Friction & Lubrication n°1

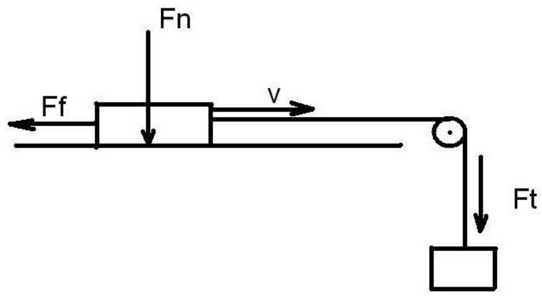
FRICTION

In this first exercise class we will study the role of friction in simple applications.

Recall: the friction "laws":

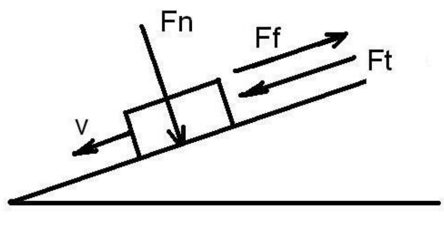
- Friction is proportional to normal load
- Friction is independent of (apparent) area of contact
- Friction is independent of velocity
- (Friction force opposes the direction of motion)

A) The static friction coefficient μ defined as: the (maximum) friction coefficient attained (just) before movement occurs.



In that case the friction coefficient μ is equal to the ratio of the tangential force Ft and the normal force Fn: μ=Ft/Fn.

B) A second way of measuring the static friction coefficient is by using an inclined surface:

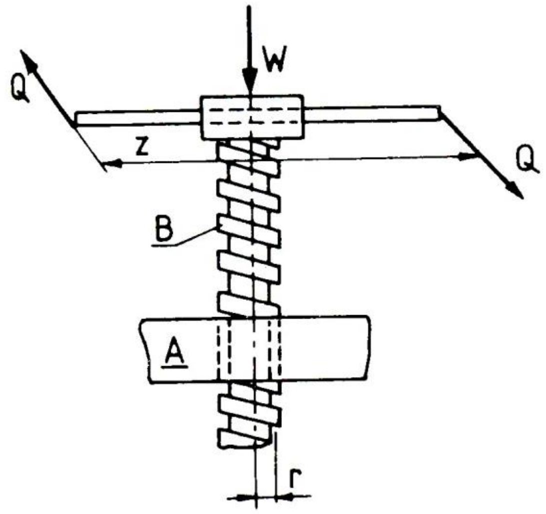


The gravity force on the mass is decomposed into two components Fn and Ft acting normal and tangential to the inclined surface. If the maximum angle (before motion) is called α, the static friction coefficient can be obtained from μ=Ft/Fn=tan(α).

C) The mechanism to be studied in this exercise is a screw jack. A load W is supported by the screw which is free to rotate. The screw is turned using a moment Q (force) times arm (z).

The pitch of the screw is p, the mean radius of the threads r and the thread angle α. Show that:

$$\tan\alpha = \frac{p}{2\pi r}$$

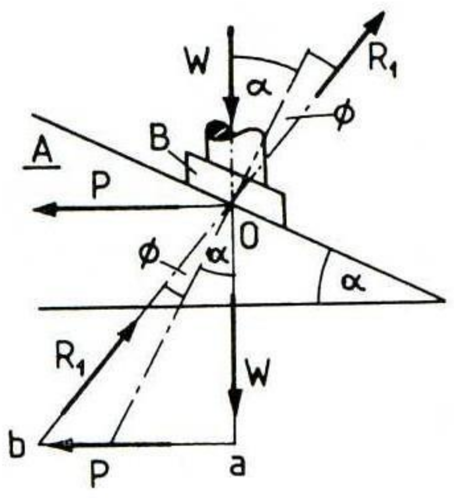


The total horizontal force is P. Show that

$$P = \frac{Qz}{r}$$

Note that all sideways forces cancel.

Show that the problem is equivalent to moving a mass W up an inclined slope with angle α.



This gives P=W tan (α+φ), where φ is the friction angle defined as the maximum angle before the mass starts to move.

The efficiency of the jack is calculated as the ratio between the useful work done and the work exerted (over one revolution).
The useful work is W.p (definition of the pitch).
The work exerted is 2πrP.

Hence the efficiency η is given by

$$\eta = \frac{Wp}{2\pi r P} = \frac{\tan(\alpha)}{\tan(\alpha+\varphi)}$$

The optimum efficiency $\eta(\alpha)$ (for a given φ) is found by differentiating η with respect to α and setting the derivative to zero.
Show that one finds

$$\alpha_{opt} = \pi/4 - \varphi/2$$

$$\eta_{opt} = \frac{\tan(\pi/4-\varphi/2)}{\tan(\pi/4+\varphi/2)}$$

For lubricated contacts, φ =10 degrees, compute the optimal angle and the optimal efficiency. However such large angles have several disadvantages and one prefers smaller angles at which the mechanism does not turn when the force P is removed. Such mechanisms are called self-locking or self-sustaining.

Give the angle α for which the system is self locking, and compute its efficiency.

Assuming the height of the nut H, and the tread width Δr, compute the mean tread pressure.

Load carrying surface $2\pi r\Delta r$ (per turn)
Number of load carrying turns H/p
Mean pressure = load/surface = $Wp/(2\pi r\Delta r\, H)$

Numerical example: car jack

W=1000 kg,
r = 1 cm
Δr=2.5 mm
p = 1 cm
H = 1 cm

Mean pressure = 64 MPa
Max pressure = 130 MPa

FROLUB – TD 2 (analyse dimensionnelle)

On s'intéresse ici à un palier incliné dont la géométrie est décrite sur la figure 1 (on suppose qu'il est infiniment large dans la direction z), entre les pièces 1 et 2. Ce palier fonctionne en mode lubrifié (à suivre dans les prochaines séances…) aussi l'espace entre les pièces, supposées rigides, est rempli d'un liquide lubrifiant de viscosité dynamique η et de masse volumique ρ. Enfin, la pièce 1 se déplace par rapport à la pièce 2 à la vitesse U suivant x, et la charge sur la pièce 1 est une force verticale W suivant y et horizontale F_T suivant $-x$ (les forces sont ici des forces par unité de largeur, donc exprimées en Nm^{-1}).

1) On imagine réaliser un essai :

- on fixe un certain nombre de paramètres *indépendants* pour fabriquer ou régler le banc d'essai (géométriques, cinématiques, chargement, matériau) ; ce sont ici les paramètres d'entrée ;
- on mesure un certain nombre de quantités (géométriques, cinématiques, chargement, matériau) ; ce sont ici les quantités de sortie.

Proposez ces paramètres et quantités dans le tableau 1.
Dans la suite, on suppose qu'on impose dans l'essai h_s et h_e. Modifiez éventuellement votre tableau en conséquence…

2) On précise l'unité SI de la viscosité dynamique : $[\eta]$ = Pa.s

a) Précisez les unités SI de tous les paramètres ou quantités mis en jeu.
b) Ecrire les quantités de sortie sous la forme d'une fonction dont vous préciserez les entrées.

3) On choisit maintenant d'utiliser comme paramètres de base : b, U et ρ

a) Montrez qu'avec ces paramètres de base, on peut retrouver les 3 unités de base du système SI (m, kg, s).
b) Avec l'analyse dimensionnelle, simplifiez au maximum les lois précédentes.
c) Une autre expression pour la force de frottement est la suivante ; est-elle compatible avec les précédentes ?

$$F_T/(\rho U^2 b) = g(\ h_e/h_s\ ,\ h_s/b\ ,\ \eta/(\rho U b)\ )$$

4) On a fait des essais avec b = 3 cm, $\Delta h = h_e - h_s$ = 0,1 mm, η = 0,1 Pa.s, ρ = 800 kg.m^{-3}, en faisant varier U de 1 m/s à 10 m/s et h_s de 0,1 mm à 1 mm. Pour chaque valeur de U et h_s, on mesure, en autres, F_T.

a) Pour une valeur fixée de h_s, on peut ainsi tracer F_T en fonction de U. Un expérimentateur avisé propose de tracer $F_T/(\rho U^2 b)$ en fonction de $z = \eta/(\rho U b)$. Est-ce judicieux ?
b) Que remarquez-vous à partir des résultats expérimentaux tracés sur la figure 2 pour la plage de valeurs testées ? Simplifiez en conséquence l'expression précédente donnant F_T.
c) Des essais plus délicats à réaliser peuvent être utilisés : on mesure F_T à h_e/h_s et U constants, en faisant varier h_s. Quel dispositif expérimental permettrait d'obtenir ces résultats ? Les résultats obtenus ont permis de voir que F_T dépend alors linéairement de $(h_s/b)^{-2}$. Simplifiez alors encore en conséquence l'expression précédente donnant F_T.
d) Enfin, des essais à h_s et U fixés, en faisant varier h_e/h_s, permettent de tracer $F_T/(\rho U^2 b)$ en fonction de h_e/h_s sur la figure 3. Comparer à la dernière expression obtenue.

5) Ayant fait ces essais sur une maquette, on souhaite maintenant prévoir l'effort de frottement F_T pour une solution technologique particulière, utilisant une autre longueur de palier, et un autre fluide : b = 10 cm, η = 0,02 Pa.s, ρ = 780 kg.m^{-3}, et en conditions de fonctionnement telles que h_s = 0,05 mm, h_e = 0,15 mm.

a) Comment peut-on y arriver ? Donnez la valeur de la force de frottement prévue.

b) Pourriez-vous proposer un abaque encore plus judicieux que celui de la figure 3, permettant de répondre plus facilement à une telle question ?

FROLUB – TD 2 (dimensional analysis)

An inclined slider with its geometry is described by figure 1 (assuming infinite width along z), is formed by parts 1 and 2. This bearing is fully lubricated (see later classes…) the spacing between the two parts is filled with a lubricating liquid with a dynamic viscosity denoted η and a density denoted ρ. Finally, part 1 moves with respect to part 2 with the velocity U along x, and load is applied on part 1 through a vertical force W and a horizontal force F_T (the forces are forces per unit width, i.e. their unit is Nm^{-1}).

1) We perform a test such that:
- several *independent* parameters are fixed for designing or tuning the test rig (geometry, kinematics, load, material); these are input parameters;
- several quantities are measured (geometry, kinematics, loading, material); these are output quantities.

Identify these parameters and quantities in table 1.
In the following section, we assume that the test prescribes the values of h_s and h_e. Change your table accordingly if necessary…

2) We recall the SI unit of dynamic viscosity: $[\eta]$ = Pa.s
   a) Give the SI units of all parameters and quantities you are using.
   b) Detail the output quantities as a function of the input parameters.

3) We now choose as basic parameters: b, U and ρ
   a) Check that these 3 parameters allow you to describe the 3 basic units of SI (m, kg, s).
   b) Using a dimensional analysis, simplify as much as possible the previous physical laws.
   c) Another expression for the friction force is outlined below; is it consistent with the previous one?

$$F_T/(\rho U^2 b) = g(\ h_e/h_s\ ,\ h_s/b\ ,\ \eta/(\rho U b)\ )$$

4) Tests are performed with b = 3 cm, $\Delta h = h_e - h_s$ = 0.1 mm, η = 0.1 Pa.s, ρ = 800 $\mathrm{kg.m^{-3}}$, letting U vary from 1 m/s to 10 m/s and h_s from 0.1 mm to 1 mm. For each value of U and h_s, one measures F_T.
   a) For a fixed value of h_s, one can plot F_T as a function of U. A smart experimenter suggests to plot $F_T/(\rho U^2 b)$ as a function of $z = \eta/(\rho U b)$. Is this clever?
   b) What do you observe concerning the experimental results shown in figure 2 for the test range? Simplify the equation found previously for F_T.
   c) More difficult tests can be performed: one measures F_T while h_e/h_s and U are held constant, and h_s changes. Can you imagine a device allowing these tests? The obtained results allow one to show that F_T is linearly dependent on $(h_s/\mathrm{b})^{-2}$. Simplify the previous expression giving F_T accordingly.
   d) Finally, tests with fixed h_s and U values, while h_e/h_s varies, allow one to plot $F_T/(\rho U^2 b)$ as a function of h_e/h_s (figure 3). Compare with the last obtained expression.

5) These tests were performed on a test rig, we now wish to predict the friction force F_T in a particular application, with different dimensions and lubricant: b = 10 cm, η = 0,02 Pa.s, ρ = 780 $\mathrm{kg.m^{-3}}$, and with operating conditions such that h_s = 0,05 mm, h_e = 0,15 mm.
   a) How can one predict the friction force? Give the predicted value.
   b) Can you suggest an even better suited abacus, replacing figure 3, and allowing an easier way to answer the current question?

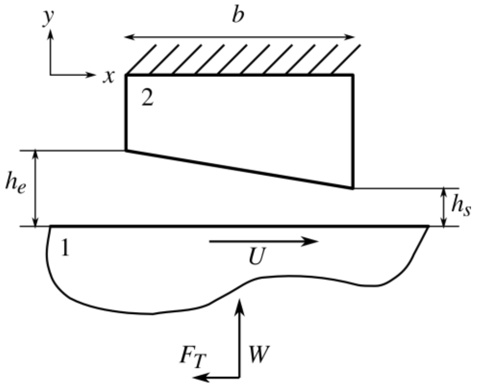


Figure 1. Principe du dispositif expérimental / sketch of the experimental device

	entrées / inputs	sorties / outputs
géométrie / geometry cinématique / kinematics chargement / load matériau / material		

Tableau 1. Entrées et sorties de l'essai / input and ouput of the test rig

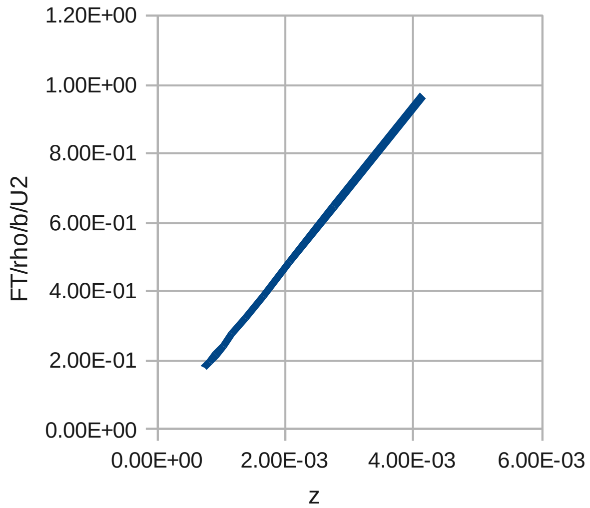


Figure 2. Evolution de $F_T/(\rho U^2 b)$ en fonction de z / evolution of $F_T/(\rho U^2 b)$ w.r.t. z (h_s = 0,1 mm)

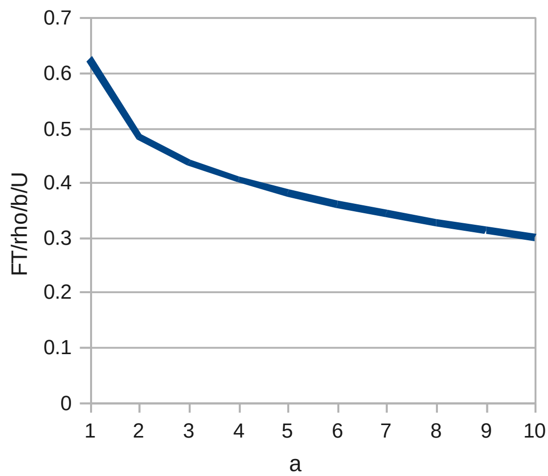


Figure 3. Evolution de $F_T/(\rho U^2 b)$ en fonction de $a = h_e/h_s$ / Evolution of $F_T/(\rho U^2 b)$ w.r.t. a (h_s = 0,1 mm, U = 2 m/s)

Friction & Lubrication n°3

VISCOMETERS
Couette flow & Poiseuille Flow

Contents

Introduction
Problem 1: Rotational Viscometer – The Couette flow
Problem 2: Capillary Viscometer – The Poiseuille flow
Appendix: Navier-Stokes equations in cylindrical coordinates

Introduction

Rheological properties of lubricants depend of 3 physical parameters: the pressure, the temperature and the shear rate imposed to the fluid. The purpose of a viscometer is to create a controlled flow of the fluid as function of these 3 physical parameters, in order to deduce its viscosity. There is 3 different families of viscometers:

- empirical viscometers,
- viscometers that require a calibration,
- and absolute viscometers.

The principle of the first viscometers consists in measuring the time necessary for a given volume of oil to flow through a calibrated hole at a regulated temperature. Different experimental devices have been used in the past that gave name to empirical viscosity units such as Engler (°E) in Continental Europe, Redwood (°R) in Great Britain and Saybold (Second Saybold Universal or SSU) in the USA. Eventually by comparing the time of flow to that of a reference fluid (often water), one can estimate the real viscosity in SI units. Because of their low performances and accuracy, these apparatuses were gradually abandoned.

Viscometers that require a calibration are mostly falling cylinder (or sphere or bullet) viscometer. The principle consists in measuring the velocity of the falling element by Doppler effect. These devices are well adapted to perform pressurized experiments up to 600 MPa or more.

Nowadays most industrial and laboratory tests are made with absolute viscometers. They are of two types: capillary or rotational. Each of these are considered separately in the coming sections.

Problem 1 – Rotational Viscometer – The Couette Flow

As shown in Fig. 1 the rotational cylindrical viscometer consists of two concentric cylinders with a fluid contained between them. The inner (or outer) cylinder is rotating and torque is measured at the outer (or inner) cylinder. Let

R1	inner cylinder radius,
R2	outer cylinder radius,
L	cylinder height,
C	radial clearance, C=R2-R1
$\omega 1$	angular velocity of inner cylinder
$\omega 2$	angular velocity of outer cylinder

1) From the Navier-Stokes equations in cylindrical coordinates (see appendix), calculate the velocity and pressure profiles between the two cylinders of common axis (both are rotating). The following assumptions are used:
   a) Viscosity and density can be considered constant.
   b) The inertia effect is small.
   c) Body forces can be neglected.
   d) The fluid flow is stationary
   e) Symmetry around θ
   f) Infinitely long in z

2) In the case of stationary outer cylinder ($\omega 2=0$), calculate the friction torque acting on both cylinders.
3) Considering that the radial clearance is small in comparison to the cylinder radius (C<<R1), show that the torque is given by the following equation:

$$T = 2\pi \cdot \mu \cdot L \cdot R^2 \frac{V}{C} \quad \text{with} \quad V = R \cdot \omega$$

Therefore
$$\mu = \frac{T \cdot C}{2\pi \cdot \omega \cdot R^3 \cdot L}$$

4) When R1=40mm, R2=40.1mm, L=100mm, $\mu=10^{-2}$Pa.s, $\omega 1$=100rad/s and $\omega 2$=0, compare the exact solution of question 2) and the approximated solution given in 3).
5) What is the power loss ?

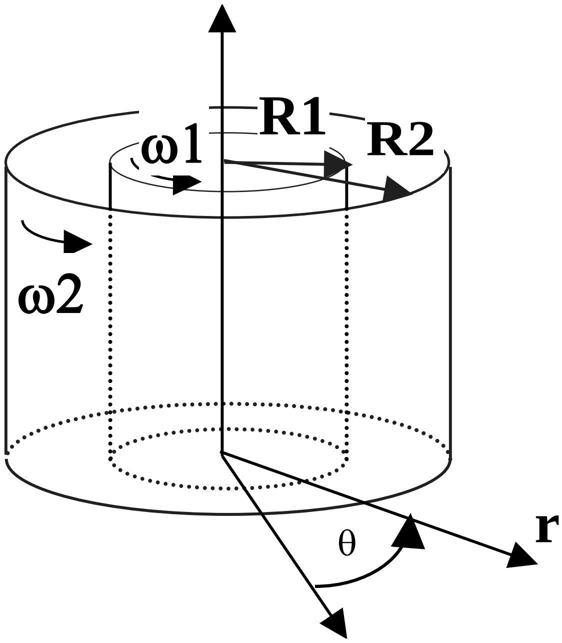


Fig. 1

Problem 2 – Capillary Viscometer – The Poiseuille Flow

This type of viscometer (see Fig. 2) is based on measuring the rate at which a fluid flows through a small-diameter tube. Usually, this takes the form of measuring the time taken to discharge a given quantity of fluid.

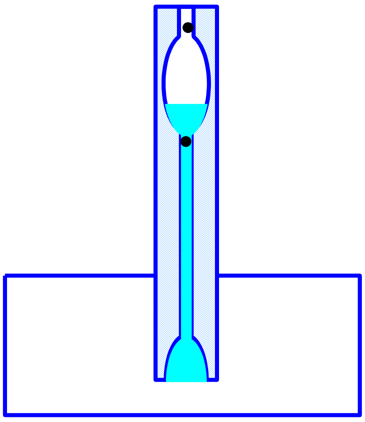


Fig. 2

Part I: Flow in a circular pipe

Consider the flow in a circular pipe as shown in Fig. 3. Cylindrical coordinates will be used with their origin at the tube center. The fluid velocity is zero at the pipe walls. The pressure at the left end of the tube is higher than that at the right end and drops gradually along the tube length. This pressure causes the fluid to flow from left to right.

The following assumptions are imposed:

g) Viscosity and density can be considered constant.
h) The inertia effect is small.
i) Body force terms can be neglected.
j) The fluid flow is stationary
k) $dp/dr=dp/d\theta=0$
l) $u=v=0$ and $w=f(r)$

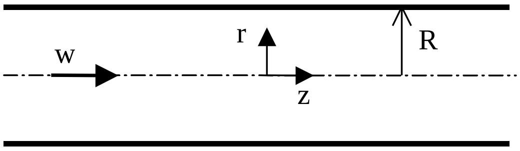


Fig. 3

a) Simplify the Navier-Stokes equations.
b) By integrating the third Navier-Stokes equation (along z), determine the velocity profile in the pipe.
c) Calculate the fluid flow and compare your solution to the Hagen-Poiseuille law given below:

Hagen-Poiseuille law

$$\boxed{Q = -\frac{\pi \cdot R^4}{8 \cdot \mu}\frac{dp}{dz}}$$

Note that a negative pressure gradient is required to get a positive flow in the z direction.

Part II: Application to a capillary viscometer (see Fig. 2)

The fluid flows down due to gravity. Thus the pressure gradient in the vertical direction corresponds to gravity. Note there are no body forces.

a) Simplify the Navier-Stokes equations.
b) By integrating the third Navier-Stokes equation, determine the velocity profile into the pipe.
c) Calculate the fluid flow
d) By considering that the fluid flow Q= – V/t with V the volume and t the time, show that the dynamic and kinematic viscosities are given by the following expressions:

<u>Dynamic viscosity</u>

$$\boxed{\mu = \frac{\pi \cdot R^4 \cdot \rho \cdot g \cdot t}{8 \cdot V}}$$

<u>Kinematic viscosity</u>

$$\boxed{\nu = \frac{\pi \cdot R^4 \cdot g \cdot t}{8 \cdot V}}$$

Appendix – Equations of Navier-Stokes

Cylindrical Coordinates – Incompressible Fluid: $div\vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \varphi} + \frac{\partial w}{\partial z} = 0$

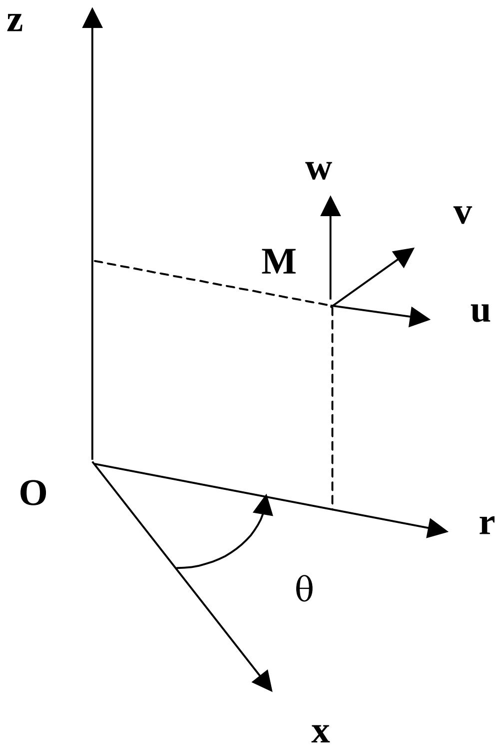
$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right] = \rho f_r - \frac{\partial p}{\partial r} + \mu\left[\Delta u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right] = \rho f_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\Delta v - \frac{v}{r^2} - \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z}\right] = \rho f_z - \frac{\partial p}{\partial z} + \mu[\Delta w]$$

with $$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

then $$\boxed{\rho\frac{d\vec{V}}{dt} = \rho\vec{F} - \overrightarrow{\mathrm{grad}}\,p + \mu\Delta\vec{V}}$$



$$\vec{V}(M)\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)_M}$$

$$\overrightarrow{OM}\begin{Bmatrix} r \\ \theta \\ z \end{Bmatrix}_{(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)_O}$$

<u>N.B. :</u> Newtonian fluid $\Leftrightarrow$ $\sigma_{ij} = [-p + \lambda\theta]\delta_{ij} + 2\mu\varepsilon_{ij}$

Appendix – Equations of Navier-Stokes

Cylindrical Coordinates – Incompressible Fluid: $div\vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \varphi} + \frac{\partial w}{\partial z} = 0$

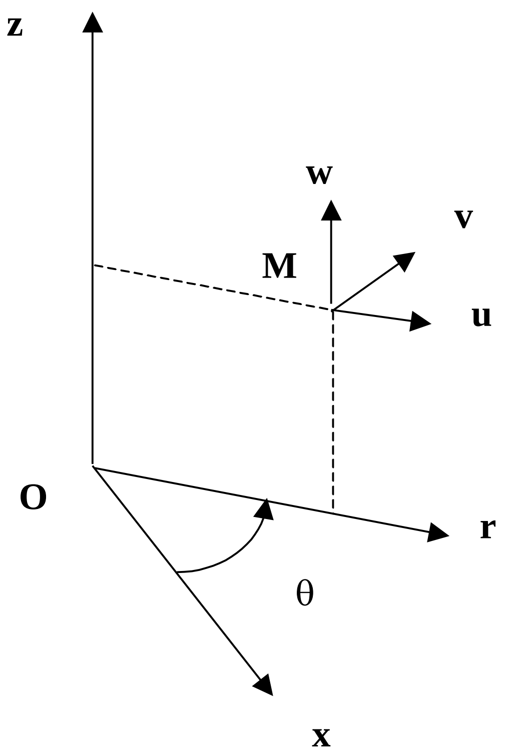
$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right] = \rho f_r - \frac{\partial p}{\partial r} + \mu\left[\Delta u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right] = \rho f_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\Delta v - \frac{v}{r^2} - \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z}\right] = \rho f_z - \frac{\partial p}{\partial z} + \mu[\Delta w]$$

with $$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

then $$\boxed{\rho\frac{d\vec{V}}{dt} = \rho\vec{F} - \overrightarrow{\text{grad}}\,p + \mu\Delta\vec{V}}$$



$$\vec{V}(M)\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)_M}$$

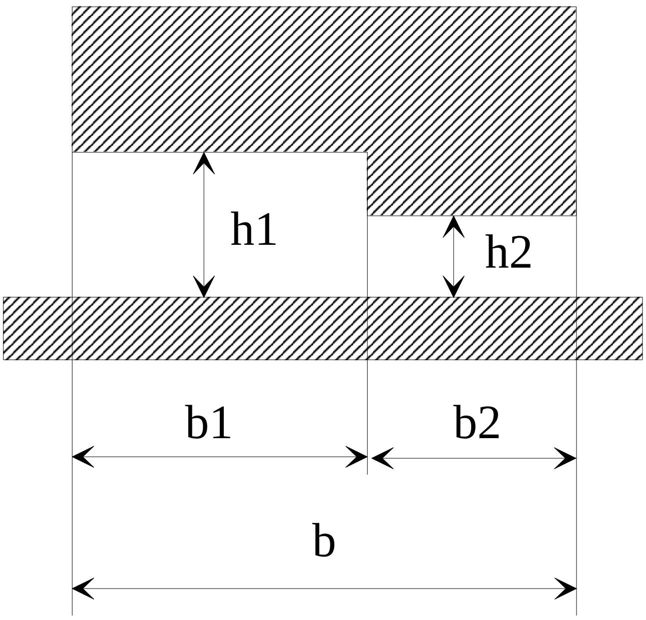
$$\overrightarrow{OM}\begin{Bmatrix} r \\ \theta \\ z \end{Bmatrix}_{(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)_O}$$

<u>N.B. :</u> Newtonian fluid $\Leftrightarrow \sigma_{ij} = [-p + \lambda\theta]\delta_{ij} + 2\mu\varepsilon_{ij}$

Contact Mechanics n°6

<u>HYDRODYNAMIC LUBRICATION:</u>

<u>Parallel-Step Slider Bearing (Lord Rayleigh's Slider Bearing)</u>



Lord Rayleigh, as long ago as 1918, demonstrated that a parallel-step geometry produced the optimum load-carrying capacity when side leakage was neglected. This bearing has not, however, enjoyed the same development and applications as the pivoted-pad slider bearing. Past neglect of this mathematically preferable configuration has been due to doubts about the relative merits of this bearing when side leakage is considered.

Assumptions:

- The side-leakage is neglected (infinite length)
- μ is the fluid viscosity
- The lower surface is moving at constant speed U
- Isothermal and isoviscous flow with an incompressible fluid

Calculate:

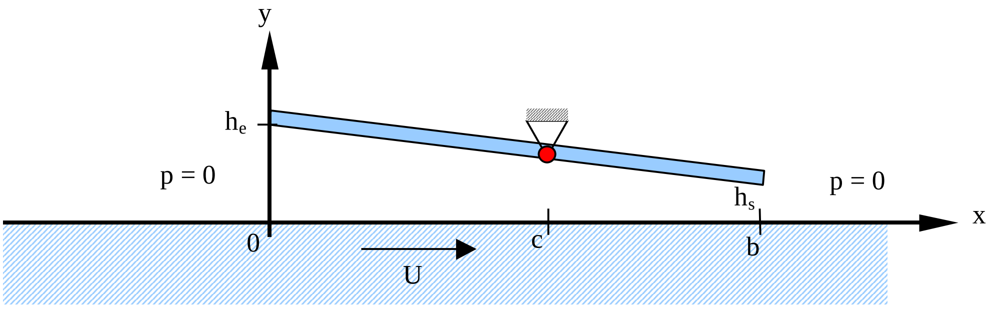
1. The simplified form of the Reynolds equation
2. The load-carrying capacity per unit length
3. The fluid flux
4. The friction force/coefficient

HYDRODYNAMIC LUBRICATION:

Pivoted-pad slider bearing

The simplest form of a pivoted-pad bearing provides only for straight-line motion and consists of a flat surface sliding below a pivoted pad as shown in the figure. If the pad is assumed to be in equilibrium under a given set of operating conditions, any change in these conditions, such as a change in load, speed, or viscosity, will alter the pressure distribution and thus momentarily shift the center of pressure, creating a moment that causes the pad to change its inclination. A pivoted-pad slider bearing is thus supported at a single point so that the angle of inclination becomes a variable and has much better stability than a fixed-incline slider under varying conditions of operation. The location of the shoe's pivot point can be found from the equilibrium of moments acting on the shoe about the point. For all practical purposes, only two significant forces may be considered in the moment equation: the resultant due to film pressure and the reaction force normal to the shoe surface. The force due to friction is ignored.

Pivoted pads are sometimes used in multiples as pivoted-pad thrust bearings or in journal bearings. Normally, a pivoted pad will only carry load if the pivot is placed somewhere between the center of the pad and the outlet edge ($b/2<c<b$). For bidirectional operation the pivot is located at the center of the pad at $c=b/2$.



h_e : input film thickness
h_s : output film thickness

Hypotheses:

- Continuous flow
- Newtonian fluid $\Rightarrow$ Reynolds equation
- Thin film
- ρ = cste , μ = cste , permanent regime

Give:

1. The simplified form of the Reynolds equation and subsequent pressure distribution
2. The load-carrying capacity per unit length and the optimum value of a.
3. The fluid flux
4. The friction force

Results will be expressed as a function of parameter a defined as: $a = \dfrac{h_e}{h_s}$

1. Simplified form of the Reynolds equation and subsequent pressure profile

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) = 6\mu U\frac{\partial h}{\partial x}$$

with $\quad h(x) = (h_s - h_e)\dfrac{x}{b} + h_e \qquad \Rightarrow \qquad h(x) = h_s\left[(1-a)\dfrac{x}{b} + a\right]$

By integrating along x

$$\frac{\partial p}{\partial x} = 6\mu U\left[\frac{h-h^*}{h^3}\right]$$

By integrating a second time

$$\frac{\partial p}{\partial h}\frac{\partial h}{\partial x} = 6\mu U\left[\frac{h-h^*}{h^3}\right] \qquad \text{equivalent to} \qquad \frac{\partial p}{\partial h} = \frac{6\mu Ub}{h_s(1-a)}\left[\frac{1}{h^2} - \frac{h^*}{h^3}\right]$$

$$\Rightarrow \qquad p(x) = \frac{6\mu Ub}{h_s(1-a)}\left[-\frac{1}{h} + \frac{h^*}{2h^2} + cte\right]$$

Boundary conditions:

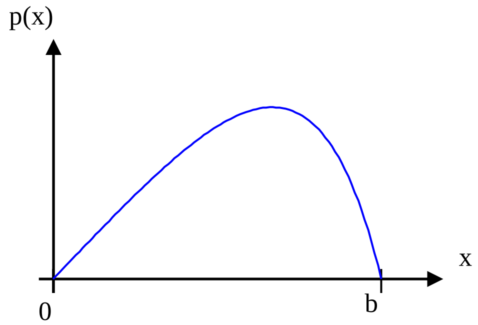
for $x = 0$, $h = h_e$ $\qquad p = 0$
for $x = b$, $h = h_s$ $\qquad p = 0$

$$\Rightarrow \qquad h^* = \frac{2h_e h_s}{h_e + h_s} \qquad \text{and} \qquad cte = \frac{1}{h_e + h_s}$$

$$\Rightarrow \qquad \frac{h^*}{h_s} = \frac{2a}{1+a} \qquad \text{and} \qquad cte.h_s = \frac{1}{1+a}$$

finaly $\boxed{p(x) = \dfrac{6\mu Ub}{h_s^2(1-a)}\left[-\dfrac{h_s}{h} + \dfrac{h_s^2}{h^2}\dfrac{a}{(1+a)} + \dfrac{1}{(1+a)}\right]}$

with $\quad \dfrac{h}{h_s} = (1-a)\dfrac{x}{b} + a$



NB: $\quad \dfrac{dh}{dx} = h_s\dfrac{1-a}{b}$

2. Calculation of the load-carrying capacity (per length unit)

$$W = \iint p(x).dx.dz \qquad \frac{W}{L} = \frac{6\mu Ub}{h_s^2(1-a)} \int \left[-\frac{h_s}{h} + \frac{h_s^2}{h^2}\frac{a}{(1+a)} + \frac{1}{(1+a)} \right] . \frac{dx}{dh} dh$$

$$\frac{W}{L} = \frac{6\mu Ub^2}{h_s^3(1-a)^2} \int \left[-\frac{h_s}{h} + \frac{h_s^2}{h^2}\frac{a}{(1+a)} + \frac{1}{(1+a)} \right] .dh$$

$$\boxed{\frac{W}{L} = \frac{6\mu Ub^2}{h_s^2(a-1)^2} \left[\log(a) - 2\frac{(a-1)}{(a+1)} \right]}$$

Which value of a ratio gives a maximum load-carrying capacity?

a = 2,19.

3. Calculation of the fluid flux

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y(y-h) + U\frac{h-y}{h} \qquad Q_x = \iint u.dy.dz$$

$$\frac{Q_x}{L} = \frac{1}{2\mu}\frac{\partial p}{\partial x} . \left[\frac{-h^3}{6} \right] + \frac{Uh}{2}$$

$$\boxed{\frac{Q_f}{L} = \frac{Uh^*}{2} = \frac{Uh_s}{2}\frac{2a}{(1+a)}}$$

4. Calculation of the friction force

$$\tau_{xy}=\mu\frac{\partial u}{\partial y} \qquad u=\frac{1}{2\mu}\frac{\partial p}{\partial x}y(y-h)+U\frac{h-y}{h} \qquad \rightarrow \qquad \tau_{xy}=\frac{1}{2}\frac{\partial p}{\partial x}(2y-h)-\mu\frac{U}{h}$$

$$F=\iint \tau_{xy}.dx.dz \qquad \frac{F}{L}=\int\left[\frac{1}{2}\frac{\partial p}{\partial x}(2y-h)-\mu\frac{U}{h}\right].\frac{dx}{dh}.dh$$

$$\frac{F}{L}=\frac{\mu Ub}{h_s(1-a)}\int\left[3\left[\frac{1}{h^2}-\frac{h^*}{h^3}\right](2y-h)-\frac{1}{h}\right].dh$$

<u>on the lower surface (y=0):</u>

$$\frac{F_{lower}}{L}=\frac{\mu Ub}{h_s(1-a)}\int\left[-3\left[\frac{1}{h^2}-\frac{h^*}{h^3}\right]h-\frac{1}{h}\right].dh=\frac{\mu Ub}{h_s(1-a)}\left\{-4\int\frac{dh}{h}+3h^*\int\frac{dh}{h^2}\right\}$$

$$\frac{F_{lower}}{L}=\frac{\mu Ub}{h_s(1-a)}\left\{-4[\ln h]_{he}^{hs}+3h^*\left[-\frac{1}{h}\right]_{he}^{hs}\right\}=\frac{\mu Ub}{h_s(1-a)}\left(4\ln a+3\frac{2h_e h_s}{h_e+h_s}\left(\frac{1}{h_e}-\frac{1}{h_s}\right)\right)$$

$$\Rightarrow \qquad \boxed{\frac{F_{lower}}{L}=\frac{\mu Ub}{h_s(a-1)}\left(-4\ln a+6\frac{a-1}{a+1}\right)}$$

<u>on the upper surface (y=h):</u>

$$\frac{F_{upper}}{L}=\frac{\mu Ub}{h_s(1-a)}\int\left[3\left[\frac{1}{h^2}-\frac{h^*}{h^3}\right]h-\frac{1}{h}\right].dh=\frac{\mu Ub}{h_s(1-a)}\left\{2\int\frac{dh}{h}-3h^*\int\frac{dh}{h^2}\right\}$$

$$\frac{F_{upper}}{L}=\frac{\mu Ub}{h_s(1-a)}\left\{2[\ln h]_{he}^{hs}-3h^*\left[-\frac{1}{h}\right]_{he}^{hs}\right\}=\frac{\mu Ub}{h_s(1-a)}\left(-2\ln a-3\frac{2h_e h_s}{h_e+h_s}\left(\frac{1}{h_e}-\frac{1}{h_s}\right)\right)$$

$$\Rightarrow \qquad \boxed{\frac{F_{upper}}{L}=\frac{\mu Ub}{h_s(a-1)}\left(2\ln a-6\frac{a-1}{a+1}\right)}$$

Check the equilibrium equation of the quantity of fluid entrapped between the upper and lower surfaces.

Contact Mechanics n°5

Circular Hydrostatic Thrust Bearing

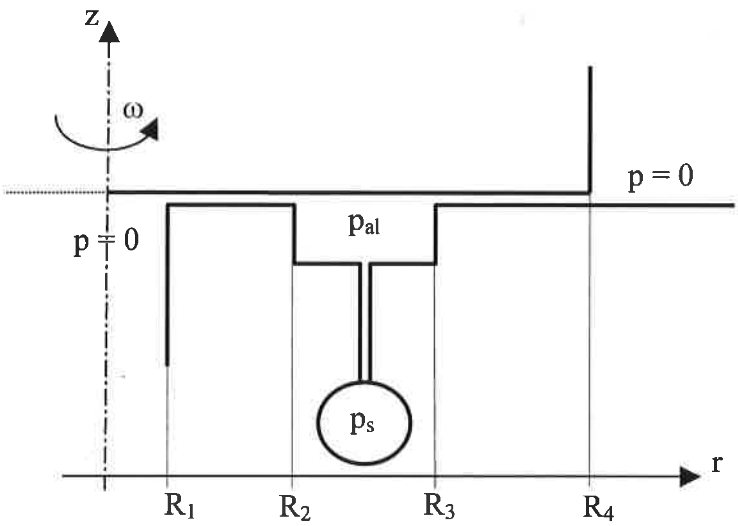
Hypotheses :

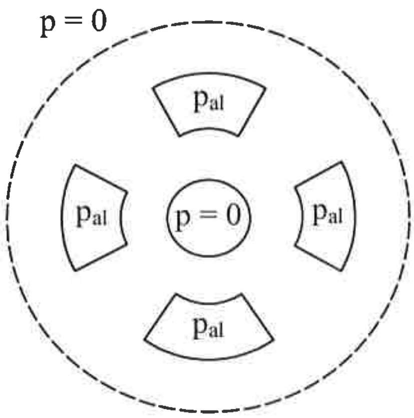
- continuum mechanics
- Newtonian fluid □ Reynolds Equation
- thin film

$\rho = cst$, $\mu = cst$, $h = cst$, stationary problem

Lubricant is supplied through 4 recesses of an angular size of $\pi/4$ (pump pressure: p_s)

One supposes that the pressure p is independent of the angle θ, and the atmospheric pressure is set to zero





Pressure distribution calculation (Reynolds)

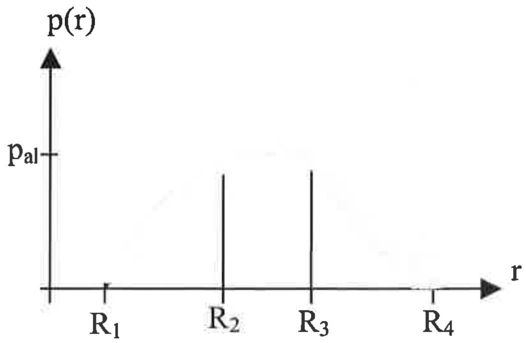
$$\frac{\partial}{\partial r}\left(\qquad \right) = 0 \qquad \square \qquad p(r) =$$

three distinct zones

$R_1 < r < R_2$ $\qquad p(r) = p_{al}$

$R_2 < r < R_3$ $\qquad p(r) = p_{al}$

$R_3 < r < R_4$ $\qquad p(r) = p_{al}$



Calculation of the load carrying capacity

$$W = \iint p(r).rd\theta.dr$$

$$W = \int_0^{2\pi}\left[\int_{R_1}^{R_2} p_{al} \qquad .rdr + \int_{R_2}^{R_3} \qquad .rdr + \int_{R_3}^{R_4} p_{al} \qquad .rdr\right].d\theta$$

$$W = \pi.p_{al}.\left[\left\{ \qquad \right\} + \left\{ \qquad \right\} + \left\{ \qquad \right\}\right]$$

$$\boxed{W = \frac{\pi.p_{al}}{2}.\left[\frac{\left(R_4^2 - R_3^2\right)}{\ln\left(R_4/R_3\right)} - \frac{\left(R_2^2 - R_1^2\right)}{\ln\left(R_2/R_1\right)}\right]}$$

remark (simplified calculation: linear approximation of the pressure)

$$W = \frac{\pi.p_{al}}{3}.\left[R_4^2 + R_3^2 - R_2^2 - R_1^2 + R_3R_4 - R_1R_2\right]$$

nb: a^3-b^3 = (a-b)(a^2+ab+b^2)

Question: what happens to the 'full' result of W *when* R_1 *tends to* R_2 *and* R_3 *tends to* R_4 *?*
what happens to the 'simplified' result when R_1 *tends to* R_2 *and* R_3 *tends to* R_4 *?*
explain physically.

Question: what happens if a load W *is applied while* p_s=0*?*
What happens if one now increases p_s *until lift-off?*
What if p_s *is again decreased until 0?*

Calculation of the frictional moment

$\tau_{z\theta} =$ $\qquad\qquad$ $v =$ $\qquad\qquad$ □ $\qquad\qquad$ $\tau_{z\theta} =$

$$C = \iint r.\tau_{z\theta}.rd\theta.dr$$

$$C = \int_0^{2\pi}\int_{R_1}^{R_2} r.\tau_{z\theta}.rd\theta.dr + \frac{1}{2}\int_0^{2\pi}\int_{R_2}^{R_3} r.\tau_{z\theta}.rd\theta.dr + \int_0^{2\pi}\int_{R_3}^{R_4} r.\tau_{z\theta}.rd\theta.dr$$

The moment generated by the recesses is neglected as the value of h is very large. Its contribution cannot be neglected in between recesses.

$$\boxed{C = \mu \frac{\omega}{h}\frac{\pi}{2}\left[R_4^4 - \frac{R_3^4}{2} + \frac{R_2^4}{2} - R_1^4\right]}$$

Mass flux calculation

$u =$ $\qquad Q_r = \iint u.rd\theta.dz$

Mass flux : $\quad Q_f = Q_{R_4} - Q_{R_1} = Q_{R_3} - Q_{R_2}$

$Q_r =$ $\qquad$ with $\quad r\frac{\partial p}{\partial r} =$ $\qquad$ for $\quad R_1 < r < R_2$

and $\quad r\frac{\partial p}{\partial r} =$ $\qquad$ for $\quad R_3 < r < R_4$

$$\boxed{Q_f = \frac{\pi p_{al} h^3}{6\mu}\left[\quad + \quad\right]}$$

Determination of the recess pressure as a function of p_s and h

The lubricant supplied to the recesses first passes through a hydraulic resistance. Only the pump pressure is imposed. Thus the recess pressure depends on the fluid flux through the mechanism.

Equality of the mass flux in the capillaries Q_c and the flux from all recesses Q_f gives:

$4\,Q_c = Q_f$

For one capillary $\quad Q_c = \frac{\pi R^4}{8L}\frac{(p_s - p_{al})}{\mu} = Kc\frac{(p_s - p_{al})}{\mu}$

Otherwise: $\quad Q_f = K_Q \frac{p_{al} h^3}{\mu}$

4 $Q_c = Q_f$ □ $4Kc(p_s - p_{al}) =$ □ $(p_s - p_{al}) =$

□ $\boxed{p_{al} = \frac{p_s}{\left[\quad\right]}}$

The film thickness h depends directly on the recess pressure, thus on the load (stiffness).

Stiffness of the thrust bearing $(\lambda = -\frac{\partial W}{\partial h})$

$\lambda = -\frac{\partial W}{\partial h} = -\frac{\partial W}{\partial p_{al}}\frac{\partial p_{al}}{\partial h}$ $W = K_w p_{al}$ □ $\lambda = -K_w\frac{\partial p_{al}}{\partial h}$

$$-\frac{\partial p_{al}}{\partial h} = \frac{p_s}{\left[\quad\right]} h^2 = 3\frac{p_{al}\,p_{al}}{p_s} h$$

$(p_s - p_{al}) =$ h □ $-\frac{\partial p_{al}}{\partial h} = \frac{3p_s}{h}$

$$\boxed{\lambda = \frac{3K_w p_s}{h}.\left[1 - \right]}$$

Optimisation of the thrust bearing

<u>Optimisation of the stiffness</u>.

For a given value of h there exists an optimal ratio $\beta = \frac{p_{al}}{p_s} = \left[1 + \frac{K_Q}{4Kc}h^3\right]^{-1}$.

$\lambda = \frac{3K_w p_s}{h}.\beta[1-\beta]$ The stiffness is maximal for $\beta = 0,5$

<u>Optimisation of the dissipated power P</u>.

$P = Q_f.p_s + C.\omega$

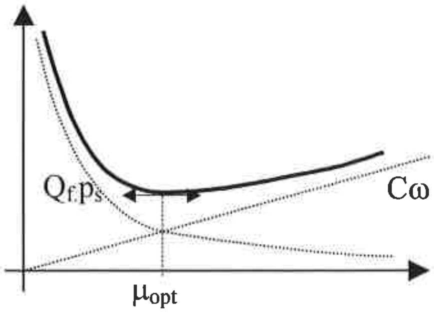
For a given value of h an optimal viscosity μ exists.

$$Q_f p_s = K_Q p_s p_{al} \frac{h^3}{\mu} \qquad C\omega = K_{Co}\omega^2 \frac{\mu}{h}$$

$$P = K_Q p_s p_{al} \frac{h^3}{\mu} + K_{Co}\omega^2 \frac{\mu}{h}$$

$$\frac{\partial P}{\partial \mu} = \quad K_Q p_s p_{al} \qquad K_{Co}$$

$$\frac{\partial P}{\partial \mu} = 0 \qquad □ \qquad \boxed{\mu_{opt} = \sqrt{\quad} .h^2}$$



μ

For the optimal viscosity one finds : $Q_f p_s = C\omega = h\sqrt{K_Q p_s p_{al} K_{Co}\omega^2}$

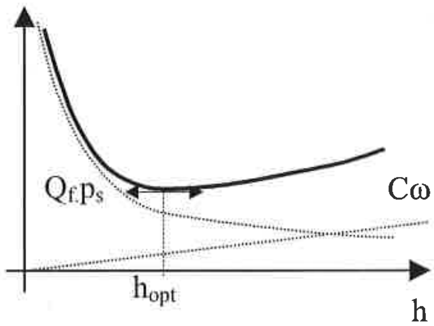
For a given viscosity an optimal film thickness value h exists.

$$Q_f p_s = K_Q p_s p_{al} \frac{h^3}{\mu} \qquad C\omega = K_{Co}\omega^2 \frac{\mu}{h}$$

$$P = K_Q p_s p_{al} \frac{h^3}{\mu} + K_{Co}\omega^2 \frac{\mu}{h}$$

$$\frac{\partial P}{\partial h} = \quad K_Q p_s p_{al} \qquad K_{Co}$$

$$\frac{\partial P}{\partial h} = 0 \qquad □ \qquad \boxed{h_{opt} = \sqrt[4]{\quad}}$$



h

For the optimal film thickness one finds: $Q_f p_s = 3.C\omega = \sqrt[4]{K_Q p_s p_{al}\left(\frac{K_{Co}\omega^2}{3}\right)^3}\sqrt{\mu}$

One concludes that the power dissipated can not be optimised simultaneously with respect to the viscosity and the film thickness. Consequently, a compromise has to be found between these two optimisations.

Contact Mechanics n°7

HYDRODYNAMIC LUBRICATION:

Journal bearings – Analytical solutions

Contents:

1. Introduction
2. Boundary conditions
3. Infinitely wide-journal-bearing
4. Short-width-journal bearing

1. Introduction

Journal bearings are used to support shafts and to carry radial loads with minimum power loss and minimum wear. The journal bearing can be represented by a plain cylindrical sleeve wrapped around the shaft (journal), but the bearings can adopt a variety of forms. The lubricant is supplied at some convenient point in the bearing through a hole or a groove. If the bearing extends around the full 360° of the journal, it is described as a "full journal bearing". If the angle of wrap is less than 360°, the term "partial journal bearing" is used.

Journal bearings rely on shaft motion to generate the load-supporting pressures in the lubricant film. The geometry of the journal bearing is shown in *figure 1*. The shaft does not normally run concentric with the bearing. The displacement of the shaft center relative to the bearing center is known as the 'eccentricity'. The shaft's eccentric position within the bearing clearance is influenced by the load that is carries. The amount of eccentricity adjusts itself until the load is balanced by the pressure generated in the converging lubricating film. The line drawn through the shaft center and the bearing center is called the "line of centers".

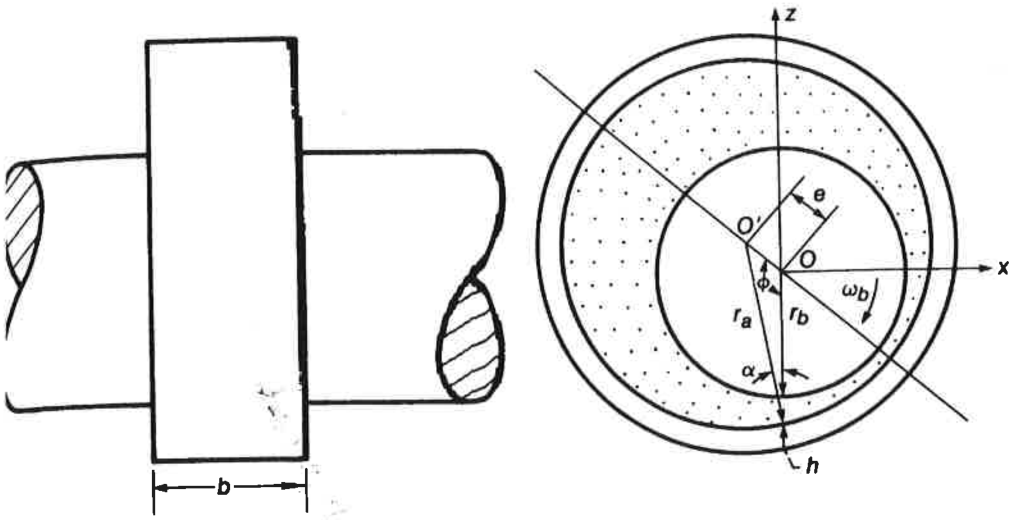


Figure 1: Hydrodynamic journal bearing geometry

The pressure generated and therefore the load-carrying capacity of the bearing depend on the shaft eccentricity, the angular velocity, the effective viscosity of the lubricant, and the bearing dimensions and clerance.

The load and the angular velocity are usually specified and the minimum shaft diameter is often predetermined. To complete the design, it will be necessary to calculate the bearing dimensions and clearance and to choose a suitable lubricant if this is not already specified.

After an important discussion on the boundary conditions that can be used, two approximate journal bearing solutions will be given: (1) for an infinite-width journal bearing (side leakage neglected) and (2) for a short-width-journal bearing.

2. Boundary conditions

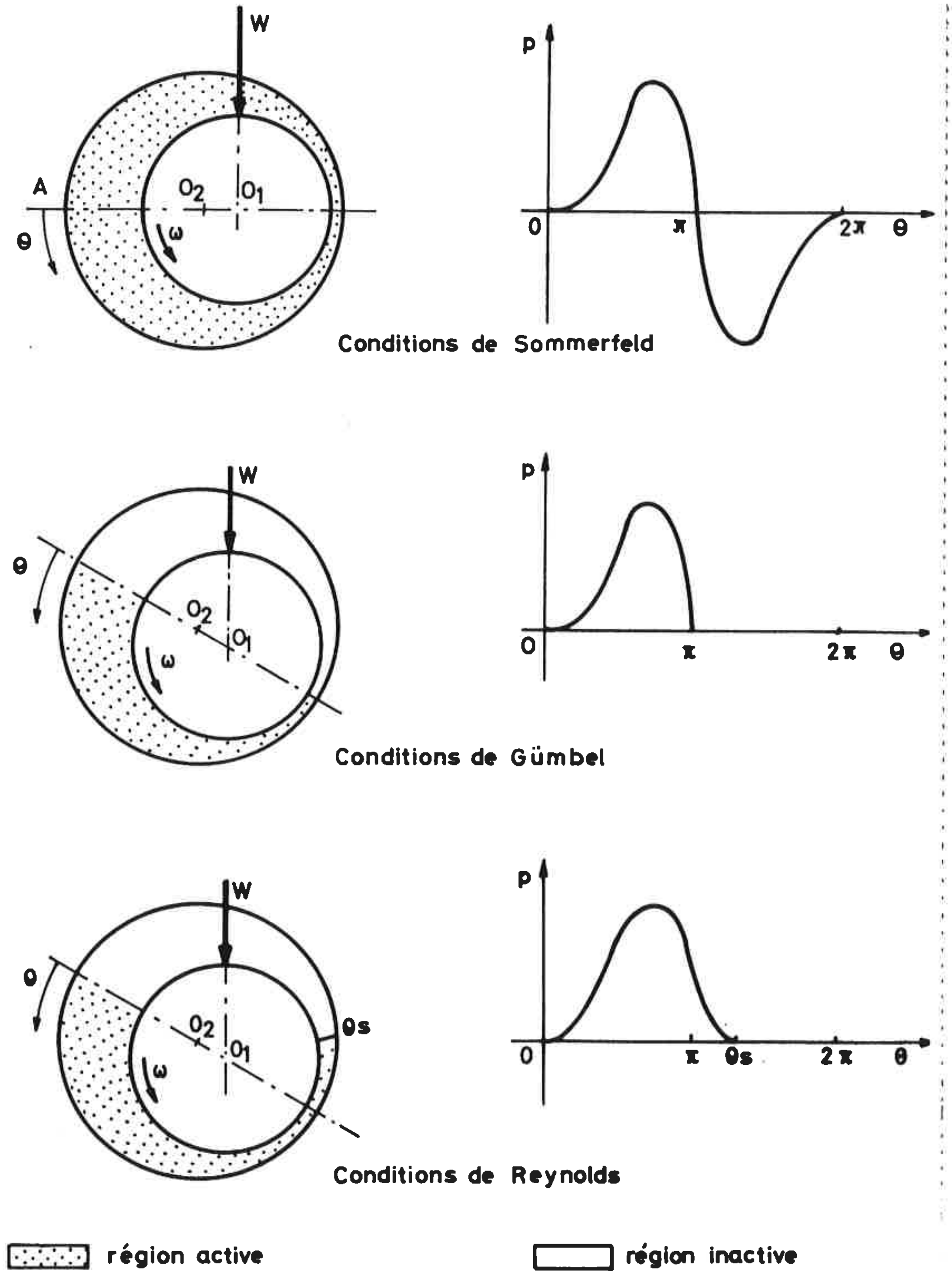


Fig. 2 : Répartition de pression pour les différentes conditions aux limites

3. Infinitely wide-journal-bearing solution

1. Show that the lubricant film thickness $h(\theta)=C(1+\varepsilon.\cos\theta)$
2. Give the simplified form of the Reynolds equation in the case where
   2.1 the reference frame is attached to the bearing
   2.2 the reference frame is attached to the shaft

4. Short-width-journal bearing theory

3. Determine the pressure profile
4. The load-carrying capacity W and the attitude angle Φ.
5. The side-leakage fluid flow
6. The friction torque
   6.1 Acting on the shaft
   6.2 Acting on bearing
   6.3 Check the equilibrium of the bearing

Exercise What are the two essential phenomena in ElastoHydrodynamic Lubrication?

Exercise How to ensure the performance of an EHL contact?

Exercise If EHL contacts work reliably today, why is it necessary to continue the research to ensure reliable operation of tomorrows EHL contacts?

Exercise In a cylindrical roller bearing, is the contact between outer ring and roller conforming or non-conforming? And the contact between inner ring and roller? What can one conclude concerning the contact pressures?

Exercise Compute the reduced radii of contact for the Figures 2.3 and 2.4, if $R_{1x} = 0.1$ m and $R_{2x} = 0.09$ m. Draw the two reduced geometries to scale.

Exercise Show that the gap between two non-conforming cylinders of radii R_{1x} and R_{2x} can be described up to second order accuracy by the equation $h(x) = h_0 + x^2/(2R_x)$, using a Taylor series development, with $h_0 = h(x = 0)$ and $1/R_x = 1/R_{1x} + 1/R_{2x}$.

Exercise What are the reduced radii of contact R_x and R_y in a non-conforming contact between a sphere of radius R and a cylinder of radius R, where the cylinder axis is aligned with the x-axis.

Exercise What are the surface velocities with respect to the contact point in the case of a cylinder rolling on a flat plane with velocity u? And what are these velocities if the cylinder slides without rotation on the plane, with the same velocity u?

Exercise Give an example in which each of the three pressure generating terms is important, and describe the details of this mechanism. What happens if all three pressure generating terms are zero?

Exercise What is the Reynolds equation describing a non-rotating cylinder of radius R falling onto a stationary plane with velocity w_0. Is it possible to instantaneously generate the same pressure profile $p(x)$ using the above stationary Reynolds equation? If so what is the relation between $u_m(x)$ and w_0 at the moment the cylinder touches the plane? Assume ρ and η to be constant.

Exercise Assuming very slow transient behaviour of period τ, for which values of τ can the transient terms in the Reynolds equation be neglected. Assume that a is a typical contact dimension. Explain how in this case the transient problem can be solved as a succession of stationary problems.

Exercise Compute the elastic deformation in a line contact loaded by a parabolical pressure distribution between $-b < x < b$: $p(x) = p_0(1 - x^2/b^2)$, $-b < x < b$, $p(x) = 0$, otherwise. Take $x_0 = b$.

Exercise Check the two asymptotes of the density pressure relation.

Exercise Assuming that $\alpha = 2 \cdot 10^{-8}$ Pa^{-1}, compute the pressure for which the viscosity is twice its atmospheric value, according to Barus. Then calculate the pressure for which the viscosity increases by a factor of 1000, then by a factor of 10^6.

Exercise Show that using the definition of α as the slope at zero (ambient) pressure for both viscosity pressure equations, one obtains indeed that $\alpha p_0/z = \ln(\eta_0) + 9.67$.

Exercise Using the figures 3.9 - 3.14 show that $\partial(\rho h)/\partial x = 0$ is indeed found in the high pressure zone. Comment on the film thickness evolution from 3.9 - 3.14. Relate this evolution to the evolution in the pressure distribution.

Exercise Show that $H(X - T) = X^2 - 2XT + T^2$ is indeed a solution of the reduced Reynolds equation for high pressures: $\partial H/\partial X + \partial H/\partial T = 0$.

Exercise Check the derivation of the three dimensionless equations.

Exercise Give the reduced forms of the Reynolds equation in the zone $\theta < 1$ and $\theta = 1$?

Exercise Derive the Reynolds equation from the mass flow continuity equation over a rectangle, comments with respect to the discrete equation?

Exercise Express the Hertzian pressure p_h in terms of w_1, E' and R only. In order to double the pressure, p_h, how much should the load w_1 change, how much the contact radius R, and how much the reduced Elastic modulus E'?

Exercise Compute δ for a contact with $b = 0.001$ m and $R = 0.05$ m. If the reduced elastic modulus $E' = 2 \cdot 10^{11}$ Pa what is the load per unit length w_1, what is p_h?

Exercise Figure 3.1 shows the deformed and the undeformed geometry for a line contact. From the difference the maximum deformation can be estimated as 0.6. Deduce which dimensionless film thickness relation has been used to obtain H.

Exercise Express the Hertzian pressure p_h in terms of w, E' and R_x only. In order to double the pressure, p_h, how much should the load w change, how much the contact radius R_x, and how much the reduced Elastic modulus E'?

Exercise Compute δ for a contact with $a = 0.001$ m and $R_x = 0.05$ m. If the reduced elastic modulus $E' = 2 \cdot 10^{11}$ Pa, what is the load w, what is the value of p_h?

Exercise Figure 3.2 shows the deformed and the undeformed geometry for a circular contact. From the difference the maximum deformation can be estimated as 1. Deduce which dimensionless film thickness relation has been used for H.

Exercise Check that for the case of a circular contact $k = 1$, the equations for a, b and δ reduce to the ones found in the previous section.

Exercise Compute a, b, p_h and δ for a contact with $R_x = 0.005$ m, $R_y = 0.05$ m, $E' = 2 \cdot 10^{11}$ Pa and $w = 10^4$ N?

Exercise Check the two asymptotes of q for $p \to 0$ and for $p \to \infty$?

Exercise Compute h^* for $R = 0.01$ m, $\eta_0 = 0.1$ Pas, $\alpha = 2 \cdot 10^{-8}$ Pa^{-1}, $u_1 + u_2 = 1$ m/s, $E' = 2 \cdot 10^{11}$ Pa, $w_1 = 10^4$ kg/m, using the parameters W_1, U and G. Same question using Figure 3.3.

Exercise Using Figure 3.3 find h^* for $R = 0.01$ m, $\eta_0 = 0.1$ Pas, $\alpha = 2 \cdot 10^{-8}$ Pa^{-1}, $u_1 + u_2 = 1$ m/s, $E' = 2 \cdot 10^{11}$ Pa, $w_1 = 10^4$ kg/m.

Exercise How much does the film thickness h change if the atmospheric viscosity η_0 is doubled? How much if the speed $u_1 + u_2$ is doubled? How much if the load per unit length w_1 is doubled? How much if the reduced elasticity E' is doubled (careful)? How much if the reduced contact radius R is doubled (careful)?

Exercise The aim of this exercise is to keep the film thickness constant. How much does one have to change the speed $u_1 + u_2$ when the viscosity η_0 is halved? How much does the load have to change when the viscosity is doubled?

Exercise Express the Hertzian contact half width b in terms of W_1 and R.

Exercise What is the dimensionless film thickness $H(X = 0)$ in the Figures 3.5 to 3.7? What is the relative evolution of $h(x = 0)$? Compare the evolution with the one predicted by Ertel-Grubin?

Exercise What is the dimensionless central film thickness $H(X = 0)$ in the Figures 3.9 to 3.14? What is the relative evolution of $h(x = 0)$, careful?

Exercise What is the dimensionless central pressure $P(X = 0)$ in the Figures 3.9 to 3.14? What is the relative evolution of $p(x = 0)$?

Exercise Compare the answers on $h(x = 0)$ and $p(x = 0)$ from the two previous exercises with the Figure 3.15.

Exercise Compute h_m for $R = 0.01$ m, $\eta_0 = 0.1$ Pas, $u_1 + u_2 = 1$ m/s, $\alpha = 2 \cdot 10^{-8}$ Pa^{-1}, $E' = 2 \cdot 10^{11}$ Pa, $w_1 = 10^4$ kg/m. Compare with the Ertel Grubin value, comments?

Exercise What are the advantages of the set M_1, L over the set W_1, U, G? What are its disadvantages?

Exercise What is the order of magnitude of H_m^D and of H_m^M? Use typical values given before for oil/steel contacts.

Exercise Compute h_m for $R = 0.01$ m, $\eta_0 = 0.1$ Pas, $u_1 + u_2 = 1$ m/s, $\alpha = 2 \cdot 10^{-8}$ Pa^{-1}, $E' = 2 \cdot 10^{11}$ Pa, $w_1 = 10^4$ kg/m. Compare with the Dowson and Higginson value and the Ertel Grubin value, comments?

Exercise Express the Ertel Grubin formula in terms of H_{min}, M_1 and L.

Exercise Check the expression of the Moes Venner formula in terms of H_m^D, W_1, U and G.

Exercise Which of the three regimes is the appropriate regime for $M_1 = 1$, $L = 0$? and for $M_1 = 100$, $L = 0$? and for $M_1 = 100$, $L = 10$? and for $M_1 = 10$, $L = 1$ (careful)? Compute for each of the cases the film thickness H_m^M.

Exercise Draw the EHL domain in Figure 3.3, where the Ertel Grubin formula is valid. Use $M_1 > 5$ and $L > 2.5$ as criteria. What happens if one uses the EG equation beyond these limits?

Exercise Calculate the dimension of the term $\eta_0 u/w_1$. What is the consequence for the minimum film thickness h_m?

Exercise Explain why the film thickness h_m is independent of E' in the I.R. regime. What about the dependence on α?

Exercise Show that for the I.E. regime $h_m \propto \eta_0^{0.4}$ and $h_m \propto w_1^{0.2}$. Show that as a consequence the friction coefficient $f \propto w_1^{-0.8}$. Derive the complete equation showing the dependence of F_t with respect to all parameters η_0, R, w_1 and E'. Show through a dimensional analysis that indeed $[F_t] = N/m$.

Exercise Compare the exponents of U and W_2 in H_c^D and H_m^D. What do you conclude. Will the difference between H_m^D and H_c^D increase or decrease with increasing values of W_2 and U?

Exercise Which of the three regimes is the appropriate regime for $M_2 = 3$, $L = 0$? and for $M_2 = 100$, $L = 0$? and for $M_2 = 100$, $L = 10$? and for $M_2 = 10$, $L = 1$ (careful)? Compute for each of the cases the film thickness H_c^M.

Exercise Calculate the film thickness assuming $R_x = R_y = 30$ mm, $w = 90$ kg, $\eta_0 = 10^{-4}$ Pa s, $\alpha = 10^{-9}$ Pa^{-1}, $u_1 + u2 = 60$ m/s, $E' = 2\,10^{11}$ Pa, careful, which regime?

Exercise Derive the dimensional film thickness equation in the I.R. regime. Comment on the absence of E'. Check the dimension.

Exercise Derive the dimensional film thickness equation in the I.E. regime. Comment on the absence of α. Check the dimension.

Exercise Calculate the film thickness H_c^D and H_c^M for $W_2 = 10^{-5}$, $U = 10^{-11}$ and $G = 4000$. Compare the two values and list another advantage of the Moes parameter set.

Reynolds equation in cylindrical coordinates:

$$\frac{d}{dr}\left(\frac{\rho r h^3}{12\eta}\frac{dp}{dr}\right)+\frac{d}{d\theta}\left(\frac{\rho h^3}{12r\eta}\frac{dp}{d\theta}\right)=\frac{rd(\rho h(u_{r1}+u_{r2}))}{2dr}+\frac{d(\rho h(u_{\theta 1}+u_{\theta 2}))}{2d\theta}+\frac{d\rho h}{dt}$$

Velocity profile

$$u(z)=\frac{1}{2\eta}\frac{dp}{dx}(z-h)z+u_1\frac{h-z}{h}+u_2\frac{z}{h}$$

Barus equation

$$\eta(p)=\eta_0\, e^{\alpha p}\ (Barus)$$

Vogel equation

$$\eta(T)=\eta_0\, e^{\frac{b}{T+\theta}}\ (Vogel)$$

Hertz:

$p_h=(2w_1)/(\pi b)$, $p_h=(3w)/(2\pi a^2)$

$b=((8w_1R)/(\pi E'))^{1/2}$, $a=((3wR)/(2E'))^{1/3}$, $\delta=a^2/R$

$2/E'=(1-\upsilon_1^2)/E_1+(1-\upsilon_2^2)/E_2$

Lubricated:

DH parameters $W_1=w_1/(E'R)$, $U=\eta_0u/(E'R)$, $G=\alpha E'$, $H=h/R$

HD parameters $W_2=w/(E'R^2)$, $U=\eta_0u/(E'R)$, $G=\alpha E'$, $H=h/R$

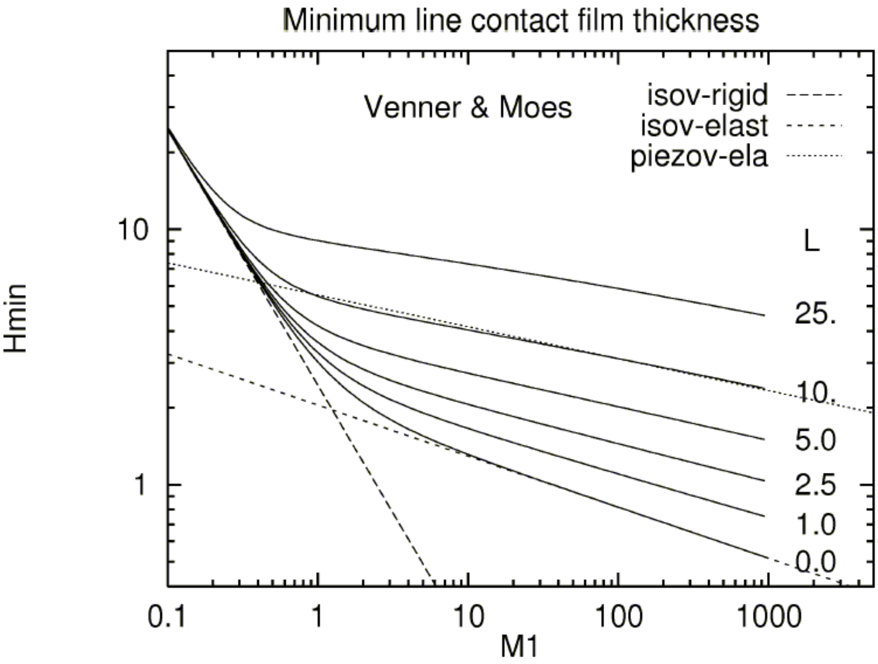
MV parameters $M_1=W_1/\sqrt{U}$, $L=GU^{1/4}$, $H=h/(R\sqrt{U})$

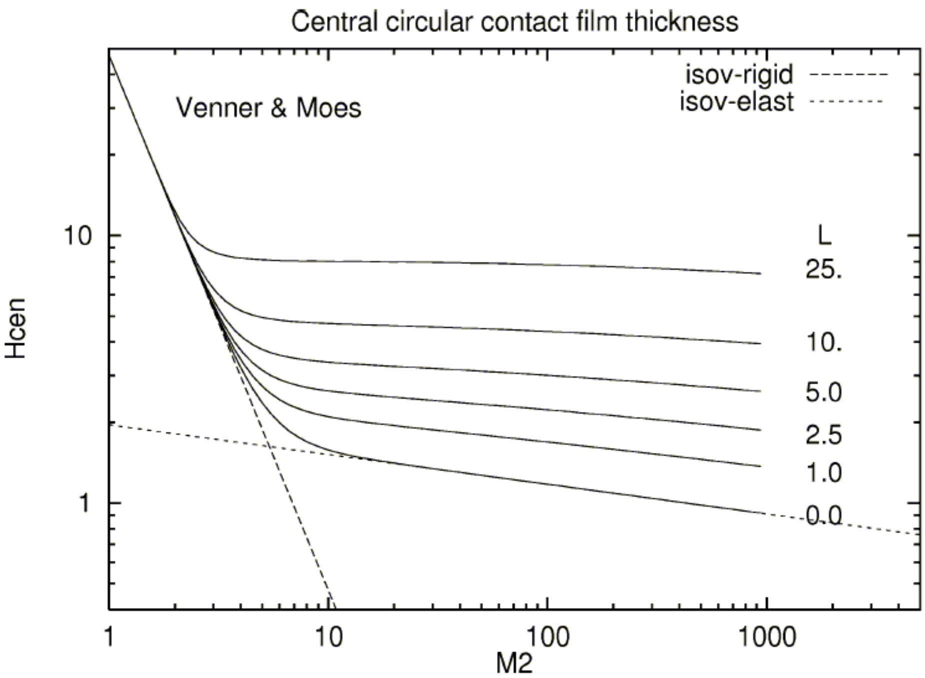
MV parameters $M_2=W_2/U^{3/4}$, $L=GU^{1/4}$, $H=h/(R\sqrt{U})$

EG: $H^*=1.31\ (UG)^{3/4}\ W_1^{-1/8}$

IR: $H=2.45/M_1$, $H=47.55/M_2^2$

IE: $H=2.05/M_1^{1/5}$, $H=1.96/M_2^{1/9}$


Minimum line contact film thickness
Venner & Moes
isov-rigid
isov-elast
piezov-ela
Hmin
10
1
L
25.
10.
5.0
2.5
1.0
0.0
0.1
1
10
100
1000
M1


Central circular contact film thickness
Venner & Moes
isov-rigid
isov-elast
Hcen
10
1
L
25.
10.
5.0
2.5
1.0
0.0
1
10
100
1000
M2