Contact Mechanics n°5

Circular Hydrostatic Thrust Bearing

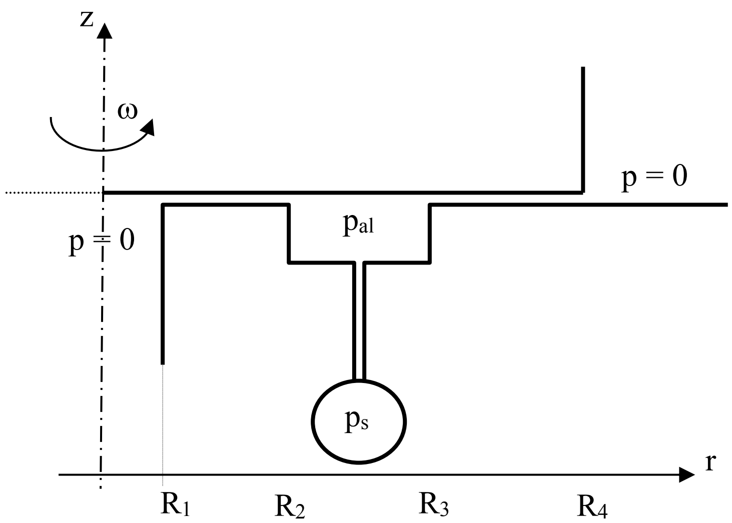
Hypotheses :

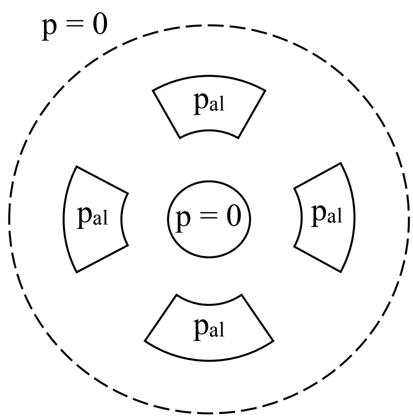
- continuum mechanics
- Newtonian fluid → Reynolds Equation
- thin film

$\rho = cst$, $\mu = cst$, $h = cst$, stationary problem

Lubricant is supplied through 4 recesses of an angular size of $\pi/4$ (pump pressure: p_s)

One supposes that the pressure p is independent of the angle θ, and the atmospheric pressure is set to zero





Pressure distribution calculation (Reynolds)

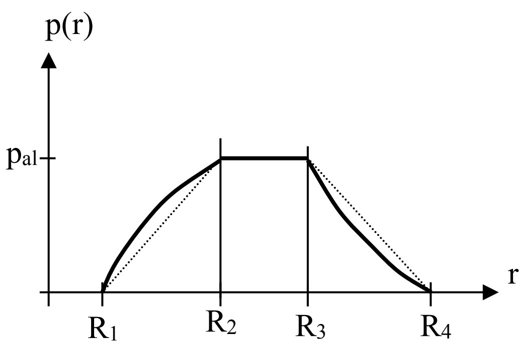
$$\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = 0 \qquad \rightarrow \qquad p(r) = A\ln(r) + B$$

three distinct zones

$R_1 < r < R_2$ $$p(r) = p_{al}\frac{\ln\left(r/R_1\right)}{\ln\left(R_2/R_1\right)}$$

$R_2 < r < R_3$ $$p(r) = p_{al}$$

$R_3 < r < R_4$ $$p(r) = p_{al}\frac{\ln\left(r/R_4\right)}{\ln\left(R_3/R_4\right)}$$



Calculation of the load carrying capacity

$$W = \iint p(r).r d\theta.dr$$

$$W = \int_0^{2\pi}\left[\int_{R_1}^{R_2} p_{al}\frac{\ln\left(r/R_1\right)}{\ln\left(R_2/R_1\right)}.rdr + \int_{R_2}^{R_3} p_{al}.rdr + \int_{R_3}^{R_4} p_{al}\frac{\ln\left(r/R_4\right)}{\ln\left(R_3/R_4\right)}.rdr\right].d\theta$$

$$W = \pi.p_{al}.\left[\left\{R_2^2 - \frac{\left(R_2^2 - R_1^2\right)}{2\ln\left(R_2/R_1\right)}\right\} + \left\{R_3^2 - R_2^2\right\} + \left\{-R_3^2 + \frac{\left(R_3^2 - R_4^2\right)}{2\ln\left(R_3/R_4\right)}\right\}\right]$$

$$\boxed{W = \frac{\pi.p_{al}}{2}.\left[\frac{\left(R_4^2 - R_3^2\right)}{\ln\left(R_4/R_3\right)} - \frac{\left(R_2^2 - R_1^2\right)}{\ln\left(R_2/R_1\right)}\right]}$$

remark (simplified calculation: linear approximation of the pressure)

$$W = \frac{\pi.p_{al}}{3}.\left[R_4^2 + R_3^2 - R_2^2 - R_1^2 + R_3R_4 - R_1R_2\right]$$

nb: $a^3-b^3 = (a-b)(a^2+ab+b^2)$

Question: what happens to the `full' result of W *when* R_1 *tends to* R_2 *and* R_3 *tends to* R_4 *?*
what happens to the `simplified' result when R_1 *tends to* R_2 *and* R_3 *tends to* R_4 *?*
explain physically.

Question: what happens if a load W *is applied while* $p_s=0$*?*
What happens if one now increases p_s *until lift-off?*
What if p_s *is again decreased until 0?*

Calculation of the frictional moment

$$\tau_{z\theta} = \mu\frac{\partial v}{\partial z} \qquad v = \omega r\frac{z}{h} \qquad \rightarrow \qquad \tau_{z\theta} = \mu\frac{\omega r}{h}$$

$$C = \iint r.\tau_{z\theta}.rd\theta.dr$$

$$C = \int_0^{2\pi}\int_{R_1}^{R_2} r.\tau_{z\theta}.rd\theta.dr + \frac{1}{2}\int_0^{2\pi}\int_{R_2}^{R_3} r.\tau_{z\theta}.rd\theta.dr + \int_0^{2\pi}\int_{R_3}^{R_4} r.\tau_{z\theta}.rd\theta.dr$$

The moment generated by the recesses is neglected as the value of h is very large. Its contribution cannot be neglected in between recesses.

$$\boxed{C = \mu \frac{\omega}{h}\frac{\pi}{2}\left[R_4^4 - \frac{R_3^4}{2} + \frac{R_2^4}{2} - R_1^4\right]}$$

Mass flux calculation

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial r} z(z-h) \qquad\qquad Q_r = \iint u.rd\theta.dz$$

Mass flux : $\quad Q_f = Q_{R_4} - Q_{R_1} = Q_{R_3} - Q_{R_2}$

$$Q_r = \frac{1}{2\mu}\frac{\partial p}{\partial r}.2\pi r.\left[\frac{-h^3}{6}\right] \qquad \text{with} \quad r\frac{\partial p}{\partial r} = \frac{1}{\ln\left(R_2/R_1\right)} \quad \text{for} \quad R_1 < r < R_2$$

$$\text{and} \quad r\frac{\partial p}{\partial r} = \frac{1}{\ln\left(R_3/R_4\right)} \quad \text{for} \quad R_3 < r < R_4$$

$$\boxed{Q_f = \frac{\pi p_{al} h^3}{6\mu}\left[\frac{1}{\ln\left(R_2/R_1\right)} + \frac{1}{\ln\left(R_4/R_3\right)}\right]}$$

Determination of the recess pressure as a function of p_s and h

The lubricant supplied to the recesses first passes through a hydraulic resistance. Only the pump pressure is imposed. Thus the recess pressure depends on the fluid flux through the mechanism.

Equality of the mass flux in the capillaries Q_c and the flux from all recesses Q_f gives:

$$4\,Q_c = Q_f$$

For one capillary $\quad Q_c = \frac{\pi R^4}{8L}\frac{(p_s - p_{al})}{\mu} = Kc\frac{(p_s - p_{al})}{\mu}$

Otherwise: $\quad Q_f = K_Q \frac{p_{al} h^3}{\mu}$

$$4\,Q_c = Q_f \quad \rightarrow \quad 4Kc(p_s - p_{al}) = K_Q p_{al} h^3 \quad \rightarrow \quad (p_s - p_{al}) = \frac{K_Q}{4Kc} p_{al} h^3$$

$$\rightarrow \quad \boxed{p_{al} = \frac{p_s}{\left[1 + \frac{K_Q}{4Kc} h^3\right]}}$$

The film thickness h depends directly on the recess pressure, thus on the load (stiffness).

Stiffness of the thrust bearing ($\lambda = -\frac{\partial W}{\partial h}$)

$$\lambda = -\frac{\partial W}{\partial h} = -\frac{\partial W}{\partial p_{al}} \frac{\partial p_{al}}{\partial h} \qquad W = K_w p_{al} \quad \rightarrow \quad \lambda = -K_w \frac{\partial p_{al}}{\partial h}$$

$$-\frac{\partial p_{al}}{\partial h} = \frac{p_s}{\left[1 + \frac{K_Q}{4Kc} h^3\right]^2} \frac{3K_Q}{4Kc} h^2 = 3 \frac{p_{al}}{p_s} \frac{p_{al}}{h} \frac{K_Q}{4Kc} h^3$$

$$(p_s - p_{al}) = \frac{K_Q}{4Kc} p_{al} h^3 \quad \rightarrow \quad -\frac{\partial p_{al}}{\partial h} = \frac{3p_s}{h} \frac{p_{al}}{p_s} \frac{(p_s - p_{al})}{p_s}$$

$$\boxed{\lambda = \frac{3K_w p_s}{h} . \frac{p_{al}}{p_s}\left[1 - \frac{p_{al}}{p_s}\right]}$$

Optimisation of the thrust bearing

<u>Optimisation of the stiffness</u>.

For a given value of h there exists an optimal ratio $\beta = \frac{p_{al}}{p_s} = \left[1 + \frac{K_Q}{4Kc} h^3\right]^{-1}$.

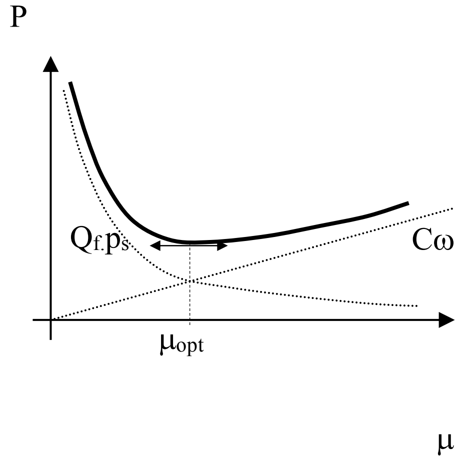
$\lambda = \frac{3K_w p_s}{h} . \beta[1 - \beta]$ The stiffness is maximal for $\beta = 0{,}5$

<u>Optimisation of the dissipated power P</u>.

$P = Q_f . p_s + C . \omega$

For a given value of h an optimal viscosity μ exists.

$$Q_f p_s = K_Q p_s p_{al} \frac{h^3}{\mu} \qquad C\omega = K_{Co}\omega^2 \frac{\mu}{h}$$



$$P = K_Q p_s p_{al} \frac{h^3}{\mu} + K_{Co}\omega^2 \frac{\mu}{h}$$

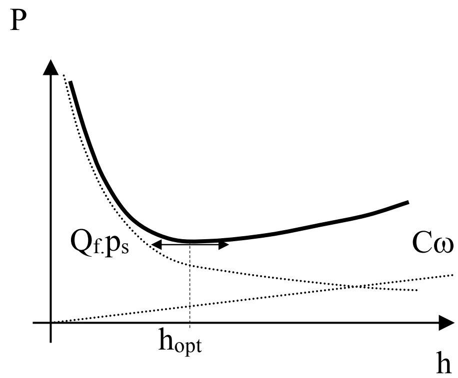
$$\frac{\partial P}{\partial \mu} = -K_Q p_s p_{al} \frac{h^3}{\mu^2} + K_{Co} \frac{\omega^2}{h}$$

$$\frac{\partial P}{\partial \mu} = 0 \qquad \rightarrow \qquad \boxed{\mu_{opt} = \sqrt{\frac{K_Q p_s p_{al}}{K_{Co}\omega^2}}.h^2}$$

For the optimal viscosity one finds : $Q_f p_s = C\omega = h\sqrt{K_Q p_s p_{al} K_{Co}\omega^2}$

For a given viscosity an optimal film thickness value h exists.

$$Q_f p_s = K_Q p_s p_{al} \frac{h^3}{\mu} \qquad C\omega = K_{Co}\omega^2 \frac{\mu}{h}$$



$$P = K_Q p_s p_{al} \frac{h^3}{\mu} + K_{Co}\omega^2 \frac{\mu}{h}$$

$$\frac{\partial P}{\partial h} = 3K_Q p_s p_{al} \frac{h^2}{\mu} - K_{Co}\omega^2 \frac{\mu}{h^2}$$

$$\frac{\partial P}{\partial h} = 0 \qquad \rightarrow \qquad \boxed{h_{opt} = \sqrt[4]{\frac{K_{Co}\omega^2\mu^2}{3K_Q p_s p_{al}}}}$$

For the optimal film thickness one finds: $Q_f p_s = 3.C\omega = \sqrt[4]{K_Q p_s p_{al}\left(\frac{K_{Co}\omega^2}{3}\right)^3}\sqrt{\mu}$

One concludes that the power dissipated can not be optimised simultaneously with respect to the viscosity and the film thickness. Consequently, a compromise has to be found between these two optimisations.