M8: Building design: multidisciplinary approach

Hydraulics

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Hydraulics-pressurized flows

Scope of the course/project

-Design and size a water distribution network (water supply/heating networks)

-Calculate flow rate distribution in a water supply network with respect of water demand and operating pressures

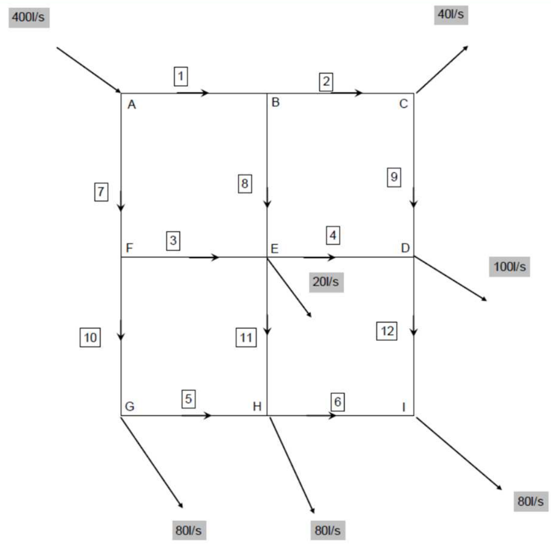
… develop the ability to implement design methods into algorithms (using Matlab)

Course overview

-Scope

-Reminders and exercise (~2-3h)

➡ Calculate the flow distributions in a meshed-type network

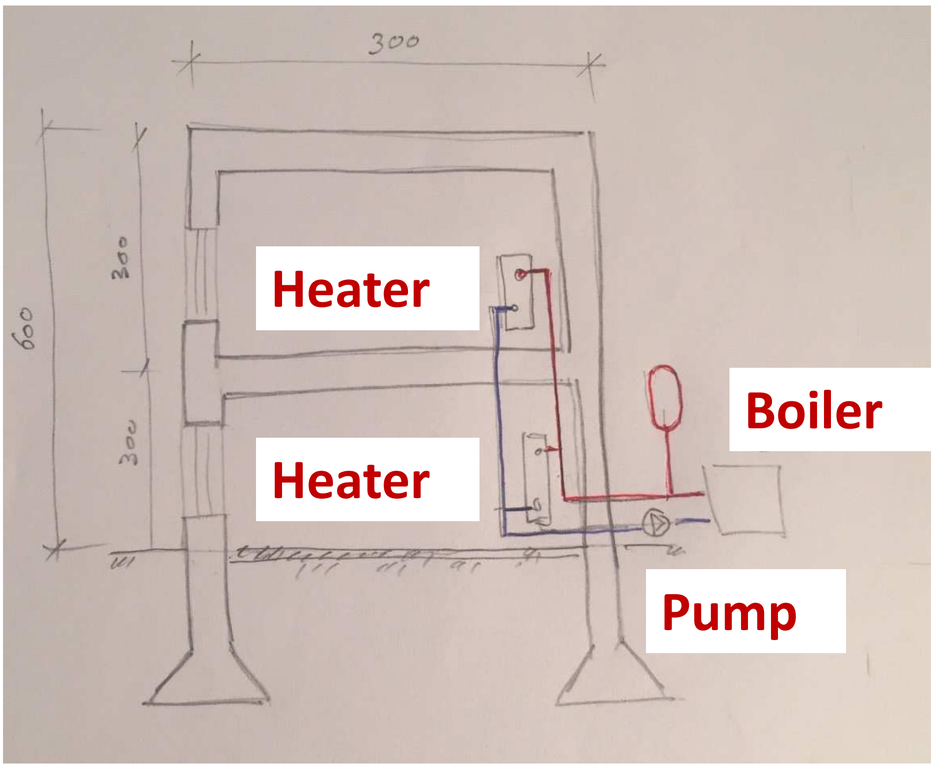


-Project

Design and size a water distribution network for the heating (2 storey building)

Project Scope

Design and size a water distribution network for the heating of two storey-building (pump, pipes…) according to the heating demand



Reminders

- Fluids properties
- Conservation equations
- Head losses
- Pipe networks
- Centrifuge pump and operating point of a system

Reminders: fluids properties

Fluid/solid

React differently to shear stress: solid deform only a finit amount, whereas fluids deform continuously until the stress is removed

Liquid/gas

Both liquids and gases are fluids: differ strongly in both degree of compressibility and formation of a free surface (interface) for the liquid

Liquids: incompressible, gases range from compressible to incompressible

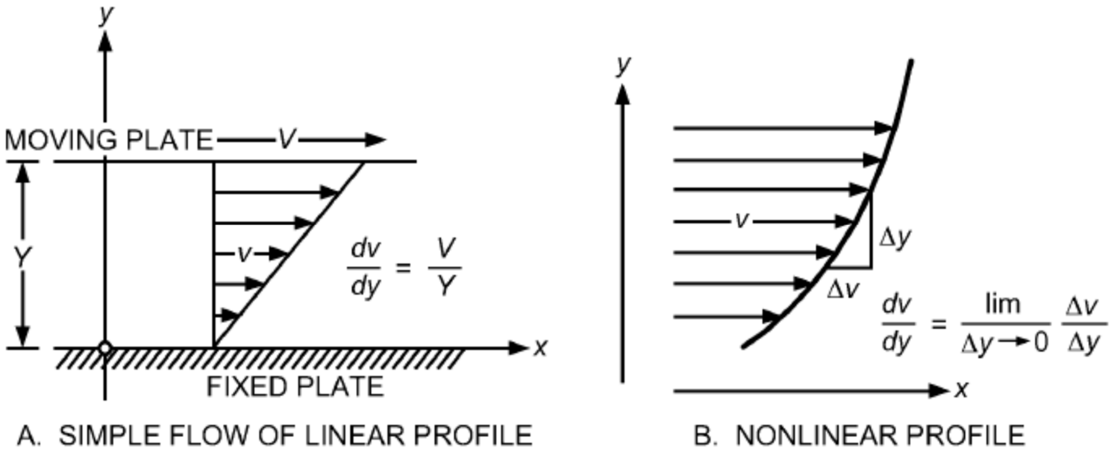
Density (at standard conditions: 20°C and 101.325 kPa-sea level atmospheric pressure):

$\rho_{water} = 998\ kg/m^3$ $\rho_{air} = 1.21\ kg/m^3$

Reminders: fluids properties

Viscosity: Resistance of adjacent fluid layers to shear.

μ is the absolute or dynamic viscosity of the fluid.



In **Newtonian fluids**, the rate of deformation proportional to the shearing stress

$$F/A = \mu(V/Y)$$

In **non Newtonian fluids**, velocity and shear stress may vary across the flow field

$$\tau = \mu \frac{dv}{dy}$$

Reminders: fluids properties

Viscosity: Resistance of adjacent fluid layers to shear.

μ is the absolute or dynamic viscosity of the fluid.

Absolute viscosity has dimensions of force × time/length^2

μ_{water} = 1.01 10^{-3} N.s/m^2

μ_{air} = 18.1 10^{-6} N.s/m^2

(At standard indoor conditions)

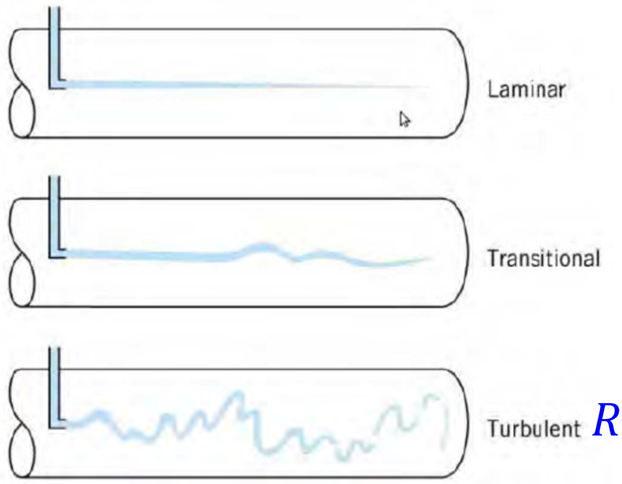
***Kinematic viscosity ν** is sometimes used in lieu of dynamic viscosity **μ**

$$\boxed{\nu = \mu/\rho}$$

ν_{water} = 1.01 10^{-6} m^2/s^{-1}

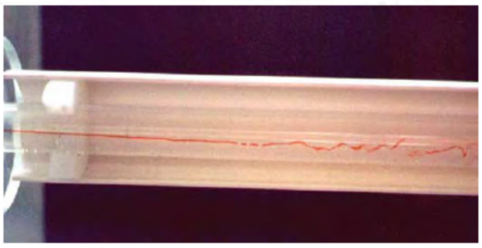
ν_{air} = 15. 10^{-6} m^2/s^{-1}

Reminders: fluids properties



$Re < 2000$

$Re > 5000$



Reynolds number:

$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

(dimensionless ratio)

$$\boxed{Re = \frac{\mathrm{VL}}{\upsilon}}$$

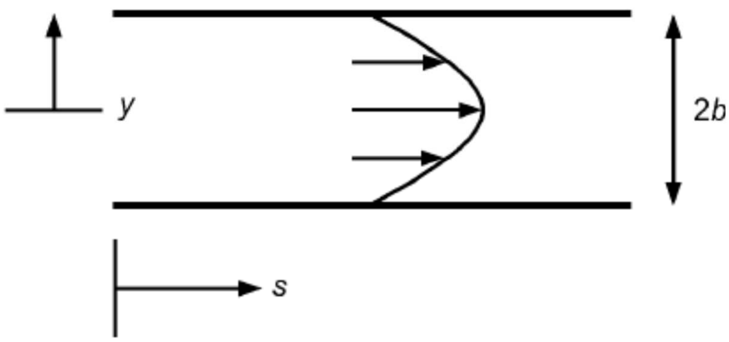
L is a characteristic length scale

ν is the kinematic viscosity of the fluid.

Reminders: fluids properties

Laminar vs turbulence flows

For steady, fully developed **laminar flow** between 2 parallel plates:



$$v = \left(\frac{b^2 - y^2}{2\mu}\right)\frac{dp}{ds}$$

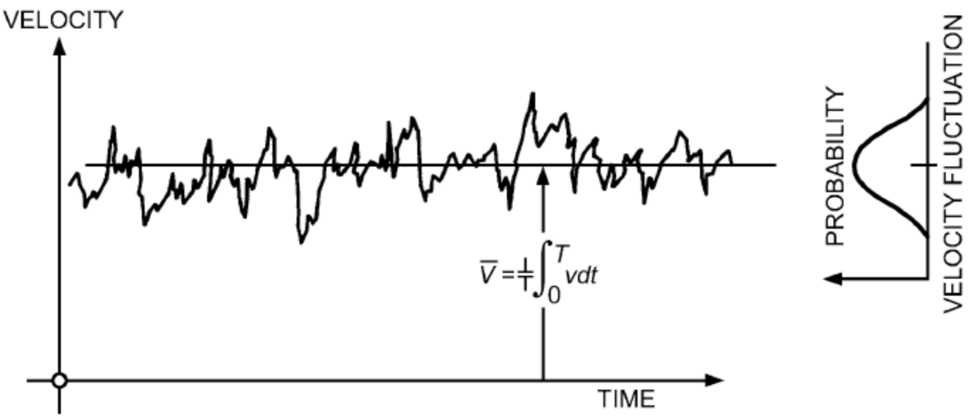
The resulting parabolic velocity profile in a wide rectangular channel is commonly called Poiseuille flow.

Maximum velocity occurs at the centerline (y = 0), and the average velocity U is 2/3 of the maximum velocity.

Reminders: fluids properties

Laminar vs turbulence flows

Fluid flows are generally **turbulent**, involving random perturbations or fluctuations of the flow (velocity and pressure):



Turbulence can be quantified statistically.

The velocity used is the **time-averaged velocity**.

In flow through pipes, tubes, and ducts, the characteristic length scale is the hydraulic diameter D_h, given by

$$\boxed{D_h = 4A/P_w}$$

A: cross-sectional area of the pipe, Pw: wetted perimeter

Reminders: mass conservation

Continuity equation

$$\dot{m} = \int \rho v \, dA = \text{constant}$$

where **$\dot{m}$ is the mass flow rate (kg/s)** across the area normal to flow, *V* is fluid velocity normal to differential area *dA*, and ρ is fluid density.

When flow is incompressible (ρ = constant) in a pipe:

$$\dot{m} = \rho V A \qquad Q = \dot{m}/\rho = \text{U x A=cst}$$

Where U, the average velocity is defined as follows:

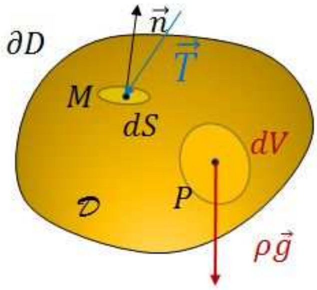
$$U = \frac{Q}{A} = \frac{1}{A}\iint_S \vec{V}.\vec{n}\, dA$$

Q is **volumetric flow rate (m^3/s)**

Reminders: momentum conservation

D: material domain

$\vec{n}$ the external normal



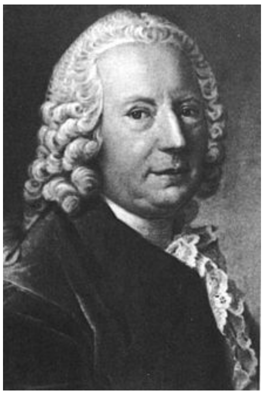
Momentum principle :

$$\sum \vec{F}_{ext} = \frac{d(m\vec{V})}{dt}$$

$\boldsymbol{m\vec{V}}$ is the momentum.

with $\mathrm{m} = \iiint_D \rho dV$

Reminders: energy conservation (Bernoulli equation)



The Bernoulli equation derives from the **conservation of momentum** and express the **conservation of energy** along a streamline;

The change in energy content ΔE (<u>per unit mass</u>) of flowing fluid is a result of the work (per unit mass) done on the system (heat absorbed or rejected neglected).

Fluid energy is composed of kinetic, potential (elevation z, pressure) energies. Per unit mass of fluid, the relation of the energy change between two sections of the system is

1700-1782

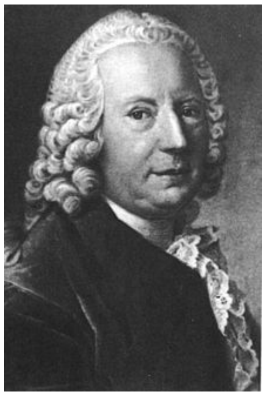
$$\Delta E_{mechanical} = \Delta\left(gz + \frac{P}{\rho} + \frac{1}{2}V^2\right) = E_{ext}$$

E_{ext} is the external work from a fluid machine (positive for a pump or blower)

V is the velocity along a streamline

Reminders: energy conservation (Bernoulli equation)

If there is no viscous frictional forces, no external work (E_{ext}=0), no heat transfer, it comes:



$$gz + \frac{P}{\rho} + \frac{1}{2}V^2 = B = cst$$

B is the Bernoulli constant

Energy (per unit mass) is conserved along a streamline.

1700-1782

Alternative form is:

$$z + \frac{P}{\rho g} + \frac{1}{2g}V^2 = \frac{B}{g} = H$$

Energy per unit weight **(Head)**

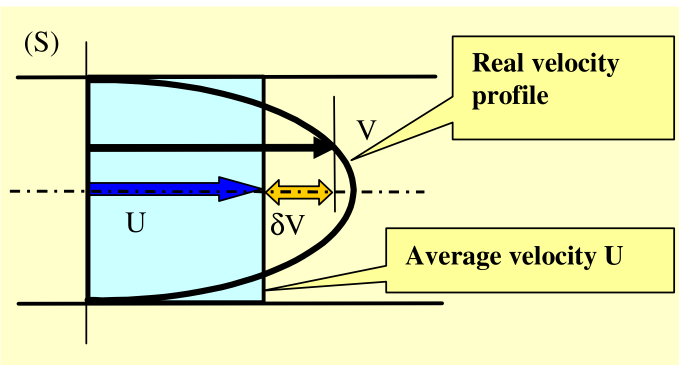
Reminders: energy conservation (Bernoulli equation)

When the **velocity V varies significantly across the cross section**, the kinetic energy term has to be correctly calculated:

$$z + \frac{P}{\rho g} + \frac{\alpha}{2g}U^2 = H$$

The velocity used is the **average velocity (U),** so that a kinetic energy factor α (α > 1) has to be calculated to take into account true kinetic energy of the velocity profile.

α the expresses the ratio of the true kinetic energy of the velocity profile to kinetic energy of the average velocity :



For laminar flow in a pipe, α = 2

For turbulent flow, α ≈ 1

(cylindrical pipes)

Reminders: energy conservation (Bernoulli equation)

Real fluid flow

If there are now **viscous frictional forces** and external work (pump), conversion of mechanical energy to internal energy may be expressed as a loss ΔE_L.

The change in the Bernoulli equation between stations 1 and 2 along the conduit can be expressed as

$$\left(gz + \frac{P}{\rho} + \frac{\alpha}{2}U^2\right)_1 + E_{pump} - \Delta E_L = \left(gz + \frac{P}{\rho} + \frac{\alpha}{2}U^2\right)_2$$

Alternative form is (energy by unit weight):

$$\left(z + \frac{P}{\rho g} + \frac{\alpha}{2g}U^2\right)_1 + H_{pump} - \Delta H_L = \left(z + \frac{P}{\rho g} + \frac{\alpha}{2g}U^2\right)_2$$

ΔH_L is the head loss through friction

Reminders: Frictional head losses

In real-fluid flow, a frictional shear occurs at bounding walls, gradually influencing flow further away from the boundary. A lateral velocity profile is produced and flow energy is converted into heat (fluid internal energy), which is generally unrecoverable (a loss). This loss in fully developed conduit flow is evaluated using the **Darcy-Weisbach equation**:

$$\boxed{\Delta H_L = f\,\frac{L}{D}\,\frac{U^2}{2g}}$$

where L is the length of conduit of diameter D and

f is the **Darcy-Weisbach friction factor**.

Frictionnal head losses (Determination of friction coefficient f)

- **Phase 1**: Poiseuille equation for a laminar regime with Re<2000, with a slope $f = \frac{64}{Re}$
- **Phase 3: Blasius law (Re<10^5)**: $f = \frac{0.316}{Re^{1/4}}$

ε/D=Relative roughness

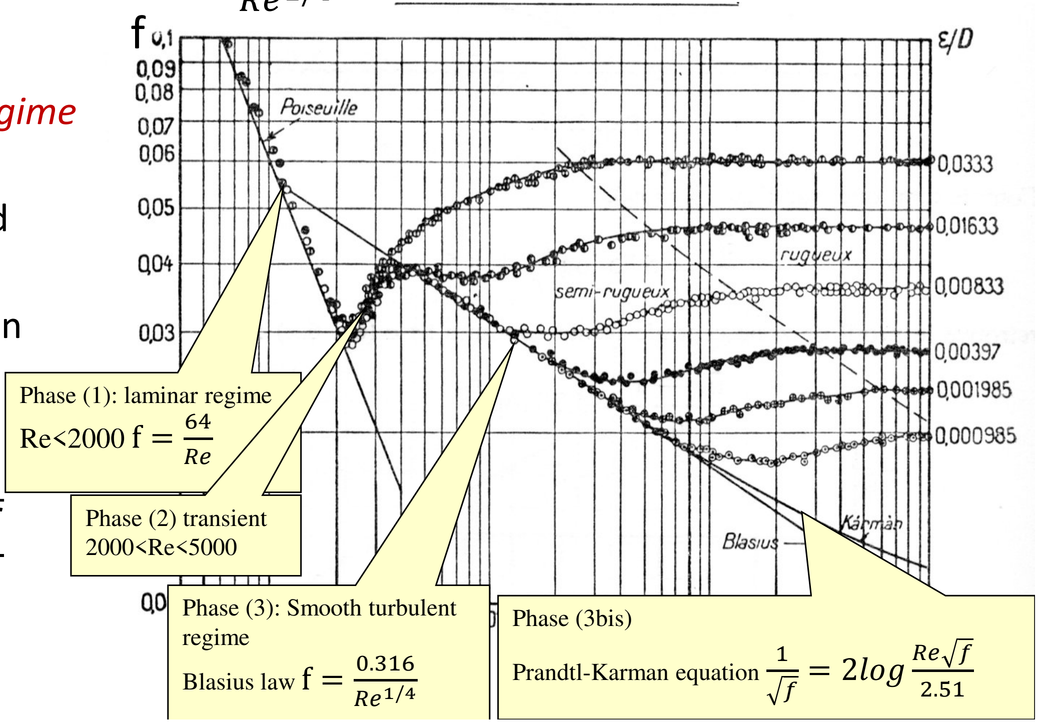
Hydraulically smooth regime

For the smallest ε/D and Re > 10^5 :

Prandtl-Karman equation better fits (phase 3bis)

Phase (3bis)

$$\frac{1}{\sqrt{f}} = 2log\frac{Re\sqrt{f}}{2.51}$$



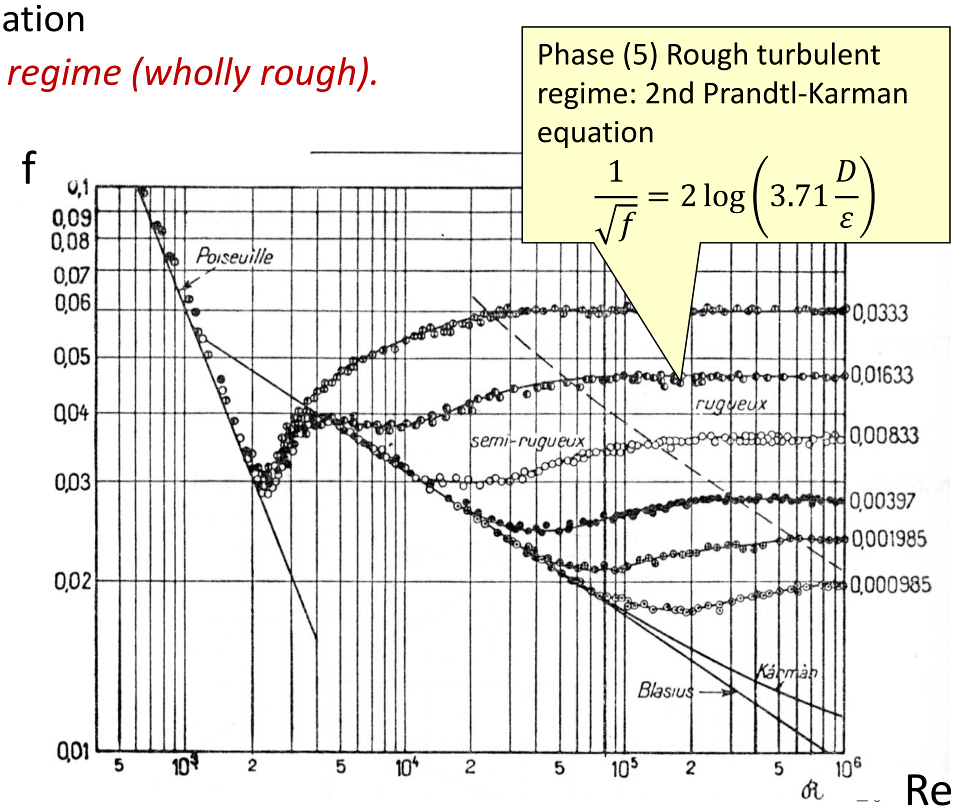
Frictionnal head losses (Determination of friction coefficient f)

- **Phase 5**, friction factor depends only on **relative roughness ε /D**;

2nd Prandtl-Karman equation

Hydraulically rough flow regime (wholly rough).

- $\frac{1}{\sqrt{f}} = 2\log 3.71\frac{D}{\varepsilon}$



Frictionnal head losses (Determination of friction coefficient f)

Phase 4: friction coefficient depends both on **Re and ε/D**.

Colebrook formula

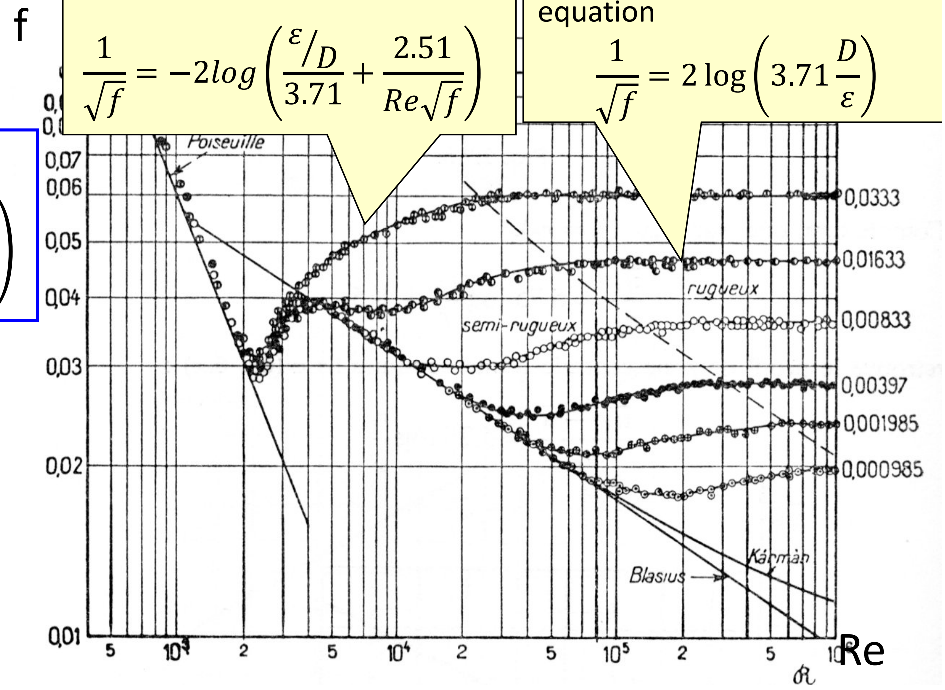
$$\frac{1}{\sqrt{f}} = -2log\left(\frac{\varepsilon/_D}{3.71} + \frac{2.51}{Re\sqrt{f}}\right)$$

Phase (4): Colebrook formula

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{\varepsilon/_D}{3.71} + \frac{2.51}{Re\sqrt{f}}\right)$$

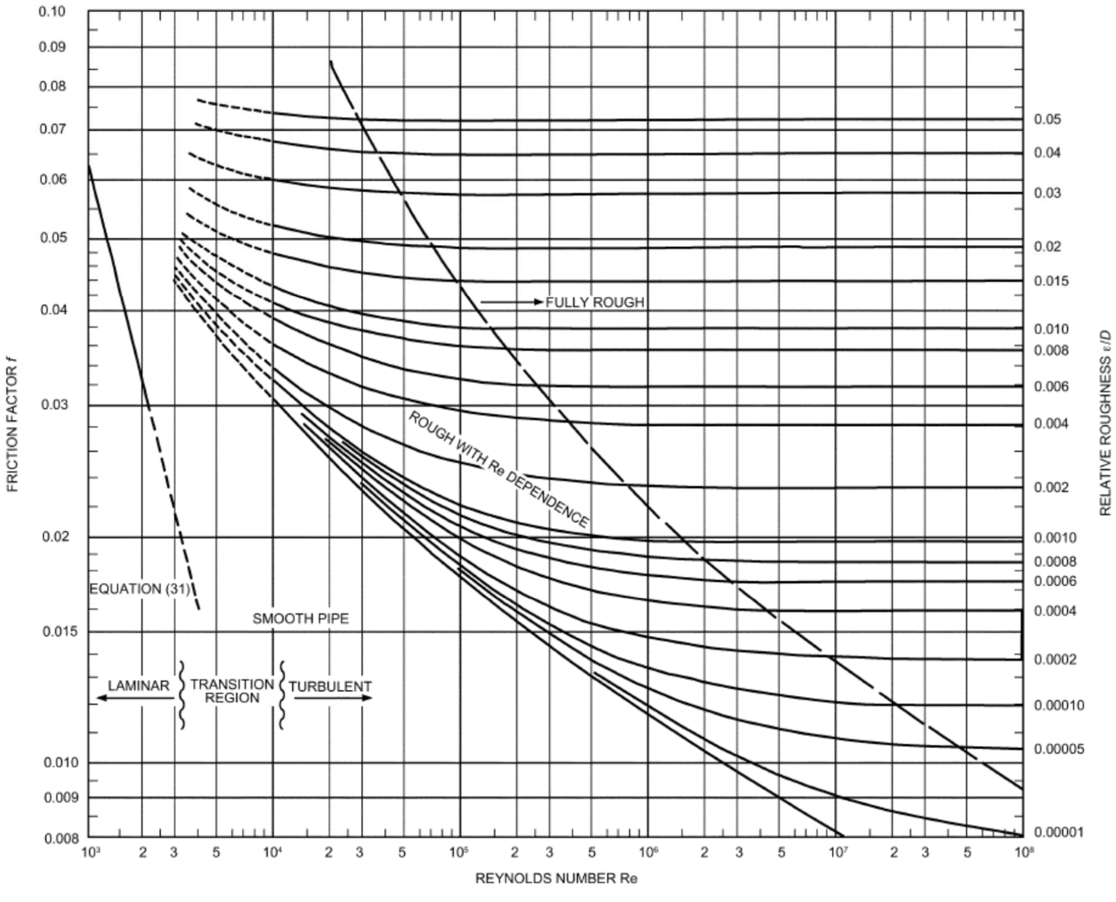
Phase (5) Rough turbulent regime: 2nd Prandtl-Karman equation

$$\frac{1}{\sqrt{f}} = 2\log\left(3.71\frac{D}{\varepsilon}\right)$$



Frictionnal head losses (Determination of friction coefficient f)

Moody diagram



Frictionnal head losses (Industrial pipes)

Other empirical formulae (not exhaustive)

Authors and conduit type	**empirical formulae** notation: U(m/s), D(m), Q (m^3/s) $j = \frac{\Delta H}{L}$	Comments
Chézy cast iron	$U = C\sqrt{\frac{D.j}{4}}$ then $\Delta H = \frac{U^2}{2.g}\frac{L}{D}\left(\frac{8.g}{C^2}\right)$	$40 < C < 100$ then $0,008 < \frac{8.g}{C^2} < 0,022$
Darcy cast iron 0,05 <D(m)<0,5	$\frac{D.j}{4} = \left(\alpha + \frac{\beta}{D}\right).U^2$ then $\Delta H = \frac{U^2}{2.g}\frac{L}{D}\left(8.g\left(\alpha + \frac{\beta}{D}\right)\right)$	$\alpha = 0,000507$ $\beta = 0,00001294$ then $0,04 < 8.g\left(\alpha + \frac{\beta}{D}\right) < 0,06$
Lévy cast iron 0,08 <D(m)<3	$U = \beta.\sqrt{\frac{D.j}{2}.\left(1 + 3\sqrt{\frac{D}{2}}\right)}$ then $\Delta H = \frac{U^2}{2.g}\frac{L}{D}\left(\frac{4.g}{\beta^2\left(1+0,75\sqrt{D}\right)}\right)$	$0,013 < \frac{4.g}{\beta^2\left(1+0,75\sqrt{D}\right)} < 0,078$ $\beta = 36,4$ (fonte neuve) $\beta = 20,5$ (fonte en service)
Flamant and Maning cast iron D(m)<1,3	$D.j = 0,00092.\sqrt[4]{\frac{U^7}{D}}$	cast iron in service

Reminders: Head losses/ Minor losses

Bends, entrances, exits, contractions, enlargements…

We treat the entire effect of a disturbance as if it occurs at a single point in the flow direction.

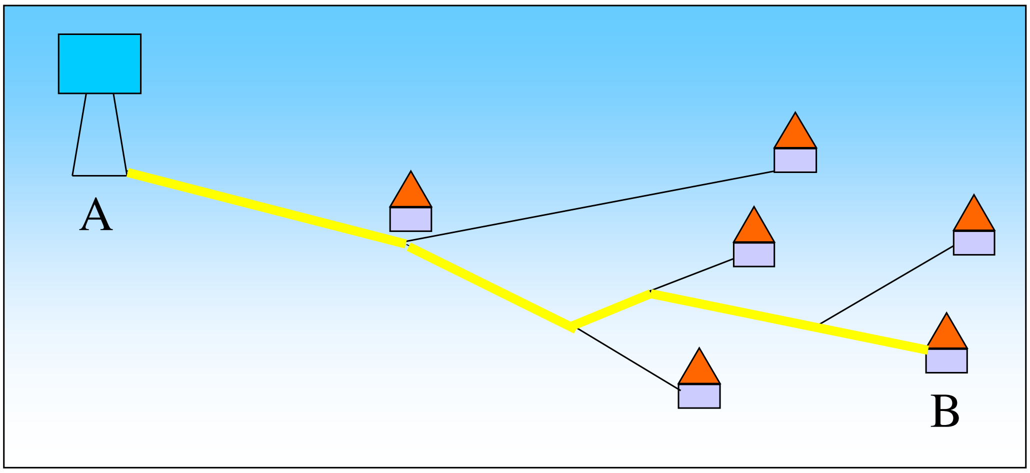
By treating these losses as a local phenomenon, they can be related to the velocity by the **loss coefficient K**:

$$\boxed{\Delta H = K\frac{U^2}{2g}}$$

K depends on the particularity of the fittings

Reminders: Pipe networks

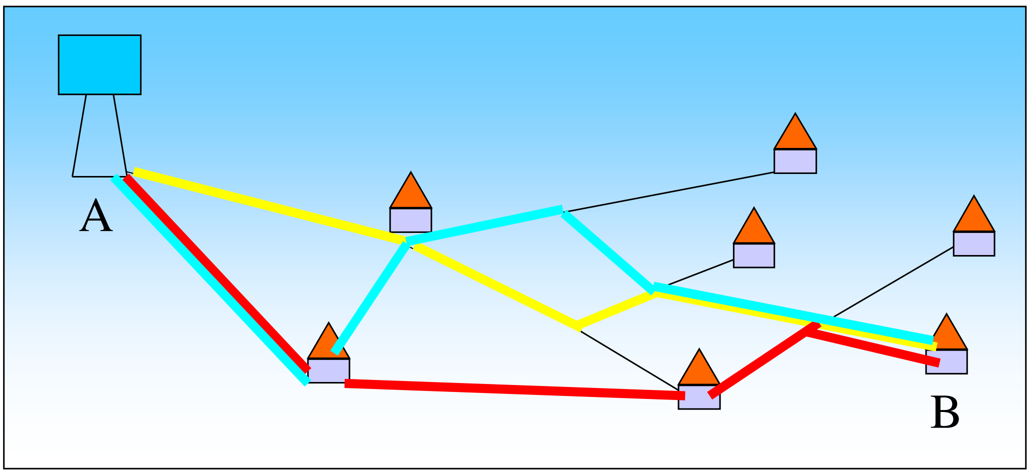
Branched network



Hydraulic characteristic	ZONE
An <u>only path</u> from A to B	**Rural area**

Reminders: Pipe networks

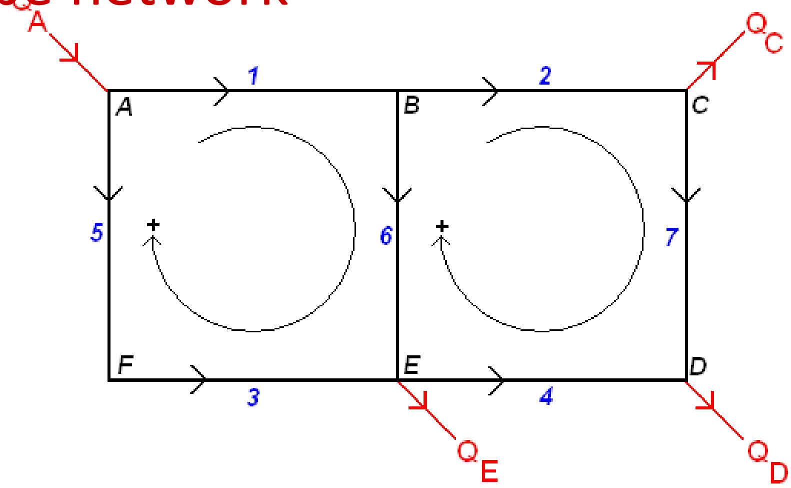
Meshed-type network (several loops)



Hydraulic characteristic	ZONE
<u>Several possible paths</u> from A to B	**Urban**

Reminders: Pipe networks

Meshed-type network

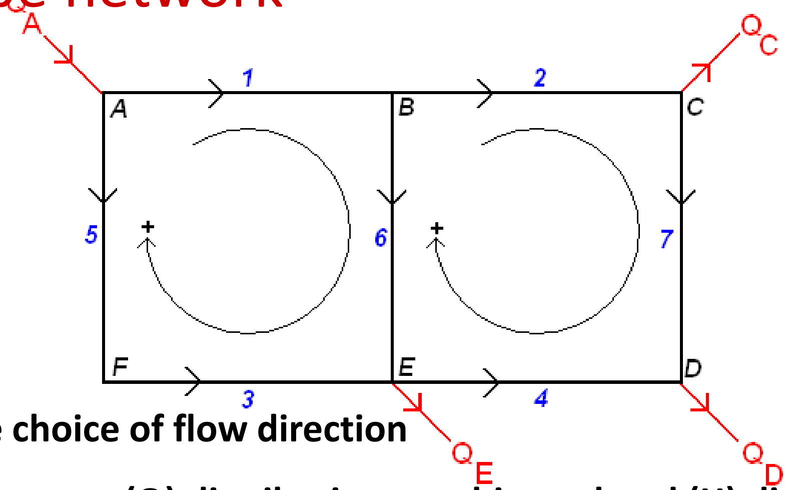


Unknown= Flow rates in sections/Nodes heads

Solution = ?

Reminders: Pipe networks

Meshed-type network



-Hypothesis on the choice of flow direction

-Arbitrary initial flow rate (Q) distribution vs arbitrary head (H) distribution

-Mass conservation for each node

-Head losses around each loop ($\sum \Delta H = 0$)

Unknown= Flow rates in sections/Nodes heads

Solution = ?

Exercise: computation of flow rates/heads in a meshed-type network

-Hypothesis on the choice of flow direction

-Arbitrary initial flow rate (Q) distribution vs Arbitrary head (H) distribution

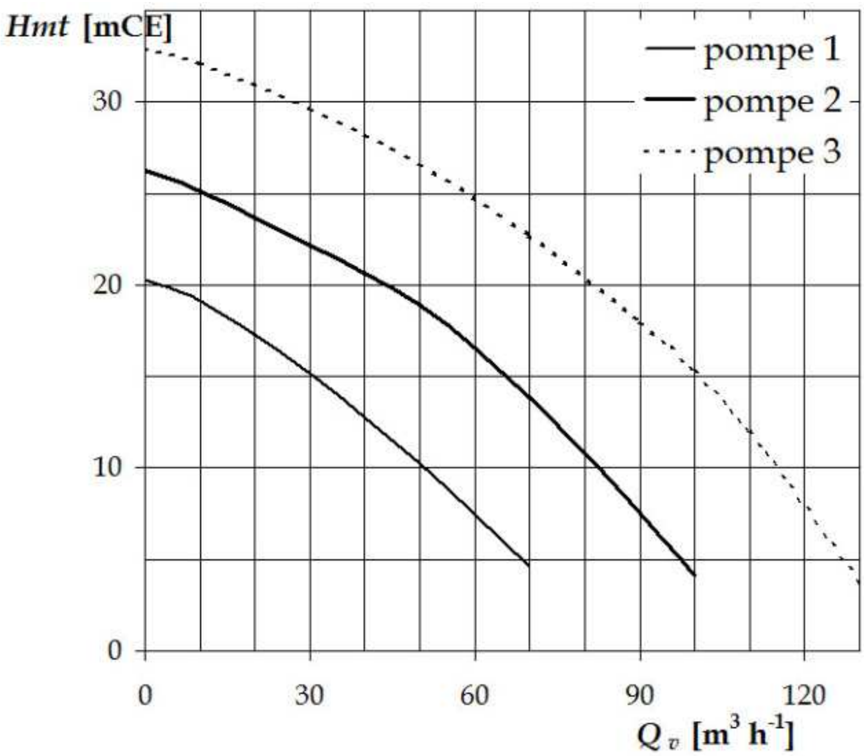
-Mass conservation for each node

-Head losses around each loop $(\sum \Delta H = 0)$

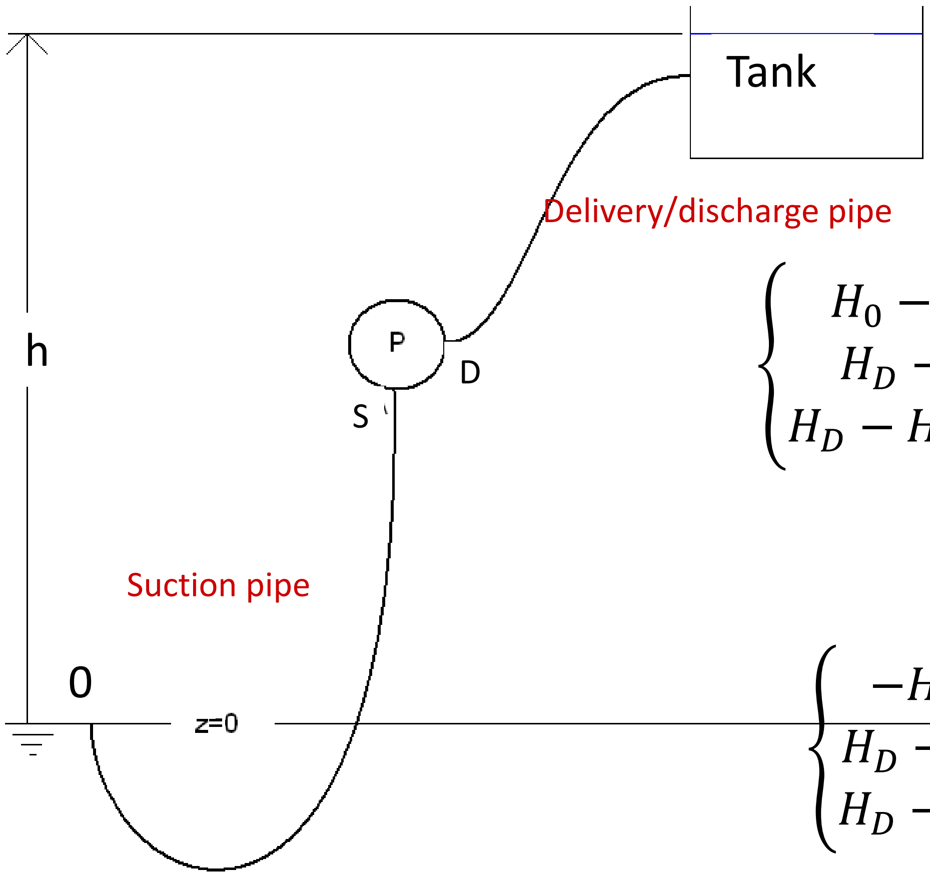
Unknown= Flow rates in section/Nodes heads

Solution = ?

Reminders: centrifuge pump



Reminders: operating point of a system



$$\begin{cases} H_0 - H_S = \Delta H_S(Q) \\ H_D - H_S = H_n(Q) \\ H_D - H_{Tank} = \Delta H_D(Q) \end{cases}$$

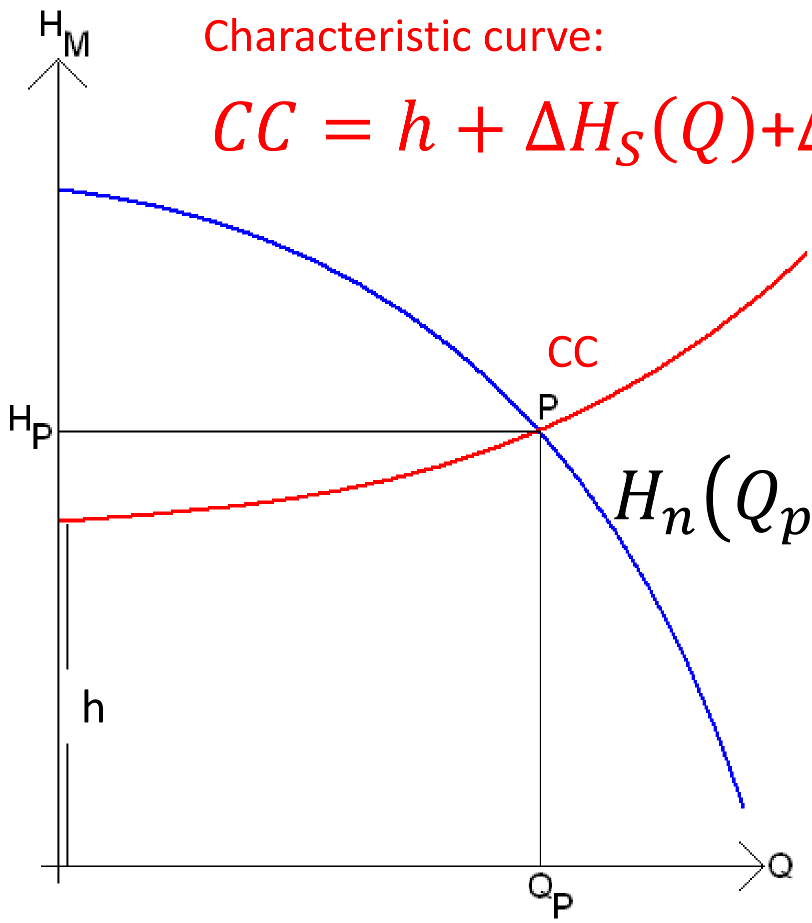
$$\Downarrow$$

$$\begin{cases} -H_S = \Delta H_S(Q) \\ H_D - H_S = H_n(Q) \\ H_D - h = \Delta H_D(Q) \end{cases}$$

Reminders: operating point of a system

Characteristic curve:

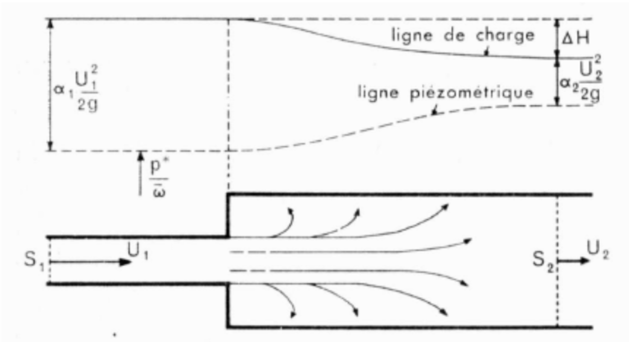
$$CC = h + \Delta H_S(Q) + \Delta H_D(Q)$$



$$H_n(Q_p) = H_n = CC(Q_p)$$

Appendix : Head losses/ Minor losses

Exemple des changements de section

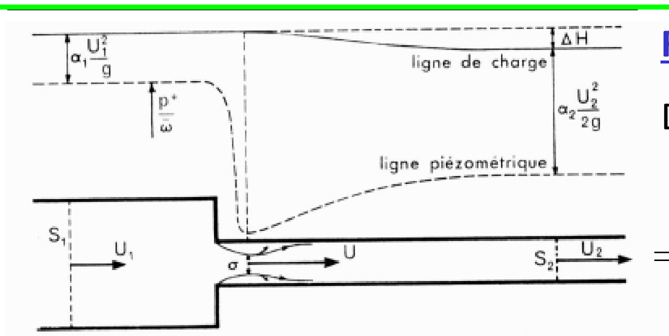


Elargissement brusque

D'après **Borda**

$$\boxed{\Delta H = \frac{(U_1 - U_2)^2}{2.g}}$$

$$\Rightarrow \Delta H = \frac{U_1^2}{2.g}.\left(1 - \frac{U_2}{U_1}\right)^2 = \frac{U_1^2}{2.g}.\left(1 - \frac{S_1}{S_2}\right)^2 \quad \text{soit} \quad \boxed{K = \left(1 - \frac{S_1}{S_2}\right)^2}$$



Rétrécissement brusque

D'après **Borda**

$$\boxed{\Delta H = \frac{(U - U_2)^2}{2.g}}$$

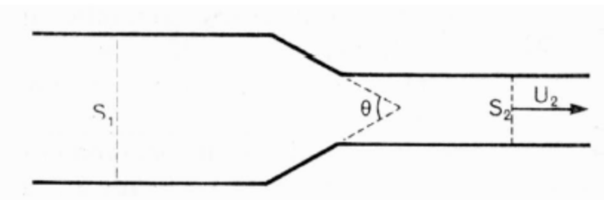
$$\Rightarrow \Delta H = \frac{U_2^2}{2.g}.\left(\frac{U}{U_2} - 1\right)^2 = \frac{U_2^2}{2.g}.\left(\frac{S_2}{\sigma} - 1\right)^2 \quad \text{soit} \quad \boxed{K = \left(\frac{1}{C_c} - 1\right)^2}$$

Dans un rétrécissement brusque **la perte de charge se localise en aval de la section contractée** telle que $\sigma = C_c . S_2$

avec $C_c = 0{,}63 + 0{,}37.\left(\frac{S_2}{S_1}\right)^3$

Appendix : Head losses/ Minor losses

Exemple des changements de section

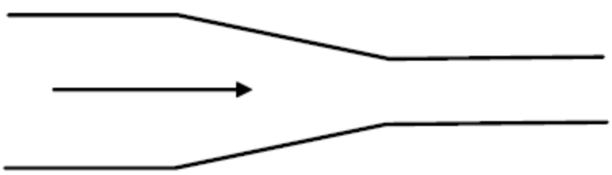


Rétrécissement en biseau

$$K = \left(\frac{1}{C_c} - 1\right)^2 .\sin\theta \quad \text{pour} \quad \theta < \frac{\pi}{2}$$

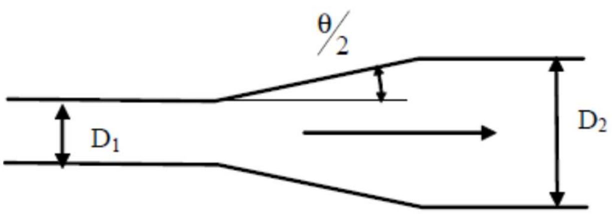
$$K = \left(\frac{1}{C_c} - 1\right)^2 \quad \text{pour} \quad \theta > \frac{\pi}{2}$$

avec Cc donné par la formule précédente



Convergent

Si le convergent est court et bien tracé, la perte de charge peut être négligée dans la plupart des applications



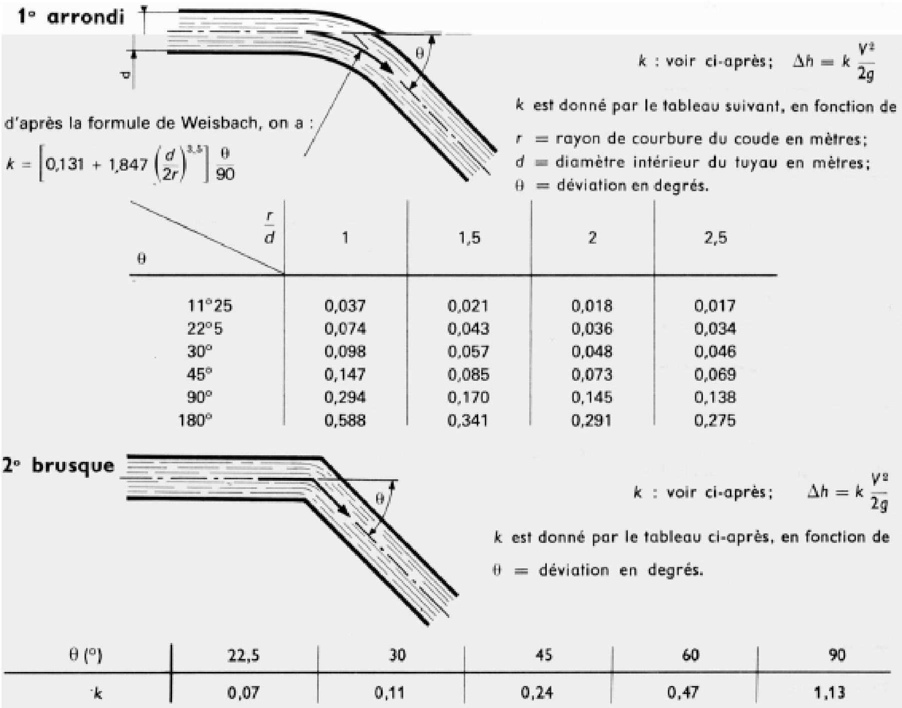
Divergent

K peut varier notablement avec l'angle d'ouverture θ. K est approximé par:

$$K = 3{,}2.\left(\operatorname{tg}\frac{\theta}{2}\right)^2.\left(1 - \left(\frac{D_1}{D_2}\right)^2\right)^2$$

Appendix : Head losses/ Minor losses

Exemple des changements de direction



1° arrondi

d'après la formule de Weisbach, on a :

$$k = \left[0{,}131 + 1{,}847\left(\frac{d}{2r}\right)^{3,5}\right]\frac{\theta}{90}$$

k : voir ci-après; $\Delta h = k\dfrac{V^2}{2g}$

k est donné par le tableau suivant, en fonction de

r = rayon de courbure du coude en mètres;
d = diamètre intérieur du tuyau en mètres;
θ = déviation en degrés.

θ \ $\frac{r}{d}$	1	1,5	2	2,5
11°25	0,037	0,021	0,018	0,017
22°5	0,074	0,043	0,036	0,034
30°	0,098	0,057	0,048	0,046
45°	0,147	0,085	0,073	0,069
90°	0,294	0,170	0,145	0,138
180°	0,588	0,341	0,291	0,275

2° brusque

k : voir ci-après; $\Delta h = k\dfrac{V^2}{2g}$

k est donné par le tableau ci-après, en fonction de

θ = déviation en degrés.

θ (°)	22,5	30	45	60	90
k	0,07	0,11	0,24	0,47	1,13

Coude arrondi

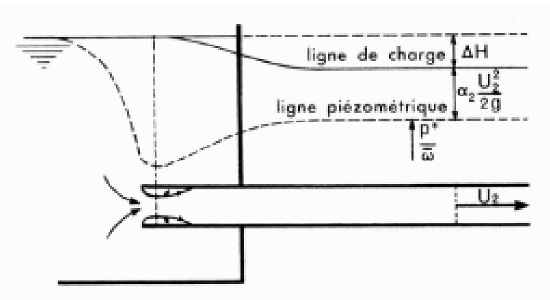
$$K = \left[0{,}13 + 1{,}847.\left(\frac{d}{2.r}\right)^{3,5}\right].\frac{\theta}{90}$$

Coude à angle vif

$$K = \sin^2\frac{\theta}{2} + \sin^4\frac{\theta}{2}$$

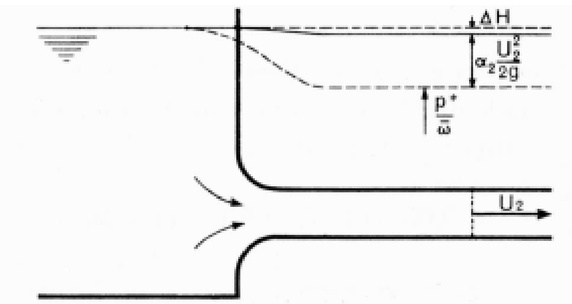
Appendix : Head losses/ Minor losses

Débouché d'une conduite dans un réservoir



Orifice de Borda

$$C_c \cong 0{,}5 \;\Rightarrow\; K \cong 1$$

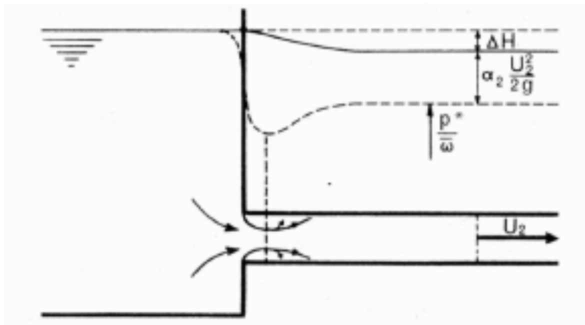


Orifice à bords arrondis et polis

$$C_c \cong 0{,}99 \;\Rightarrow\; K \cong 0$$

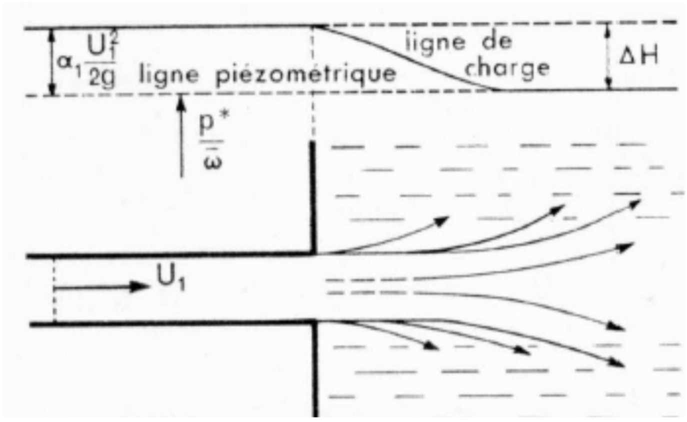
Appendix: Head losses/ Minor losses

Débouché d'une conduite dans un réservoir



Orifice à bords vifs

$$C_c \cong 0{,}6 \;\Rightarrow\; K \cong 0{,}5$$



Arrivée dans un réservoir

Il s'agit du cas particulier **de l'élargissement brusque** traité avec la formule de Borda $\Rightarrow\; K = 1$

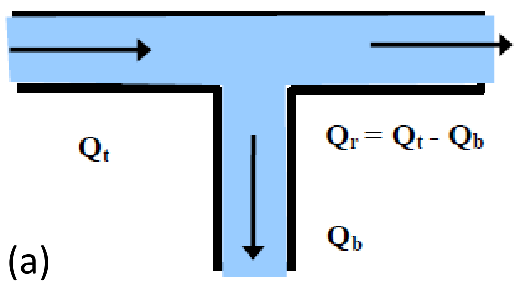
La ligne piézométrique se raccorde avec la surface libre du réservoir.

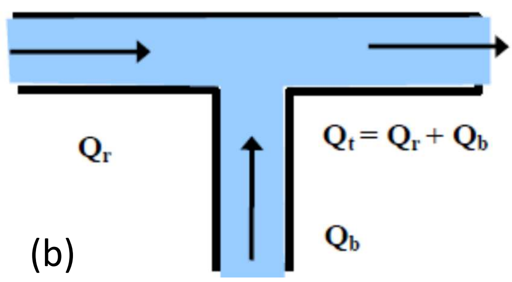
Appendix : Head losses/ Minor losses

Branchements à 90°

Des coefficients de perte de charge K_r et K_b sont respectivement affectés à la conduite rectiligne (indice r) et à la conduite de branchement (indice b) en fonction du rapport Q_b / Q_t. **Dans tous les cas la perte de charge est ensuite calculée avec la vitesse totale Vt.**

$$\Delta H_r = K_r \cdot \frac{V_t^2}{2g} \qquad \Delta H_b = K_b \cdot \frac{V_t^2}{2g}$$





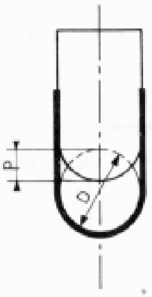
cas (a)	Q_b / Q_t	0	0,2	0,4	0,6	0,8	1
	K_r	0	0,02	0,06	0,07 à 0,15	0,15 à 0,21	0,35 à 0,4
	K_b	0,95 à 1	0,88 à 1,01	0,89 à 1,05	0,95 à 1,15	1,10 à 1,32	1,28 à 1,45

cas (b)	Q_b / Q_t	0	0,2	0,4	0,6	0,8	1
	K_r	0 à 0,04	0,17 à 0,27	0,30 à 0,46	0,41 à 0,57	0,51 à 0,60	0,55 à 0,60
	K_b			0,08 à 0,26	0,47 à 0,62	0,72 à 0,94	0,91 à 1,20

Appendix : Head losses/ Minor losses

Exemple de robinets

Robinet-vanne



Le tableau suivant donne des valeurs expérimentales moyennes de k, en fonction de

p = distance de pénétration de l'obturateur dans la section, supposée circulaire, offerte par le robinet-vanne au passage du liquide, exprimée en mètres;

D = diamètre de cette section (diamètre intérieur du robinet-vanne), en mètres.

$\frac{p}{D}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
k	0,07	0,26	0,81	2,1	5,5	17	98

Robinet à papillon

k : voir ci-après; $\Delta h = k\,\frac{V^2}{2g}$

Le tableau suivant donne des valeurs expérimentales moyennes de k, en fonction de

α = angle formé par le papillon et l'axe de la conduite, en degrés.

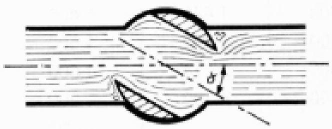
α	5	10	15	20	30	40	45	50	60	70
k	0,24	0,52	0,90	1,5	3,9	11	19	33	120	750

Appendix : Head losses/ Minor losses

Exemple de robinets

Robinet à tournant

3° Robinets à tournant



k : voir ci-après; $\Delta h = k\,\frac{V^2}{2g}$

Le tableau suivant donne des valeurs expérimentales moyennes de k, en fonction de

α = angle formé par l'axe de la lumière du boisseau — supposée à section circulaire et de même diamètre que l'intérieur du robinet — et l'axe de la conduite, en degrés.

α	5	10	15	25	35	45	55	65
k	0,05	0,29	0,75	3,1	9,7	31	110	490