}+h\frac{\partial}{\partial x}\left(\rho\frac{u_1+u_2}{2}\right)+h\frac{\partial}{\partial y}\left(\rho\frac{v_1+v_2}{2}\right)+\rho w_2+h\frac{\partial \rho}{\partial t}$$

or

$$\frac{\partial}{\partial x}\left(\rho h\frac{u_1+u_2}{2}\right)+\frac{\partial}{\partial y}\left(\rho h\frac{v_1+v_2}{2}\right)+\frac{\partial}{\partial t}(\rho h)$$

(more terms, but same usage)

Reynolds equation: summary
Equation de Reynolds : résumé

- The Reynolds equation describes the mass flow continuity in the thin full film regime.
- It relates the geometry $h(x,y)$ to the pressure distribution $p(x,y)$ using the contact conditions (u_1, u_2 & W) and the lubricant properties (viscosity η and density ρ).

In the given form, still valid if η and ρ are not constant…
But one has to add constitutive relations!

Lubricant properties

Propriétés des lubrifiants

The majority of lubricants are liquids, but two other subgroups exist:

- gas (air…)
- solids (coatings, MoS_2…)

In general though, liquids lubricate the majority of technical applications, gasses and solids are used as “niche” lubricants.

A special sub group is formed by greases which are semi-solids: lubricant + 10 to 20% thickener (see later on).

From now on if we say lubricant we mean liquid.

Example of lubricant functions

Exemples de fonctions réalisées par un lubrifiant

In an engine the lubricant:

- lubricates: piston ring, bearings, valves…
- cleans: piston ring, valves
- protects: (anti-rust)
- removes: varnish, soot and sludge
- protects: seals
- all this for many hours & km’s (anti-oxidant)

Lubricants consist of:

- a base oil (mineral or synthetic 80 to 95%)
- an additive package

Lubricant base oil

Huile de base

The base oil provides the lubricant with its physical lubrication properties based on:

- viscosity η
- density ρ

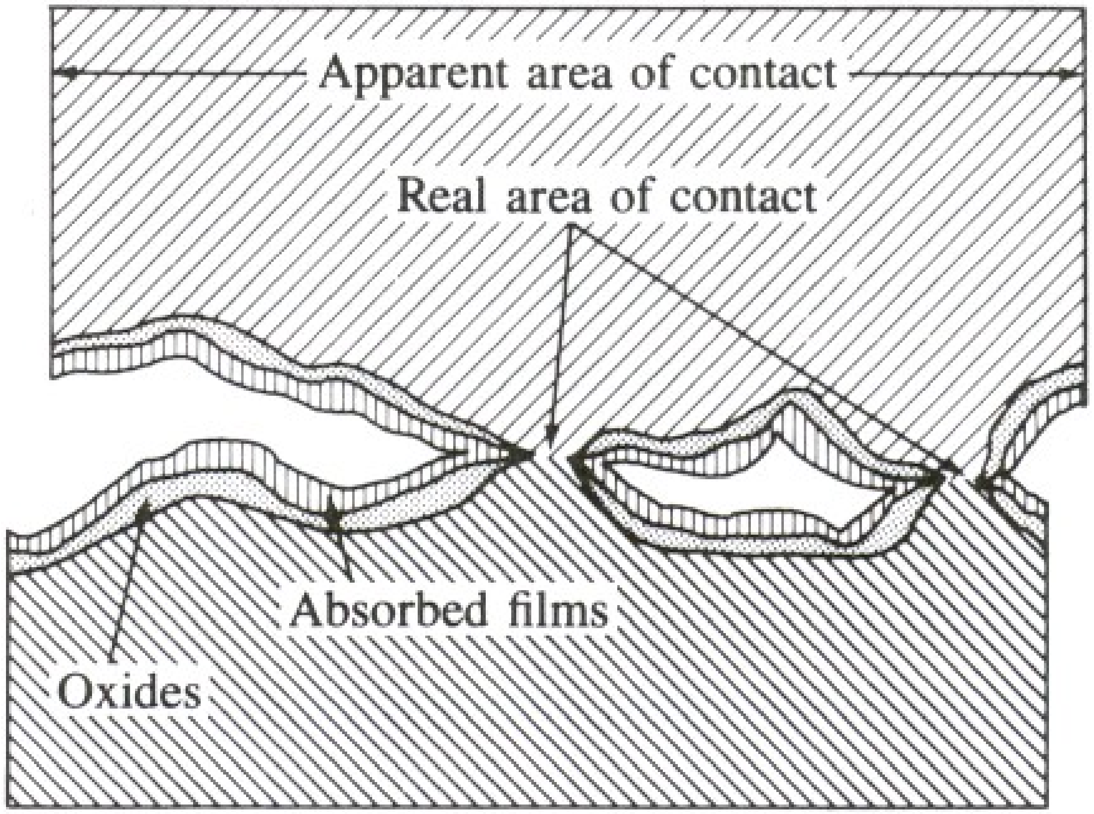
The base oil origin is either mineral (of petrol origin) or synthetic (mono-molecular origin) such as PAO's, polyglycols…

Lubricant additives

Additifs

In order to improve the lubricating performance additives (chemicals) are added:

- extreme pressure
- anti-wear
- VI improvers
- detergents
- dispersants
- emulsifiers
- anti-oxidants
- anti-foam
- etc.



Base oil properties

Propriétés de l’huile de base

The lubrication properties depend on:

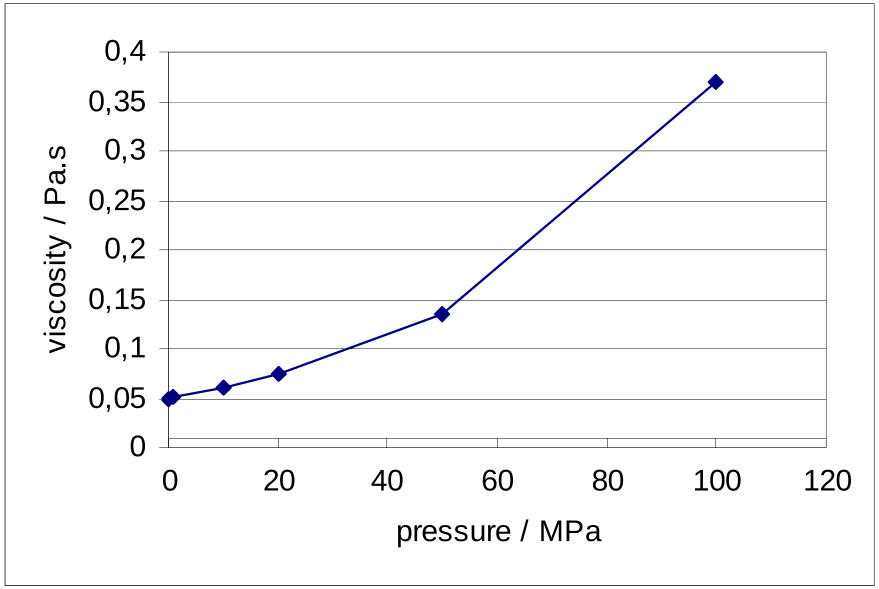
- viscosity (pressure, temperature, shear rate)
- density (pressure, temperature)

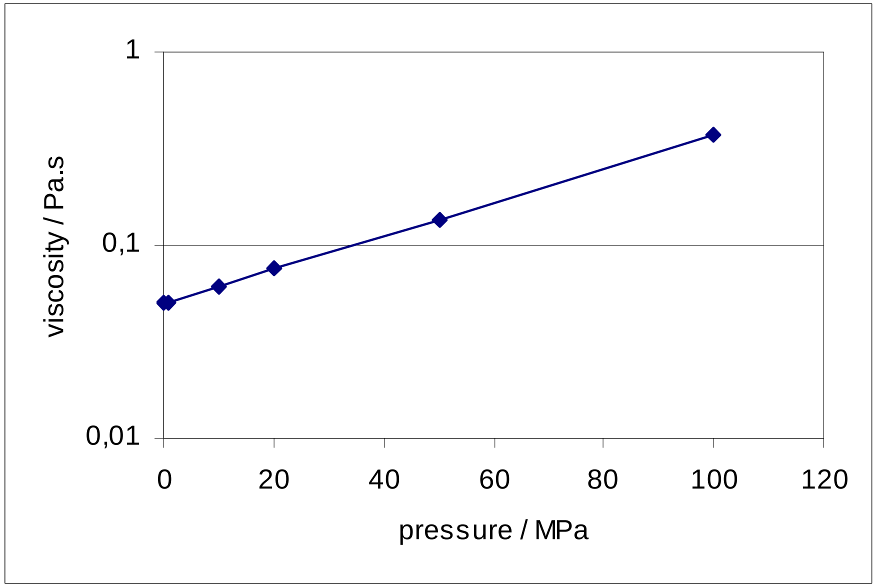
Viscosity – pressure

Viscosité – pression

p / MPa	0.1	1.0	10	20	50	100
η / Pa.s	0.050	0.051	0.061	0.075	0.136	0.369

Viscosity η as a function of pressure p





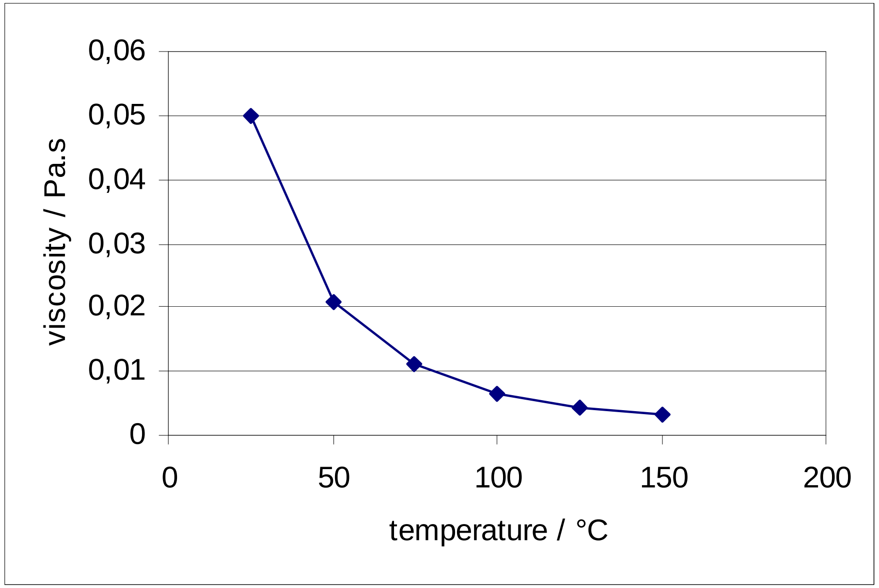
$$\eta(p) = \eta_0 \, e^{\alpha p} \text{ (Barus)}$$

Viscosity – temperature

Viscosité – température

T / ºC	25	50	75	100	125	150
η / Pa.s	0.050	0.021	0.011	0.0064	0.0043	0.0031

Viscosity η as a function of temperature T



$$\eta(T) = \eta_0 \, e^{\frac{b}{T+\theta}} \quad \text{(Vogel)}$$

Viscosity – temperature: viscosity index

Viscosité – température : index de viscosité

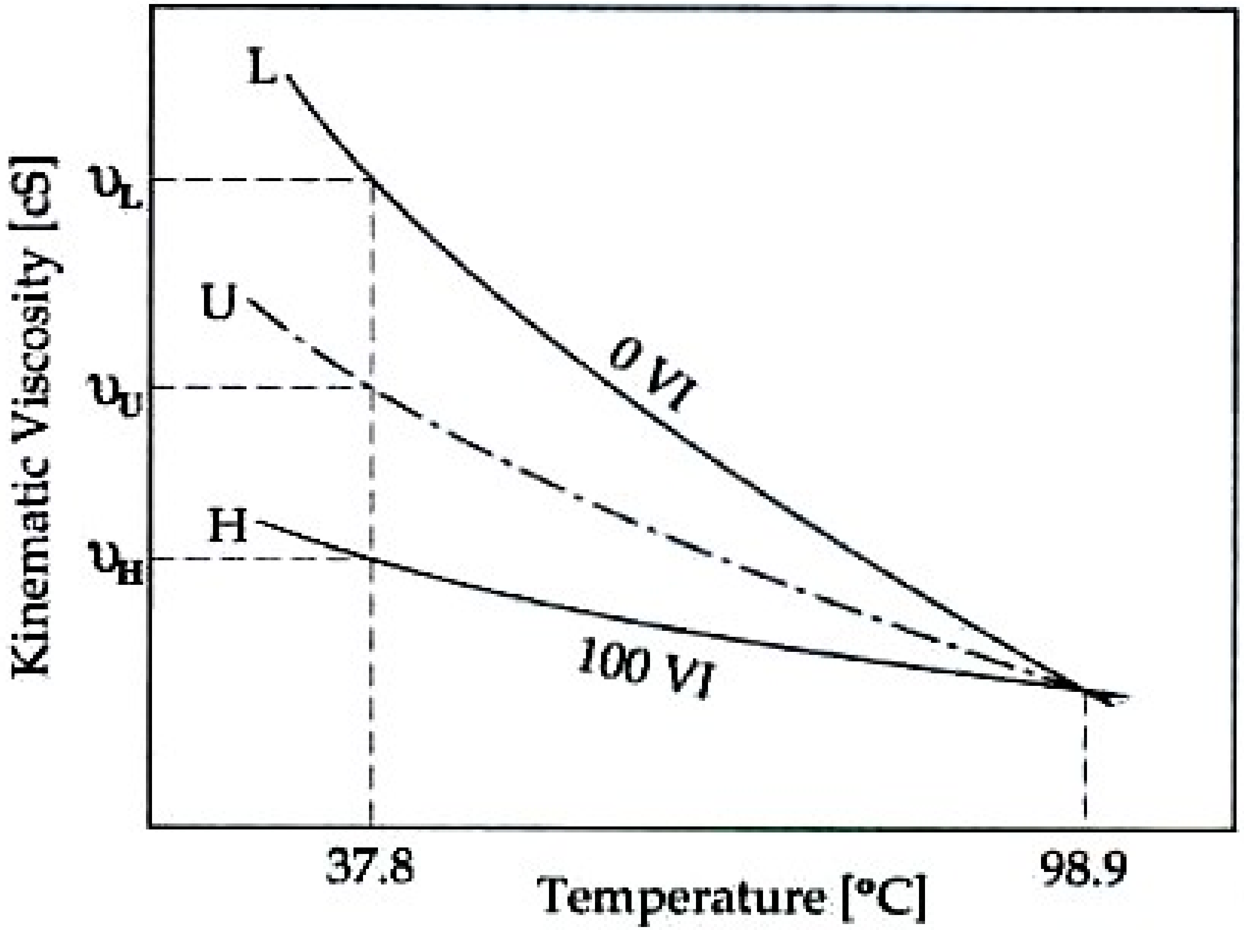
The viscosity index proposed in 1929 is still widely used, it gives an indication of the viscosity temperature relation using two reference oils with the same viscosity at 100ºC (*P*), these oils have a viscosity *H* and *L* at 40ºC, while our oil has a viscosity of *U* at 40ºC.

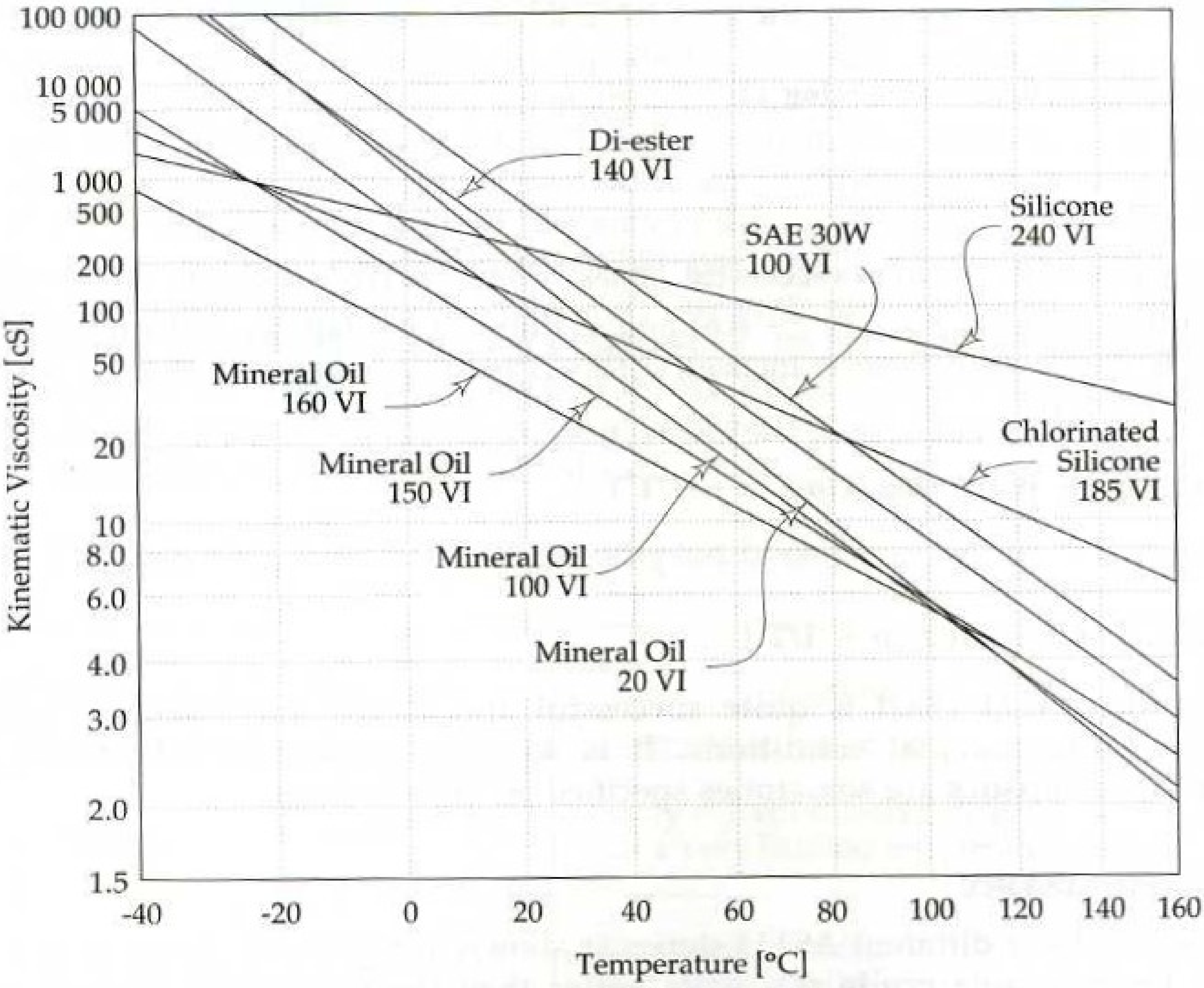
$$V.I. = 100\frac{L-U}{L-H}$$

Viscosity – temperature: viscosity index

Viscosité – température : index de viscosité

$$V.I.=100\frac{L-U}{L-H}$$



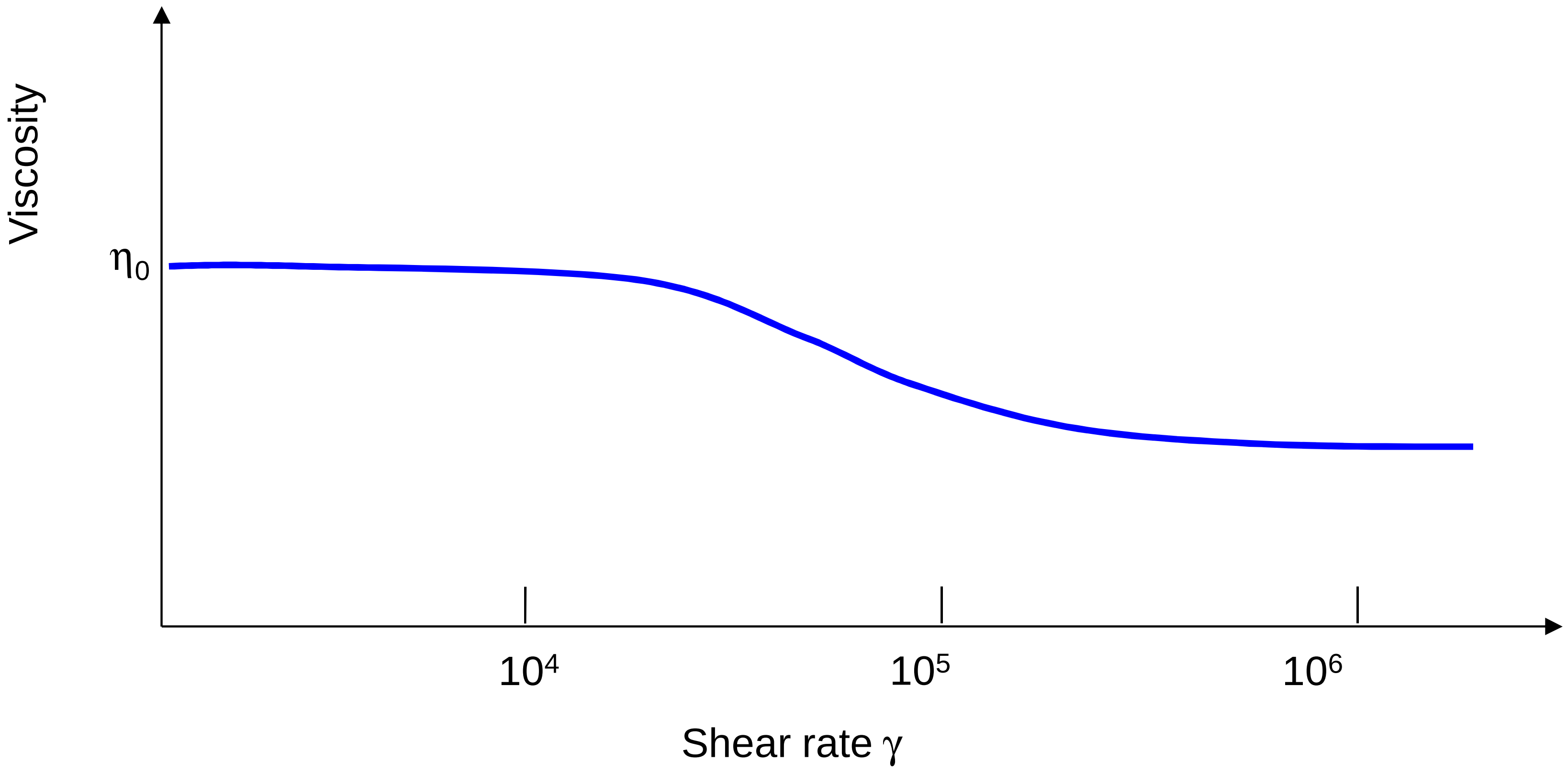

100 000
10 000
5 000
1 000
500
200
100
50
20
10
8.0
6.0
4.0
3.0
2.0
1.5
Kinematic Viscosity [cS]
-40
-20
0
20
40
60
80
100
120
140
160
Temperature [°C]
Di-ester
140 VI
Silicone
240 VI
SAE 30W
100 VI
Mineral Oil
160 VI
Chlorinated
Silicone
185 VI
Mineral Oil
150 VI
Mineral Oil
100 VI
Mineral Oil
20 VI

Viscosity – shear rate

Viscosité – taux de cisaillement

γ / s^{-1}	10^3	10^4	10^5	10^6	10^7	10^8
η / Pa.s	0.050	0.050	0.050	0.030	0.010	0.010

Viscosity η as a function of shear rate γ



Viscosity – shear rate: Example

Exemple de dépendance viscosité – taux de cisaillement

Velocity difference $\Delta u = u_2 - u_1 = 1$ m/s

Film thickness $h = 1$ µm $= 10^{-6}$ m

Shear rate $= \Delta u \,/\, h = 10^{+6}\ \mathrm{s}^{-1}$

Density – pressure

Masse volumique – pression

p / MPa	0.1	1.0	10	20	50	100
ρ / kg/m³	800	800	805	809	821	839

Density ρ as a function of pressure p

$$\rho(p) = \rho_0 \frac{0.5910^9 + 1.34\,p}{0.5910^9 + 1.0\,p} \text{(Dowson – Higginson)}$$

Beware that this one is not a physical law expression: It requires p to be in Pa…

Density – temperature

Masse volumique – température

Thermal (volume) dilatation coefficient α

$\Delta V / V = \alpha \, \Delta T$

$\rho = m / V, \quad m = \text{cst}$

$\Delta\rho / \rho \approx -\Delta V / V = -\alpha \, \Delta T$

For mineral oils, $\alpha \approx 6 \; 10^{-4} \; \mathrm{K}^{-1}$

Conclusion

Hydrodynamic lubrication:

- Viscosity is (almost) constant with pressure,
- Viscosity is very dependent on temperature,
- Viscosity is independent of shear rate
  (except at very high shear rates)
- Density is (almost) constant with pressure,
- Density is (almost) constant with temperature

Full set of equations

Jeu complet d'équations

Reynolds equation

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{\partial}{\partial x}\left(\rho h\frac{u_1+u_2}{2}\right)+\frac{\partial}{\partial y}\left(\rho h\frac{v_1+v_2}{2}\right)+\frac{\partial}{\partial t}(\rho h)$$

Geometry & kinematics

$h(x,y,t)$, $u_1(x,y,t)$, $u_2(x,y,t)$

Boundary conditions (problem dependant)

Constitutive relations

$\eta(p,T,\gamma)$, $\rho(p,T)$

and… $p \geq 0$

Physical explanation

Explication physique

Liquids can not endure tension (negative pressure):

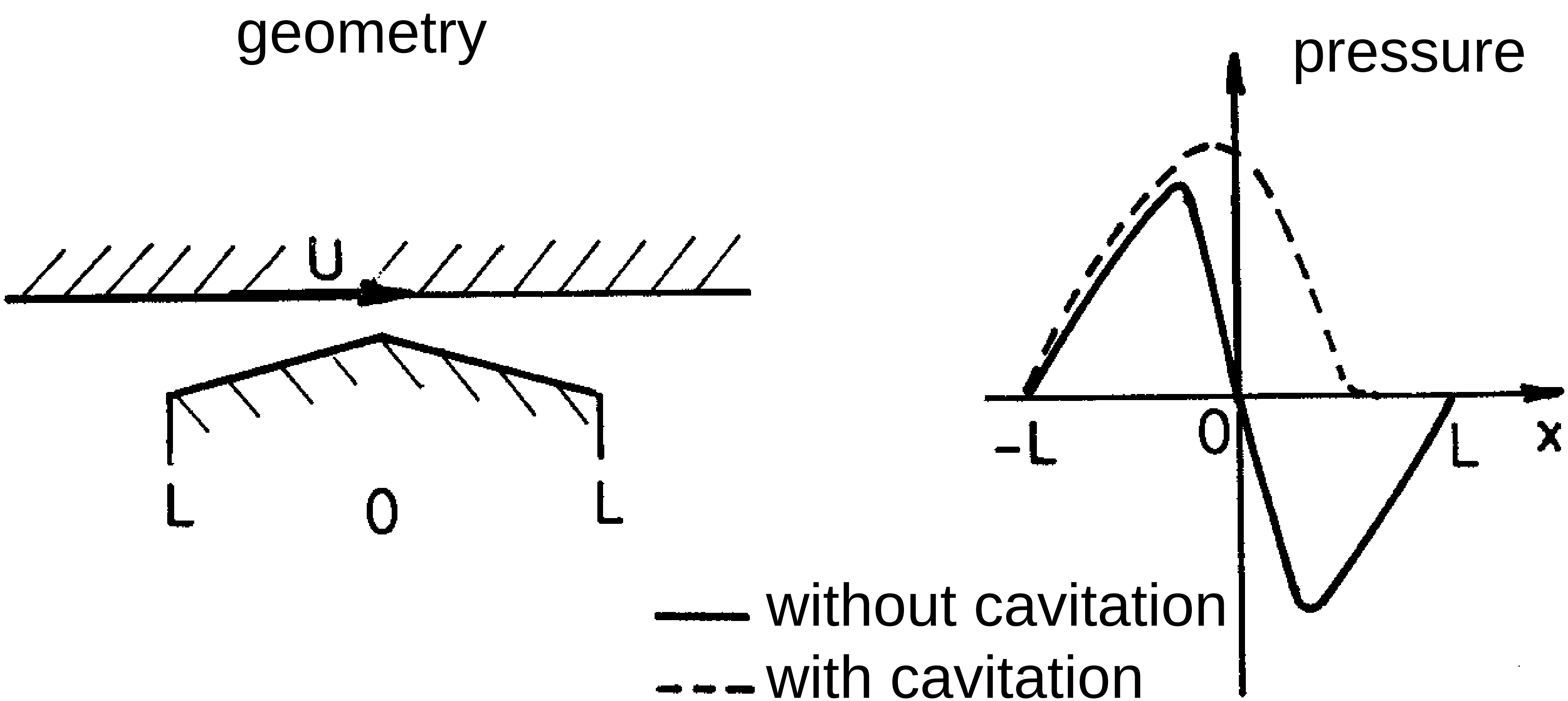
they evaporate…

The minimum pressure a fluid can sustain is called the vapor pressure

The vapor pressure is small compared to the hydrodynamic pressures (and is set to zero)

The phenomenon is called “cavitation”

Example

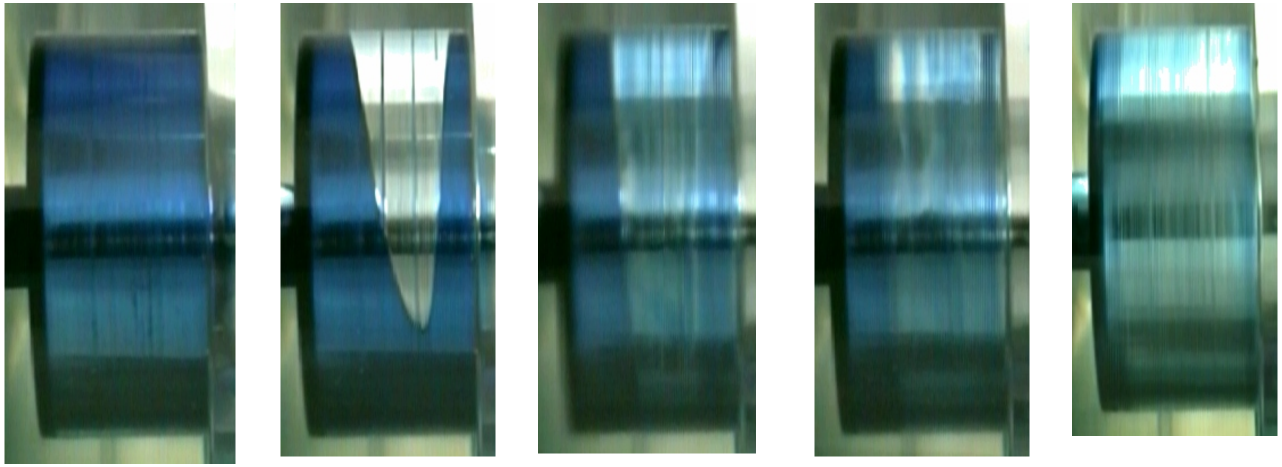

geometry
U
L
0
L
pressure
–L
0
L
x
without cavitation
with cavitation

In reality…

The cavitation phenomenon is (of course) much more complicated…

It is a free boundary problem, where the position of the free boundary is determined by the kinematic conditions and the surface tension of the liquid.

In reality…



0 Hz 10 Hz 30 Hz 40 Hz 60 Hz

After http://phn.tamu.edu/TRIBGroup/
Rotordynamics Laboratory, Texas A&M University

Physical conditions

Conditions à la frontière

At the cavitation boundary the pressure gradient normal to the boundary has to be zero:

$$\frac{\partial p}{\partial n} = 0$$

in order to satisfy mass flow continuity!

Physical conditions

Conditions à la frontière

As a result the position of the (free) boundary is difficult to predict.

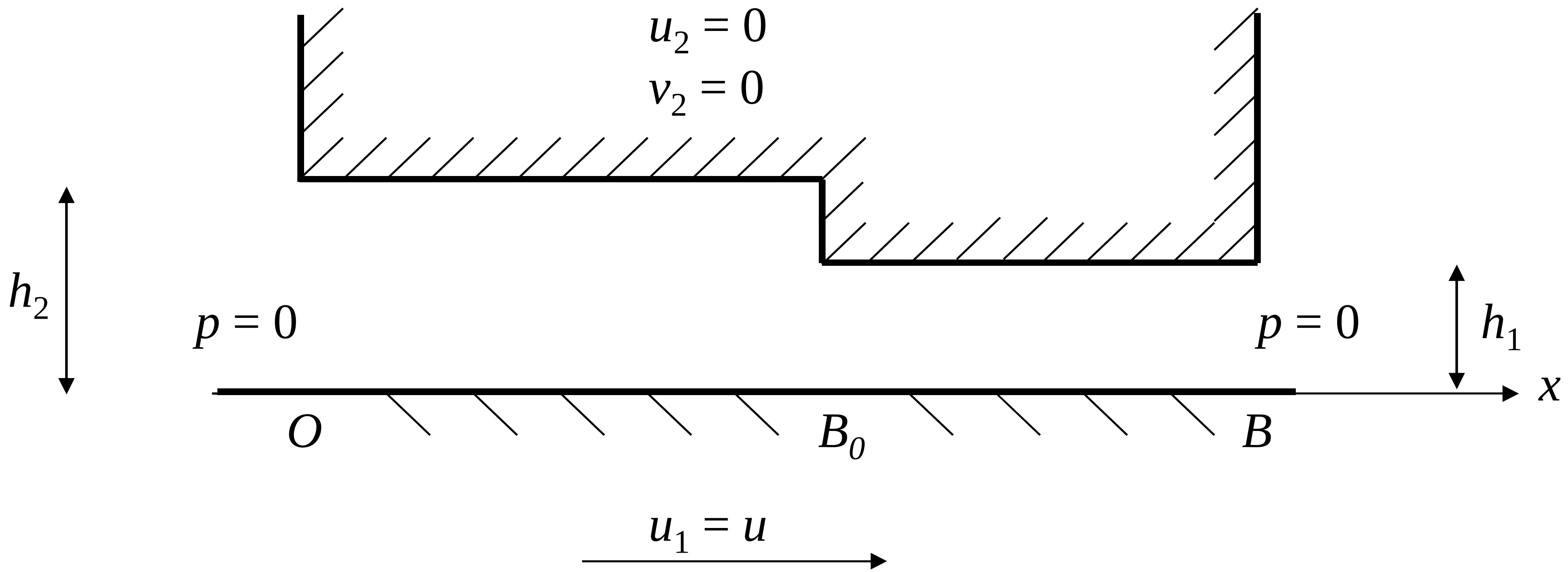
As a general rule one can say that it is beyond the point where zero pressure occurs in the non-cavitating case.

Hence boundary conditions are generally chosen to “fit” the geometry to limit this problem.

Numerically this problem can be solved;
however, it remains a “research” problem

Example: Step bearing

Exemple : palier à saut

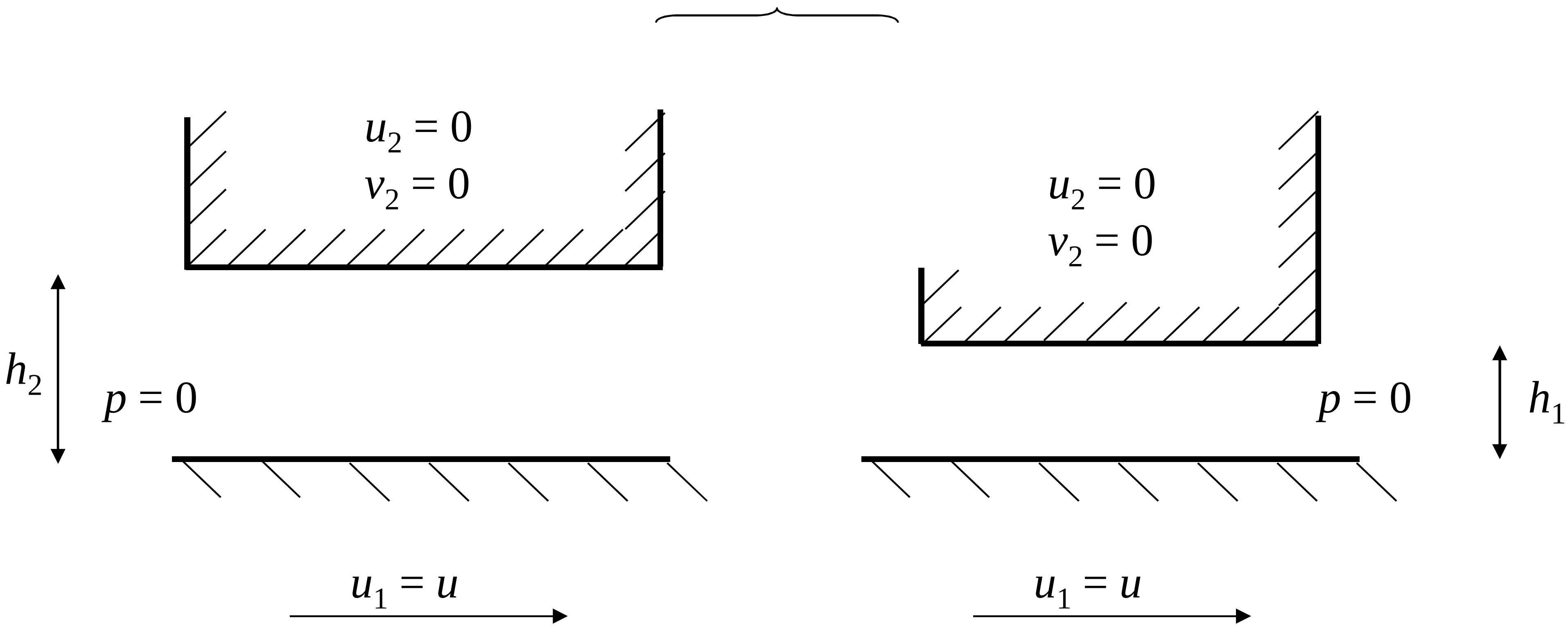


Example: Step bearing

Exemple : palier à saut

Gluing the two sub-problems:

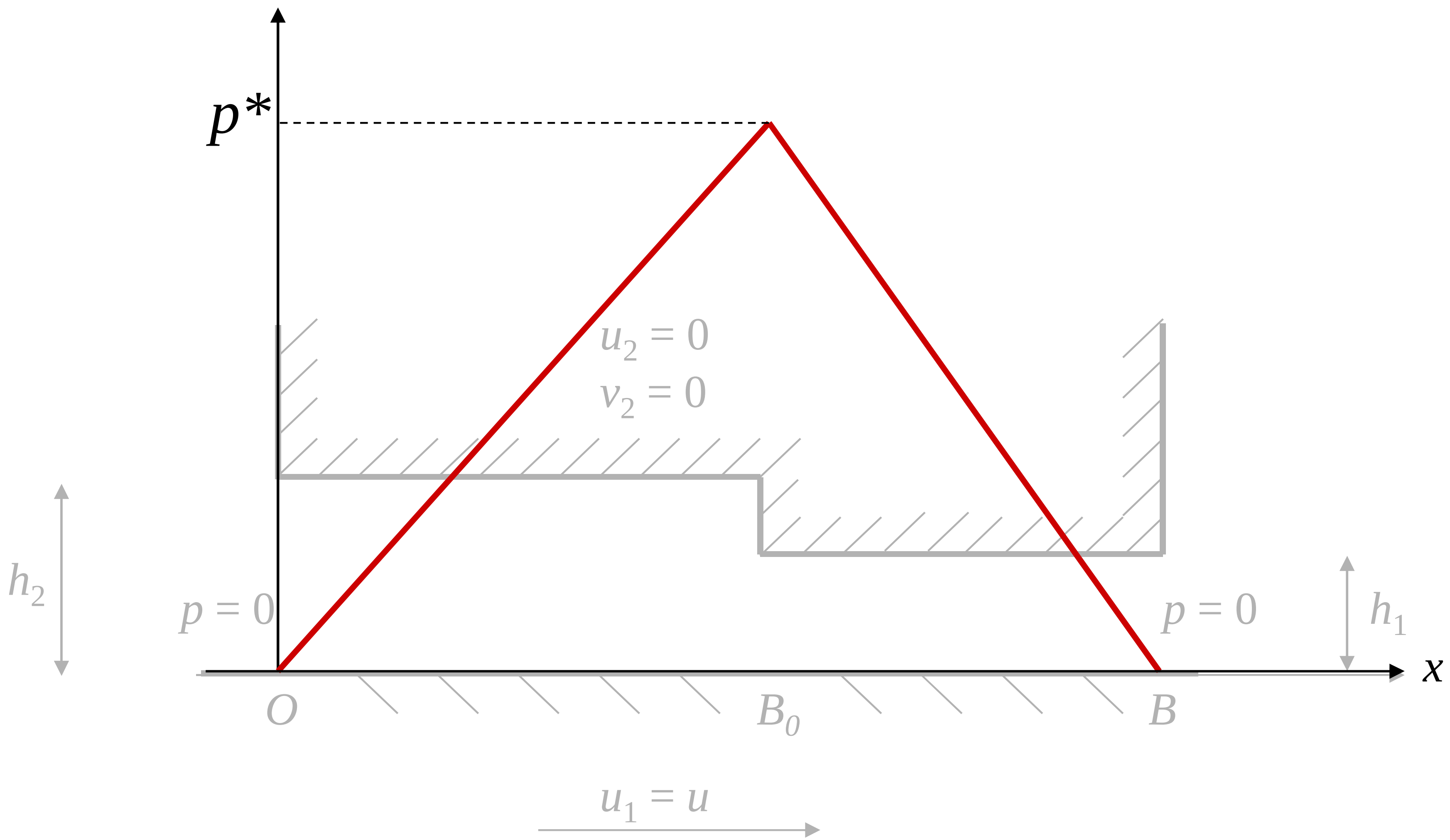
- Equilibrium: same pressure p^*
- Mass conservation: same global flow rate q



Example: Step bearing

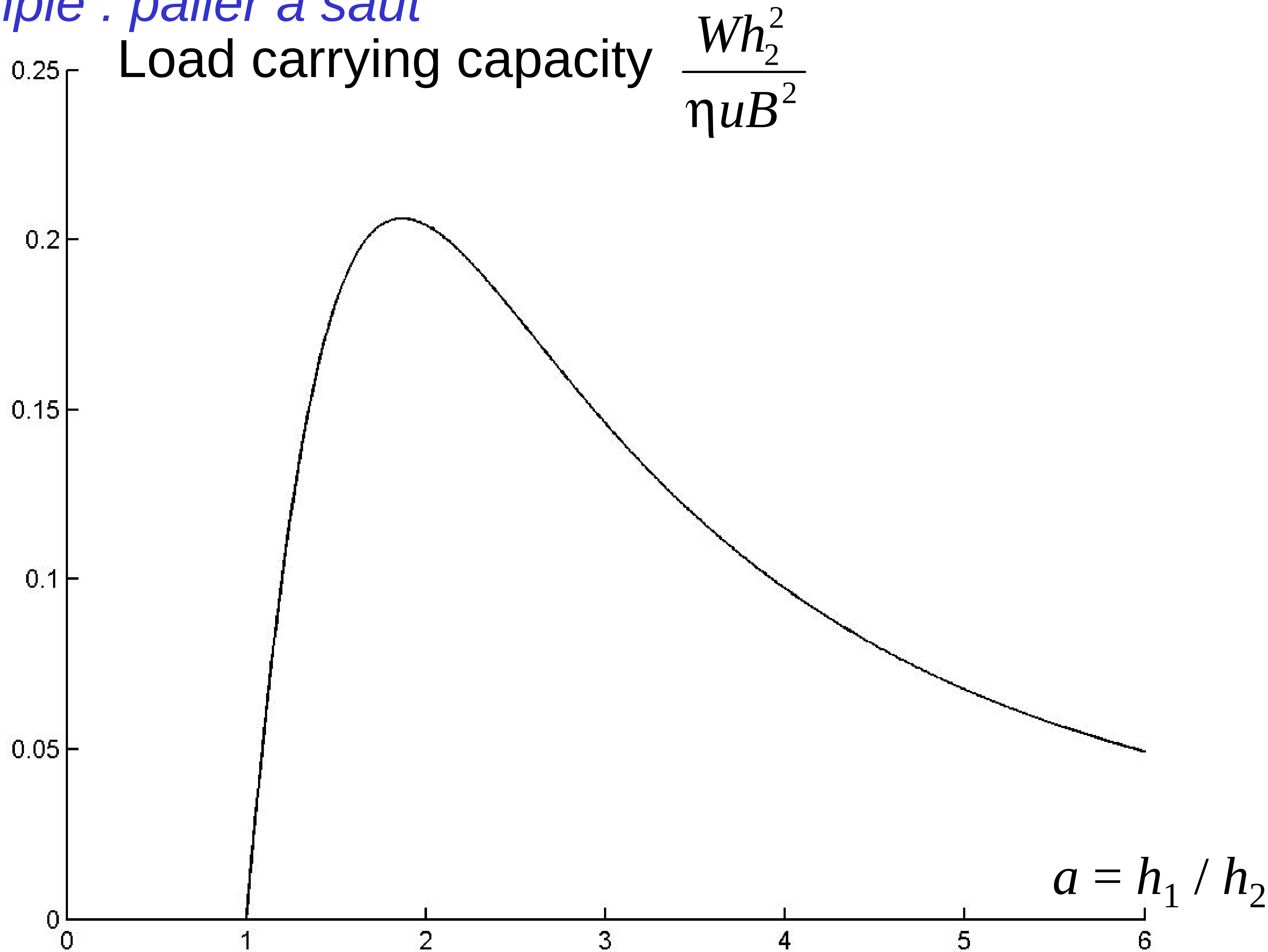
Exemple : palier à saut

Pressure distribution



Example: Step bearing

Exemple : palier à saut



Example: Step bearing

Exemple : palier à saut

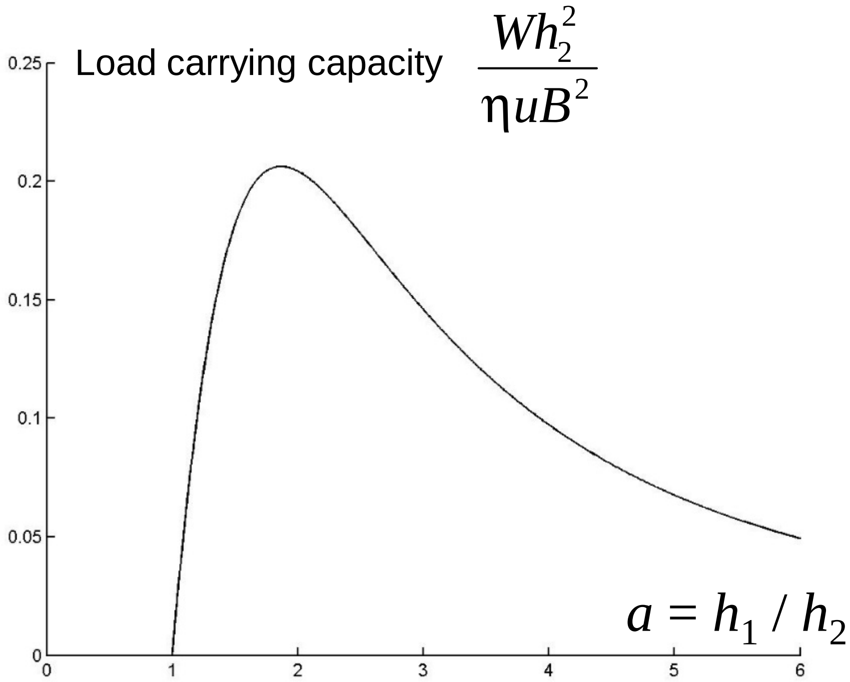
This is a unique curve.

Check it with dimensional analysis.

The optimum load carrying capacity is obtained for $a = h_1 / h_2 \approx 2.2$. Can this be obtained for different conditions (such as velocity u)?

What happens to W if one

- Doubles the velocity u
- Doubles the length B
- Halves the height h_2, keeping a constant

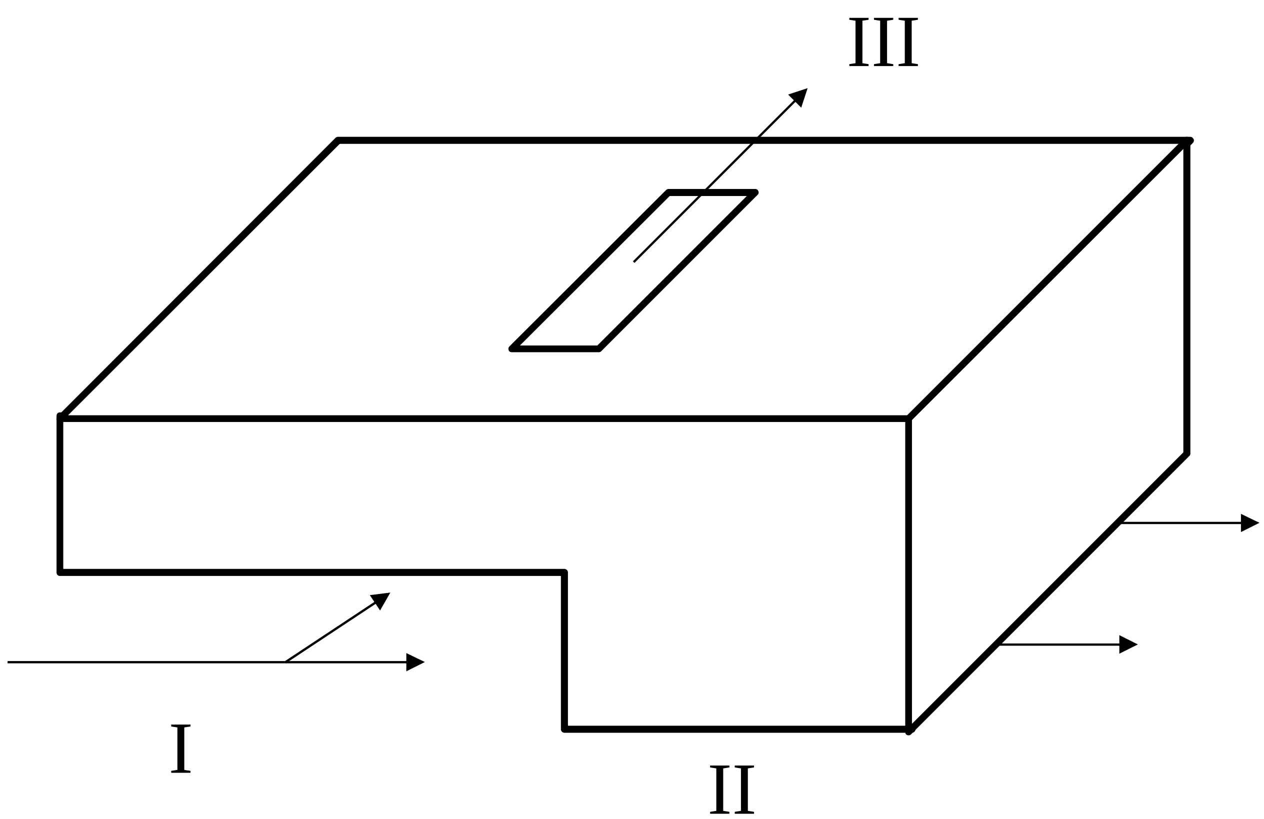


Example: Step bearing pump

Exemple : pompe à palier à saut

Gluing the three sub-problems:

- Equilibrium: same pressure p_0
- Mass conservation: $q_{\mathrm{I}} = q_{\mathrm{II}} + q_{\mathrm{III}}$



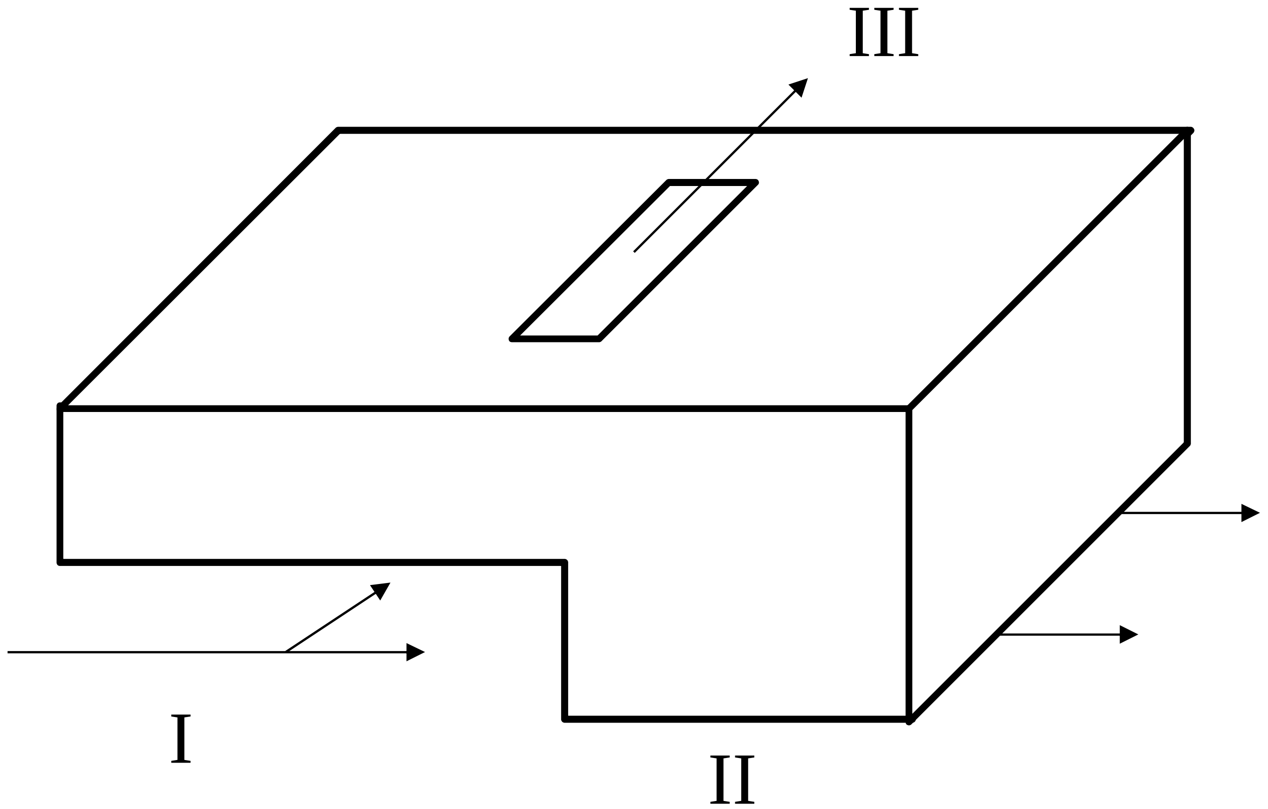
Example: Step bearing pump

Exemple : pompe à palier à saut

$$q_{III} = -\frac{p_0}{12\eta}\left[\frac{h_1^3}{B_0} + \frac{h_2^3}{B - B_0}\right] - \frac{u}{2}\left[h_2 - h_1\right]$$

What should happens if

$p_0 = p^*$ (previously obtained)?



Slider bearing conclusion

Conclusions sur les paliers à glissement

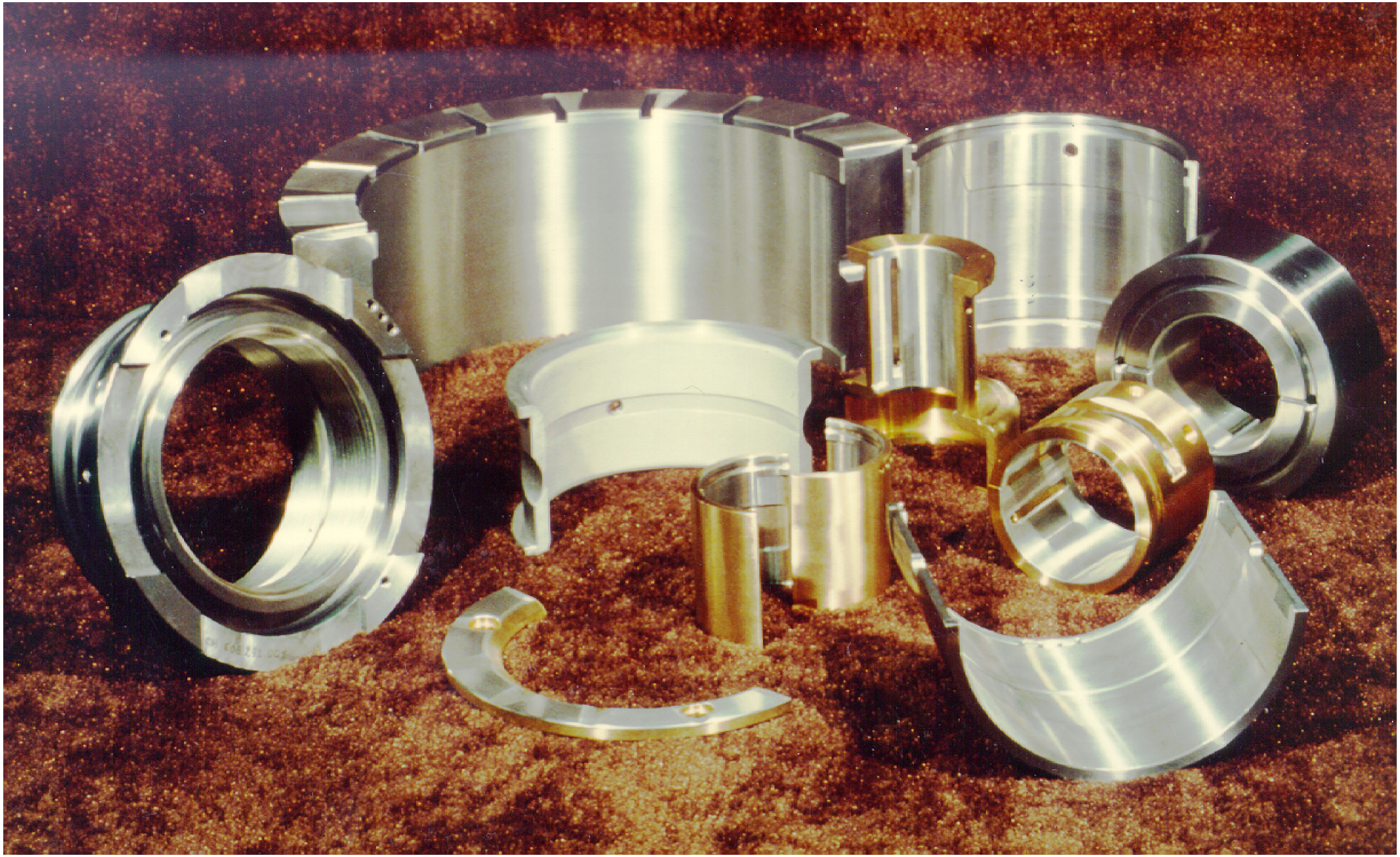
Load carrying capacity generated by a converging gap associated with a velocity.

Load carrying capacity proportional to the viscosity · velocity product.

Load carrying capacity inversely proportional to the gap height (h^{-2}).

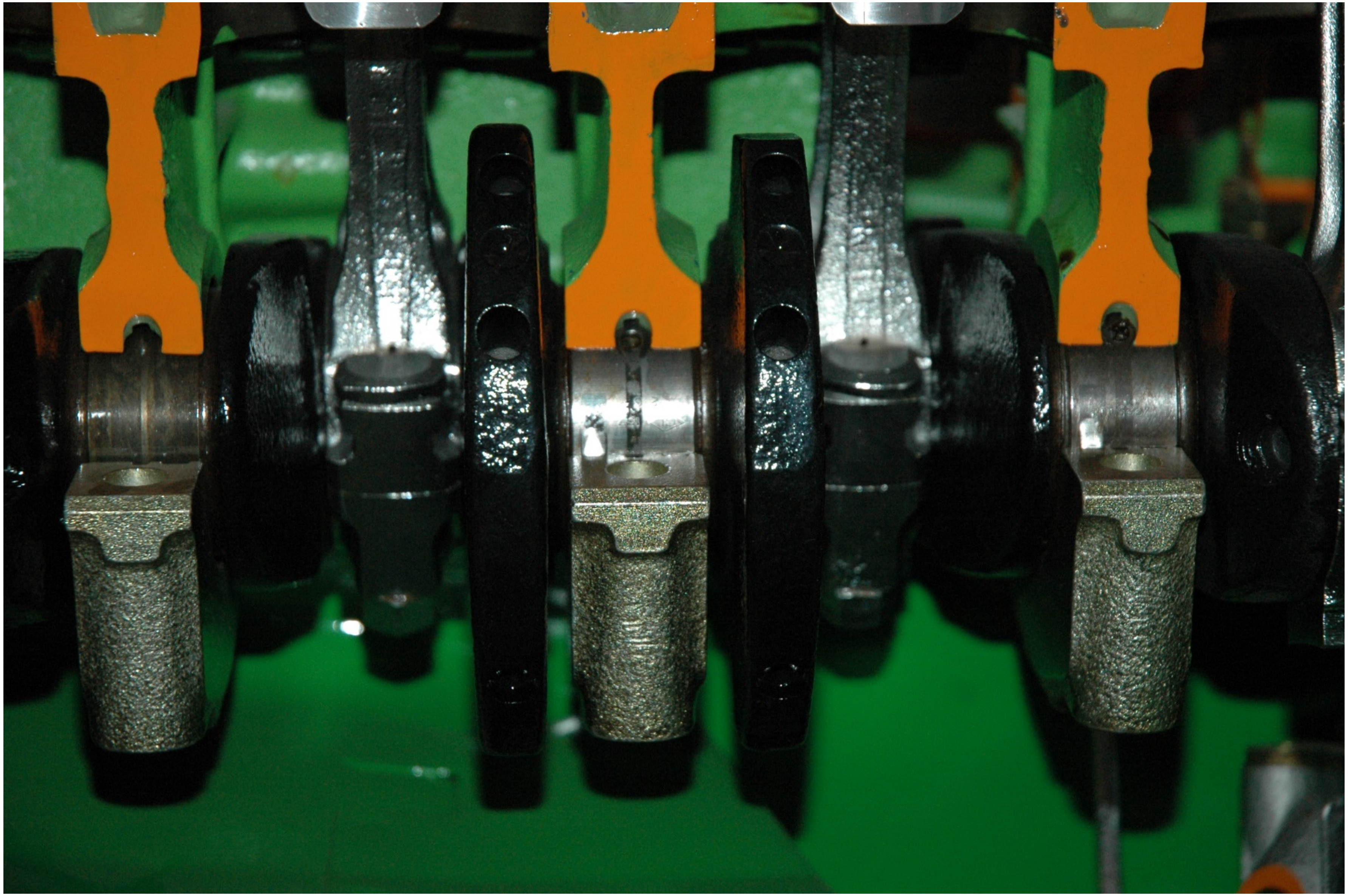
Example: Journal bearing

Exemple : palier lisse



Example: Journal bearing

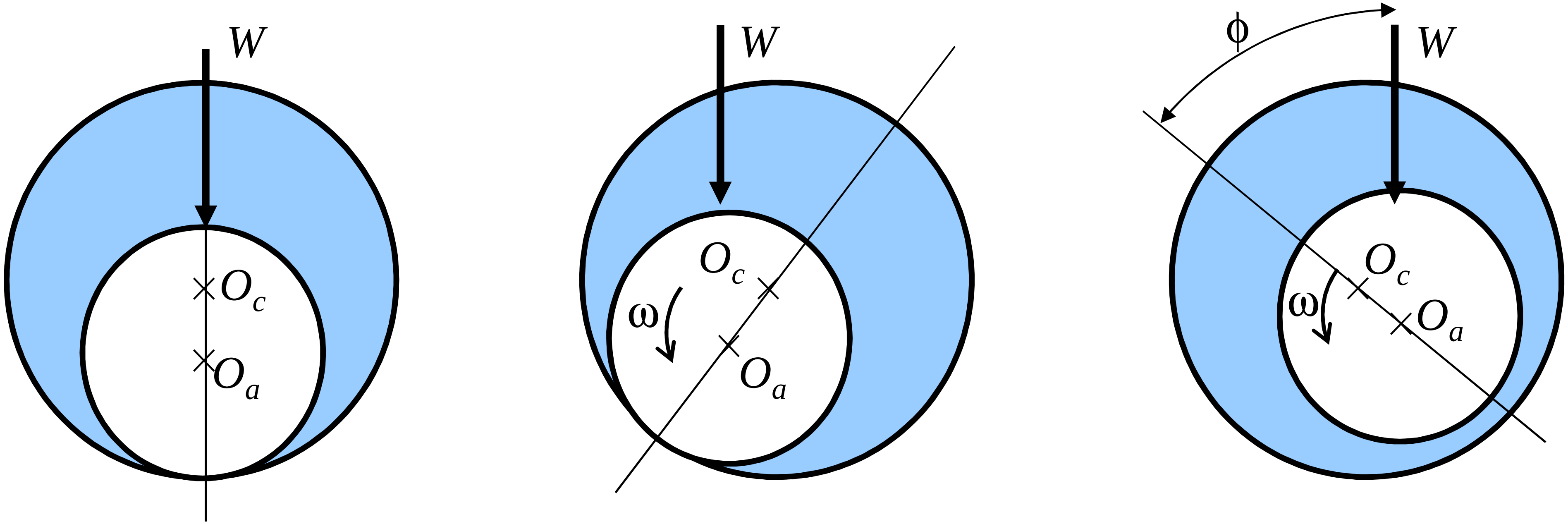
Exemple : palier lisse



Operation

Fonctionnement

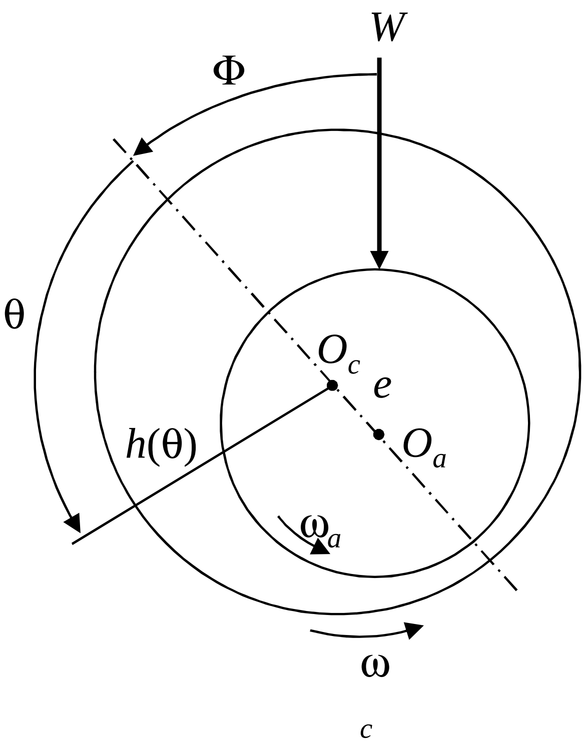
Start-up



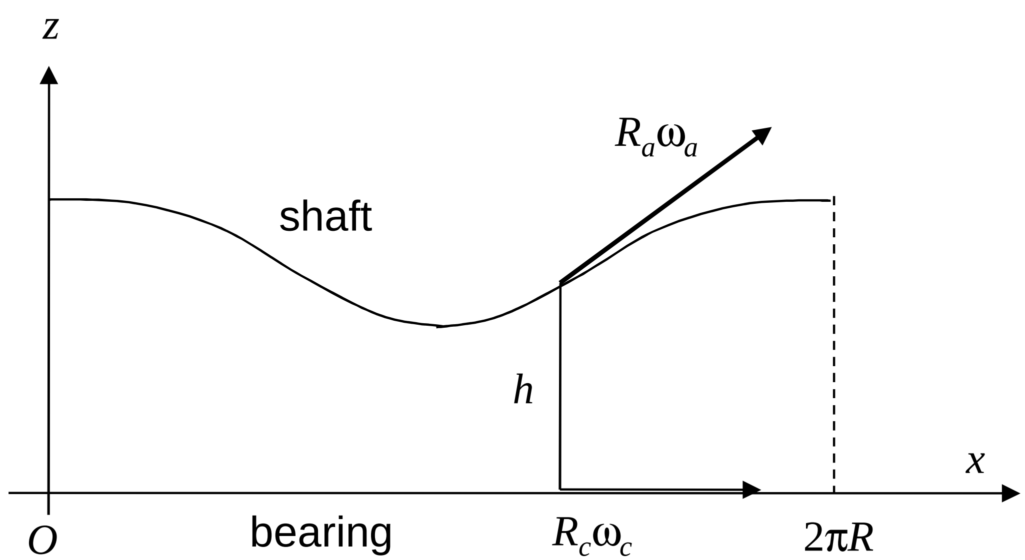
Explain!

Geometry

Real



Simplified



$$h(\theta) = c\,(1 + \varepsilon \cos\theta)$$

c: clearance

e: eccentricity

$$\varepsilon = \frac{h_{\max} - h_{\min}}{h_{\max} + h_{\min}} = \frac{e}{c}$$

Infinitely long bearing

Palier infiniment long

1D Reynolds
Stationary

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)=\frac{\partial}{\partial x}\left(\rho h\frac{u_1+u_2}{2}\right)+\frac{\partial}{\partial t}(\rho h)$$

Constant viscosity and density

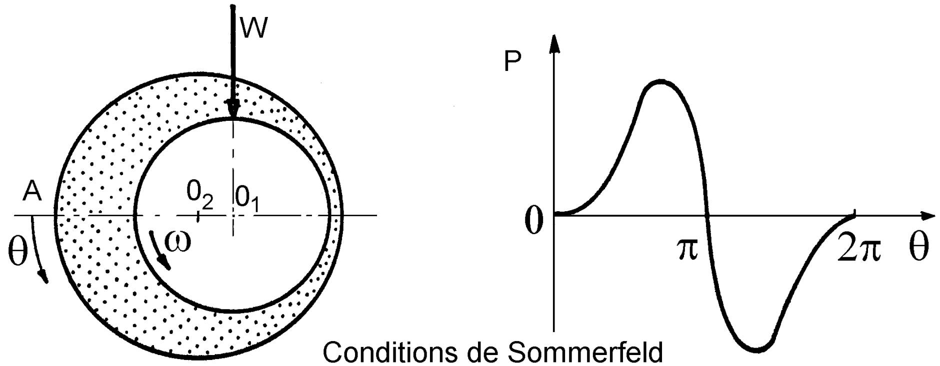
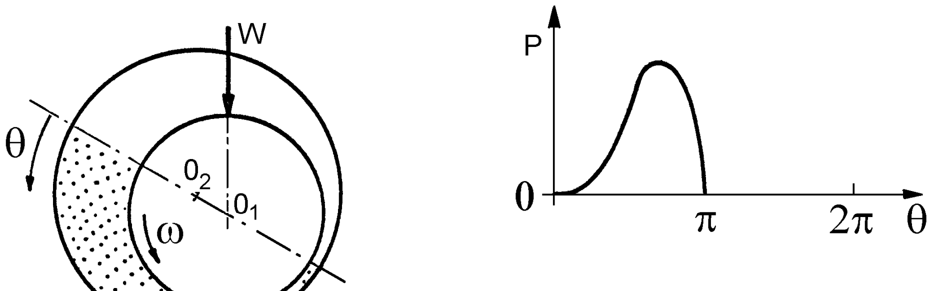
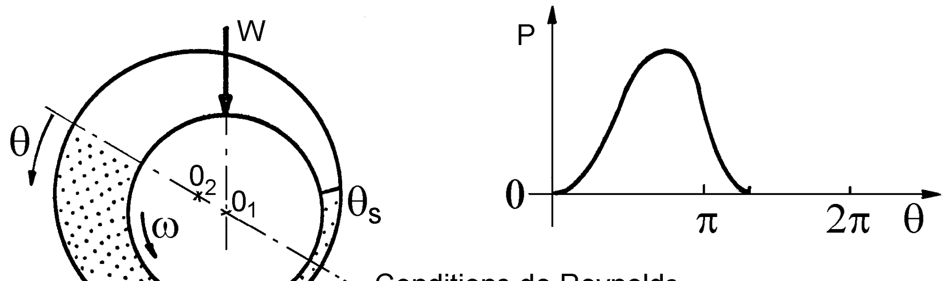
$$u_1=R\omega_1$$
$$u_2=R\omega_2$$

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right)=12\eta R\frac{\omega_1+\omega_2}{2}\frac{dh}{dx}$$

$$x=R\theta$$

$$\frac{d}{d\theta}\left(h^3\frac{dp}{d\theta}\right)=12\eta R^2\frac{\omega_1+\omega_2}{2}\frac{dh}{d\theta}$$

and boundary conditions:

Problem	equation	solution
 Conditions de Sommerfeld	1D Reynolds without cavitation	Sommerfeld
 Conditions de Gumbel	id + cut off negative pressure	Gümbel
 Conditions de Reynolds ou de Swift et Stieber	id + free surface for cavitation	Reynolds

Région active   Région inactive

Sommerfeld solution

It is possible to solve the infinitely long Reynolds equation analytically:

$$\frac{d}{d\theta}\left(h^3 \frac{dp}{d\theta}\right) = 6\eta R^2 \omega \frac{dh}{d\theta} \qquad h(\theta) = c\,(1 + \varepsilon\cos\theta)$$

using the following transformation:

$$1 + \varepsilon\cos\theta = \frac{1-\varepsilon^2}{1-\varepsilon\cos\psi}$$

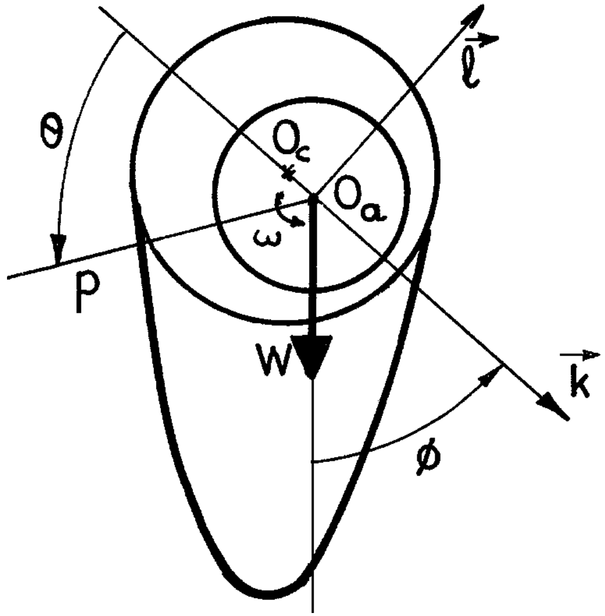
The resulting pressure distribution is:

$$p(\psi) = \frac{6\,\eta\,\omega\,(R/c)^2}{\left(1-\varepsilon^2\right)^{3/2}}\left\{\psi - \varepsilon\sin\psi - \frac{2\psi - 4\,\varepsilon\sin\psi + \varepsilon^2\,\psi + \varepsilon^2\sin\psi\,\cos\psi}{2+\varepsilon^2}\right\}$$

Sommerfeld pressure – load relation

Relation pression – chargement

A loaded (W, per unit width) journal bearing will have an eccentricity ε at an angle Φ with respect to the load, given by the following integrals:



$$W\cos\phi + \int_0^{2\pi} p(\theta)\cos\theta \, R d\theta = 0$$

$$-W\sin\phi + \int_0^{2\pi} p(\theta)\sin\theta \, R d\theta = 0$$

Sommerfeld load carrying capacity

Capacité de charge

one finds: $$W = 12\,\eta\,\omega \frac{R^3 L}{c^2} \frac{\pi\,\varepsilon}{\left(2+\varepsilon^2\right)\sqrt{1-\varepsilon^2}}$$

$W \propto (R/c)^2$

What is the limit for $\varepsilon \to 0$?

Explain!

Sommerfeld frictional torque

Couple de frottement

Integrating the shear stress: $$C_a = L\int_0^{2\pi} R\tau_{xy}\left(\theta, z = h\right) R d\theta$$

Shear stress: $$\tau_{xy} = \eta \frac{du}{dz}$$

Fluid velocity: $$u(z) = \frac{1}{2\eta}\frac{dp}{dx}\; z\left(z - h\right) + \frac{R\omega\, z}{h}$$

Sommerfeld frictional torque

Couple de frottement

Frictional torque on the shaft:

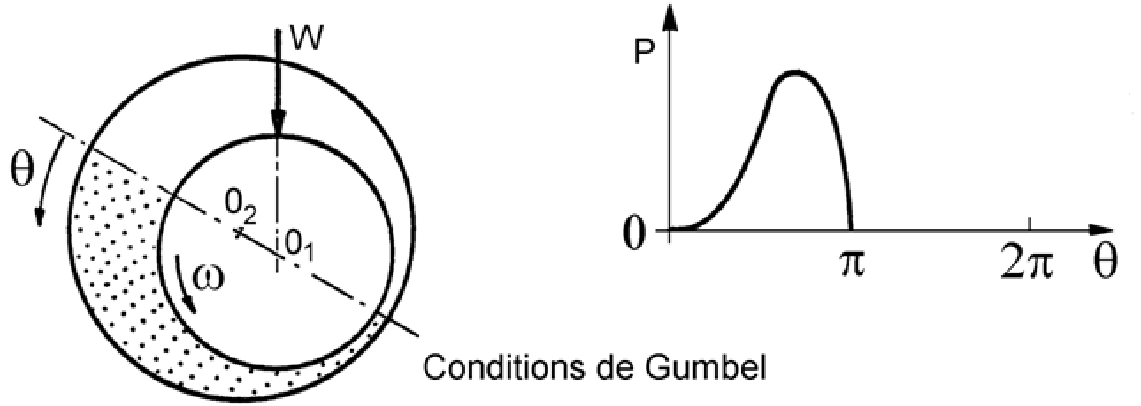
$$C_a = \frac{4\eta\omega R^3 L\pi}{c}\left[\frac{1+2\varepsilon^2}{\left(2+\varepsilon^2\right)\sqrt{1-\varepsilon^2}}\right]$$

$C_a \propto (R/c)$

What is the value of C_a when $\varepsilon = 0$? Explain the result!

What is the value of C_a when $\varepsilon \to 1$? Explain the result!

Gümbel solution

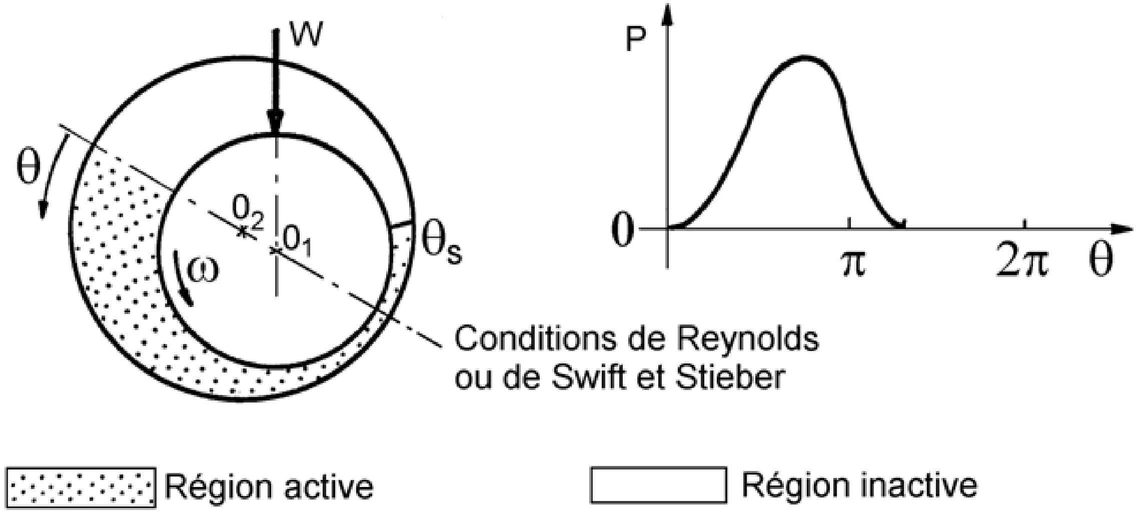


The load is: $$W = 6\eta\omega RL\left(R/c\right)^2 \frac{\varepsilon\sqrt{4\varepsilon^2 + \pi^2\left(1-\varepsilon^2\right)}}{\left(2+\varepsilon^2\right)\left(1-\varepsilon^2\right)}$$

The angle is: $$\phi = \arctan\frac{\pi\sqrt{1-\varepsilon^2}}{2\varepsilon}$$

The frictional torque is: $$C_a = \frac{\pi\eta\omega LR^3}{c\sqrt{1-\varepsilon^2}}\left\{\frac{2+\varepsilon}{1+\varepsilon} + \frac{3\varepsilon^2}{2+\varepsilon^2}\right\}$$

Reynolds solution

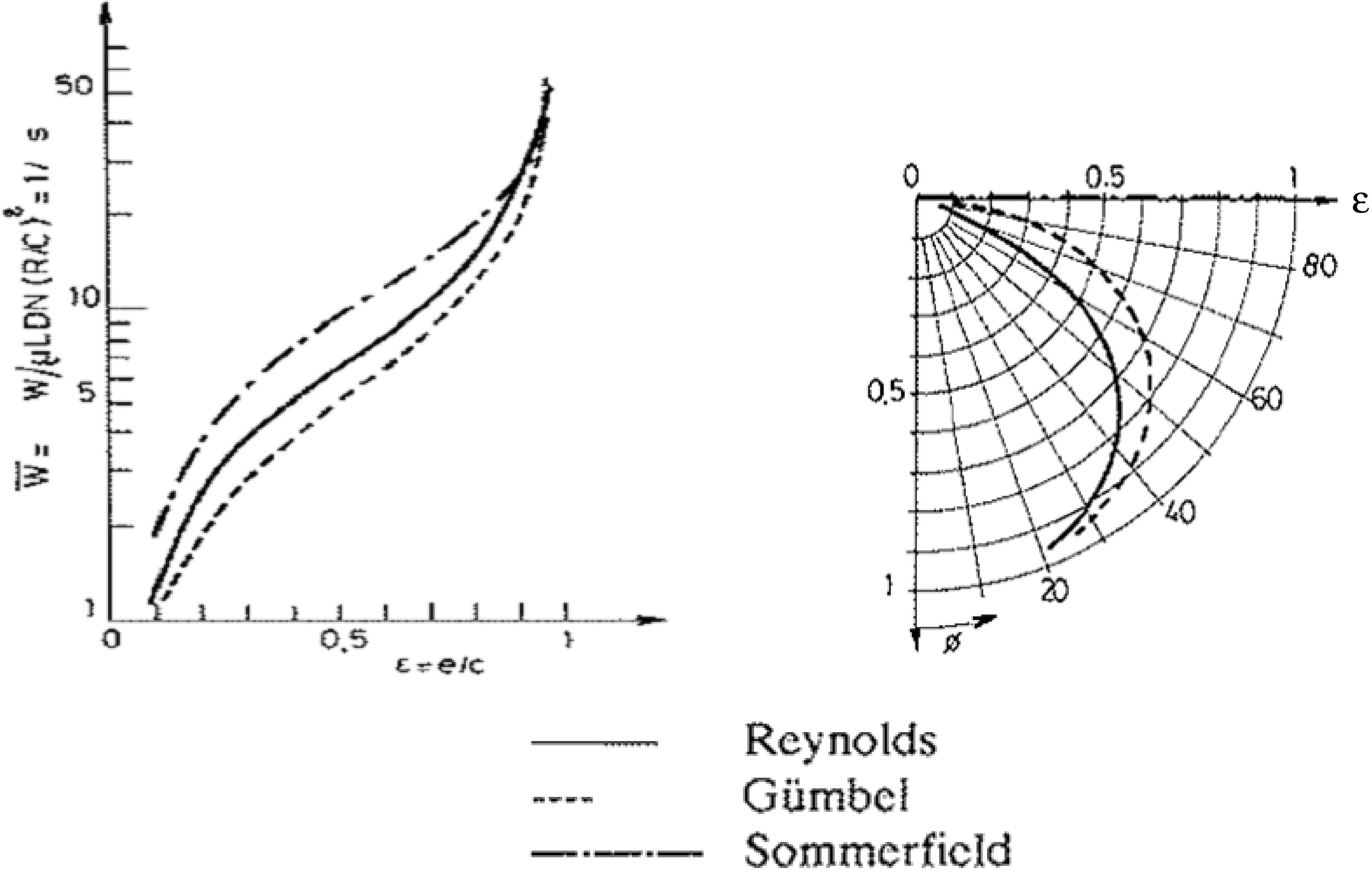


Even more intricate, but still analytical.

One can get the expressions of load, angle, frictional torque as well.

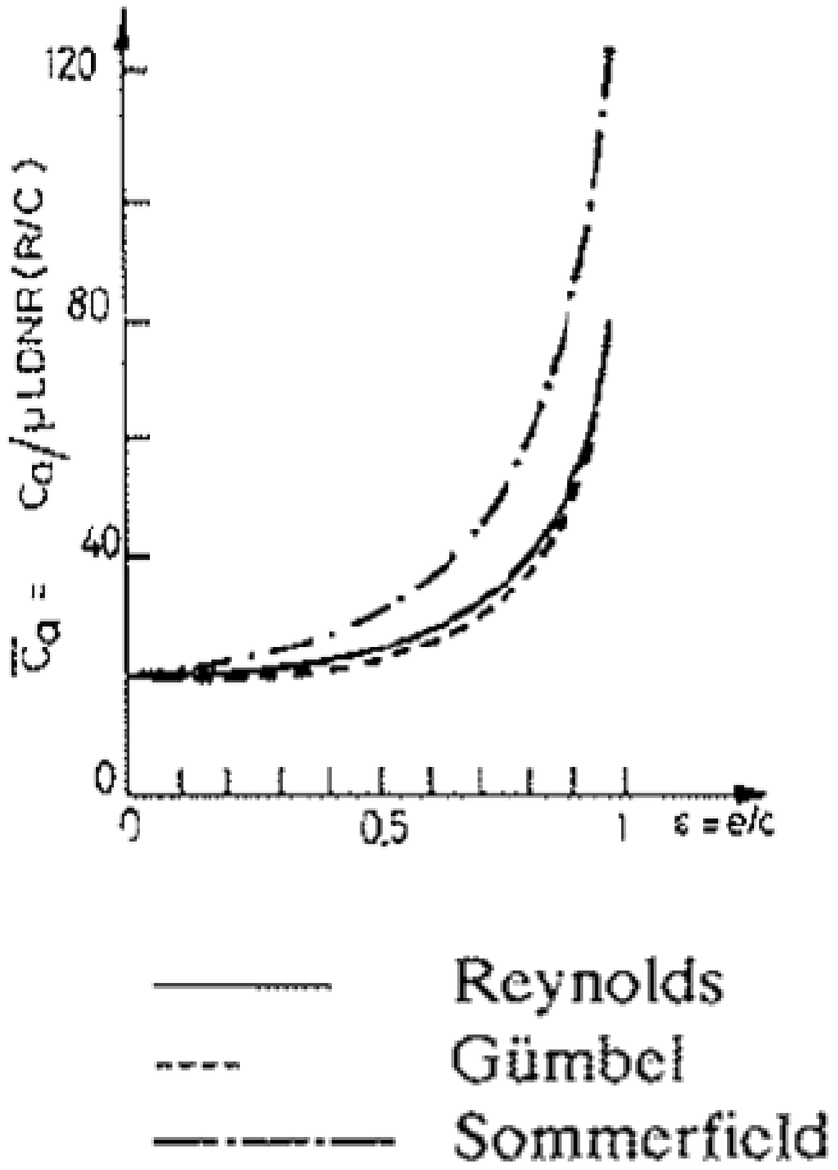
Models comparison – load

Comparaison de modèles - chargement



Models comparison – friction torque

Comparaison de modèles – couple de frottement



Infinitely short bearing

Palier infiniment court

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(h^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)=6\eta\omega\frac{dh}{d\theta}$$

$$p(\theta, y)=-\frac{3\eta\omega}{c^2}\left(y^2-\frac{L^2}{4}\right)\frac{\varepsilon\sin\theta}{(1+\varepsilon\cos\theta)^3}$$

$$W=\eta LR\omega\left(\frac{L}{D}\right)^2\left(\frac{R}{c}\right)^2\frac{\varepsilon}{\left(1-\varepsilon^2\right)^2}\sqrt{16\varepsilon^2+\pi^2(1-\varepsilon^2)}$$

$$\phi=\arctan\left(\frac{\pi}{4}\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\right)$$

Finite length bearings

Palier de largeur intermédiaire

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y} \right) = 6\eta R\omega \frac{\partial h}{\partial x}$$

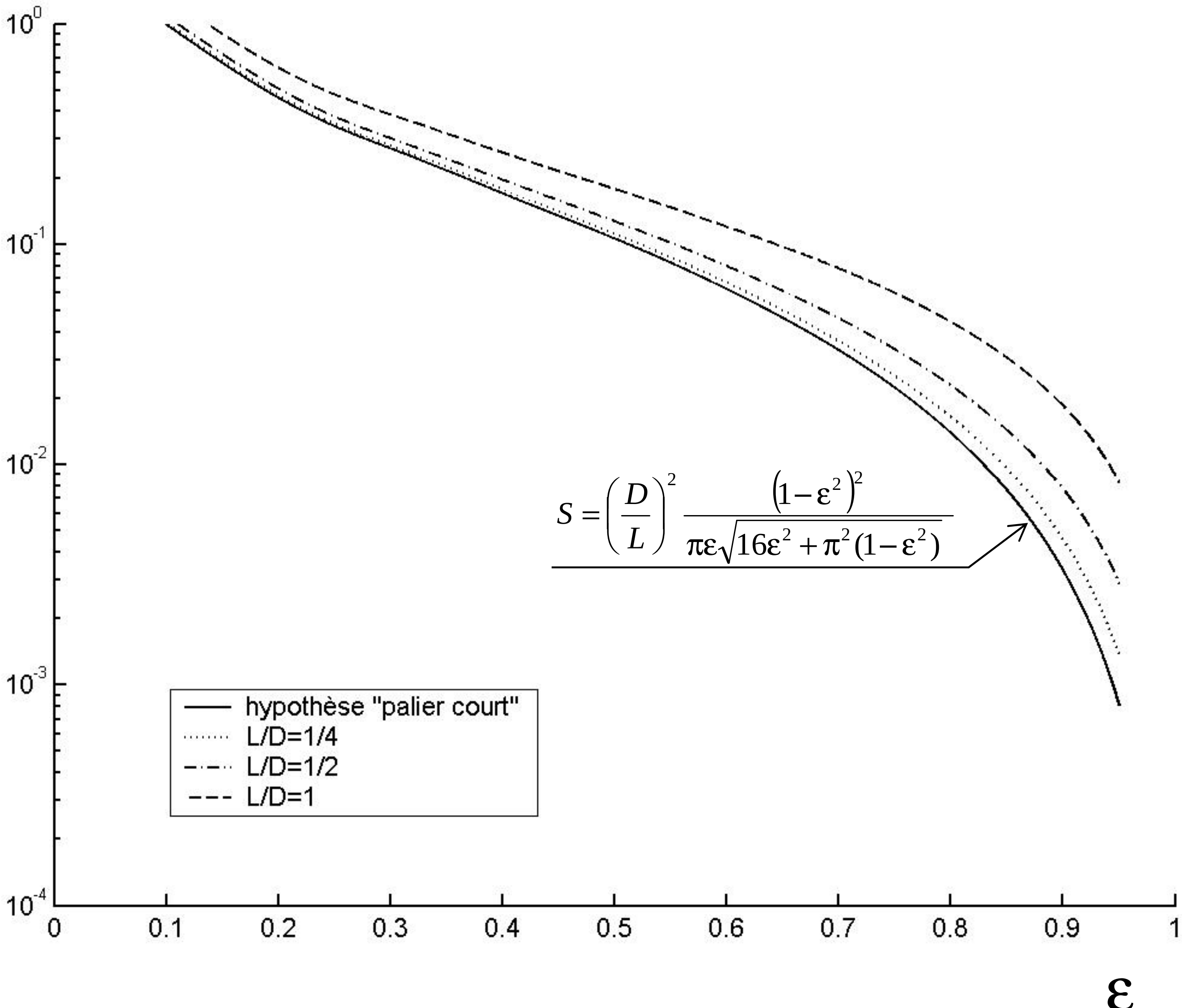
$\Rightarrow$ Numerical solution is necessary

Finite length bearings

Palier de largeur intermédiaire

Influence of the width L

$$S^* = S\left(\frac{L}{D}\right)^2$$



Results in tables

Résultats tabulés

Dimensionless frictional torque $\bar{C}_a = \dfrac{C_a}{\eta L D \omega R^2 / c} = \dfrac{1}{S}\dfrac{R}{c} f_a = \dfrac{f}{S}$

Dimensionless flow $\bar{Q} = \dfrac{Q}{L c \omega R}$

as a function of the dimensionless Sommerfeld number

$$S = \frac{\eta L R \omega}{\pi W}\left(\frac{R}{c}\right)^2$$

Example

Viscosity	η = 0.015 Pa.s
Density	ρ = 860 kg/m3
Specific heat	C_P = 2 000 J/kg/°C
Length	L = 100 mm
Radius	R = 50 mm
Clearance	c = 0.075 mm
Load	W = 50 000 N
Angular velocity	ω = 50 rev/s

Sommerfeld number

$$S = \frac{\eta L D \omega}{W}\left(\frac{R}{c}\right)^2 = 0.0666$$

L / D = 1

Example

ε	0,1	0,2	0,3	[illegible]	[illegible],6	0,7	0,8	0,9	0,95
S	1,33	0,631	0,388	0,[illegible]	[illegible],120	0,0776	0,0443	0,018[illegible]	0,00831
ϕ	79,5	74	68	6[illegible]	[illegible]0,5	44	36	26	19
$\frac{R}{C} f_a$	25,36	11,87	7,35	5[illegible]	[illegible],70	1,99	1,40	0,859	0,563
Q/(LCV)	0,0801	0,159	0,237	0,[illegible]	[illegible],466	0,542	0,616	0,688	0,721
$\bar{C}_a$	19,06	18,81	18,94	1[illegible]	[illegible]2,5	25,64	31,6	46,43	67,75

Example

Interpolating, one finds

- relative eccentricity: ε = 0.733
- angle: ϕ = 41.3°
- friction number: $f_a \dfrac{R}{C} = 1.794$
- dimensionless moment: $\overline{C}_a = 27.63$

From these values one deduces the dimensional values

- $h_{min} = c(1 - \varepsilon)$ = 20 μm
- C_a = 6.9 Nm
- Q = 67 10^{-6} m^3/s

Journal bearings conclusion

Conclusions pour les paliers lisses

- Analytical solutions for ∞ long and ∞ short bearings only
- Bearing characteristics for other bearing dimensions in tables
- Tables based on dimensionless numbers, provide eccentricity, friction, flow rate…
- Careful with grooved bearings.