Building design: multidisciplinary Approach

Structural part

Optional Lecture
GCU - S8 – M8
Civil Engineering and Urban Planning Department

INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

GEOMAS

Outline

~~**1. Introduction**~~

2. FEA in *quasi*-static conditions

2.1. Theoretical aspects

2.2. Algorithmic aspects

~~**3. FEA in dynamic conditions**~~

~~3.1. Theoretical aspects~~

~~3.2. Algorithmic aspects~~

~~**4. Projets aims**~~

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1. Introduction

2. FEA in *quasi*-static conditions

2.1. Theoretical aspects (**Remainder**)

2.2. Algorithmic aspects

3. FEA in dynamic conditions

3.1. Theoretical aspects

3.2. Algorithmic aspects

4. Projets aims

FEA in *quasi*-static conditions

⇒ Formulation of Euler-Bernoulli beam FE in 2D

Displacement field interpolation

$$\begin{bmatrix} u_0(x) \\ v_0(x) \\ \beta(x) \end{bmatrix} = \begin{bmatrix} N_1(x) & 0 & 0 & N_2(x) & 0 & 0 \\ 0 & H_1(x) & H_2(x) & 0 & H_3(x) & H_4(x) \\ 0 & \frac{\partial H_1}{\partial x} & \frac{\partial H_2}{\partial x} & 0 & \frac{\partial H_3}{\partial x} & \frac{\partial H_4}{\partial x} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \beta_1 \\ u_2 \\ v_2 \\ \beta_2 \end{bmatrix}$$

where

$$\left\{ \begin{array}{l} q_e^T = [u_1 \; v_1 \; \beta_1 \; u_2 \; v_2 \; \beta_2] \\ \mathbb{N} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & H_1 & H_2 & 0 & H_3 & H_4 \\ 0 & H_1' & H_2' & 0 & H_3' & H_4' \end{bmatrix} \end{array} \right.$$

$$\beta = \frac{\partial v_0}{\partial x}$$

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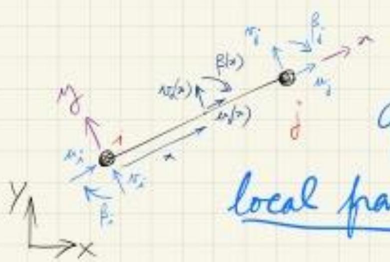
FEA in *quasi*-static conditions
⇒ Formulation of Euler-Bernoulli beam FE in 2D

Local $(\vec{x}, \vec{y})$

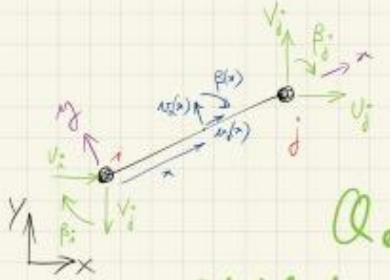
Global frames $(\vec{X}, \vec{Y})$

Within FEM (beam FE)

$$\underset{(\text{local})}{q_e} \longrightarrow \underset{(\text{global})}{Q_e} = P_6\, q_e \quad \text{where} \quad \begin{cases} P_6 = \begin{bmatrix} [P] & \begin{matrix} 0 \\ 0 \end{matrix} & \\ \begin{matrix} 0 & 0 \end{matrix} & 1 & 0_{3\times3} \\ 0_{3\times3} & & \begin{matrix} [P] & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{matrix} \end{bmatrix} \\ P = [\vec{e}_x \;\; \vec{e}_y] \end{cases}$$



q_e local frame



Q_e global frame

$$\underbrace{\begin{bmatrix} U_1 \\ V_1 \\ \beta_1 \\ U_2 \\ V_2 \\ \beta_2 \end{bmatrix}}_{Q_e} = \underbrace{\begin{bmatrix} [P] & \begin{matrix} 0 \\ 0 \end{matrix} & \\ \begin{matrix} 0 & 0 \end{matrix} & 1 & 0_{3\times3} \\ 0_{3\times3} & & \begin{matrix} [P] & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{matrix} \end{bmatrix}}_{P_6} \underbrace{\begin{bmatrix} u_1 \\ v_1 \\ \beta_1 \\ u_2 \\ v_2 \\ \beta_2 \end{bmatrix}}_{q_e}$$

Thus, at the element scale within the local frame

$$f_e = k_e \, q_e$$

- f_e → nodal forces
- k_e → stiffness matrix of the FE
- q_e → nodal displacements

It can be move to the global frame through

$$\begin{cases} Q_e = P_6 \, q_e \\ F_e = P_6 \, f_e \end{cases} \Rightarrow \quad P_6^T F_e = k_e \, P_6^T Q_e$$

Rq: $P_6^{-1} = P_6^T$

thus $$F_e = \underbrace{P_6 \, k_e \, P_6^T}_{K_e} \, Q_e$$

K_e : stiffness matrix of FE within the global frame
⇒ can be assembled

FEA in *quasi*-static conditions

⇒ Formulation of Euler-Bernoulli beam FE in 2D

Elementary stiffness matrix of EBFE in the local frame

$$k_e = \begin{bmatrix} \frac{ES}{L_e} & 0 & 0 & -\frac{ES}{L_e} & 0 & 0 \\ 0 & \frac{12EI}{L_e^3} & \frac{6EI}{L_e^2} & 0 & -\frac{12EI}{L_e^3} & \frac{6EI}{L_e^2} \\ 0 & \frac{6EI}{L_e^2} & \frac{4EI}{L_e} & 0 & -\frac{6EI}{L_e^2} & \frac{2EI}{L_e} \\ -\frac{ES}{L_e} & 0 & 0 & \frac{ES}{L_e} & 0 & 0 \\ 0 & -\frac{12EI}{L_e^3} & -\frac{6EI}{L_e^2} & 0 & \frac{12EI}{L_e^3} & -\frac{6EI}{L_e^2} \\ 0 & \frac{6EI}{L_e^2} & \frac{2EI}{L_e} & 0 & -\frac{6EI}{L_e^2} & \frac{4EI}{L_e} \end{bmatrix}$$

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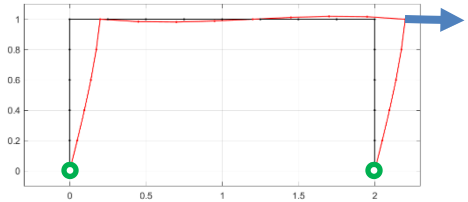
FEA in *quasi*-static conditions
⇒ Formulation of Euler-Bernoulli beam FE in 2D

How to do it in MATLAB ?

Moodle :

EB_ByHand
EB_CantileverBeam
EB_ComplexGeom
EB_ABAQUS

Example of a 2D portal frame
Loading : ponctual load
Boundary conditions :
articulation the supports



INPUTS (Geometry, Material, etc.)

↓

Meshing
(XY, TC, DOF)

↓

FE data
(E, I, L_e, g, rho, *etc.*)

↓

Matrix Assembling
(M, K, C)

↓

Loading and Boundary conditions

↓

Resolution

↓

Post Processing

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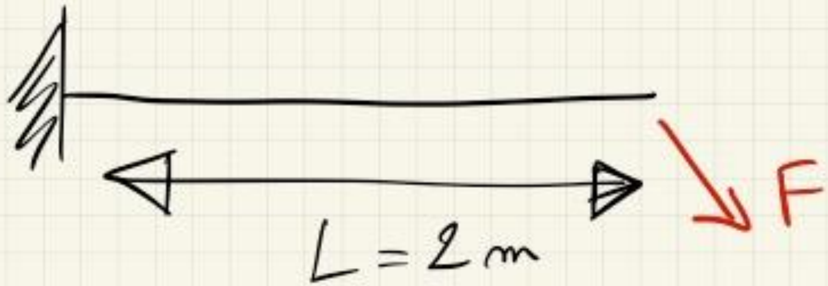
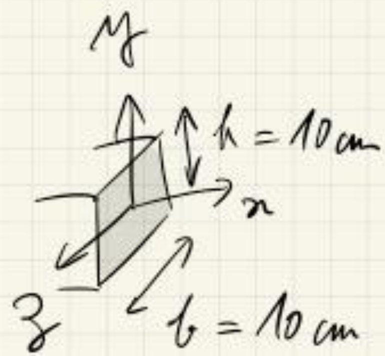
EB_ByHand

Before: Simple "By Hand" examples

Example ①:

To do with MATLAB





→ $(x, y) = (X, Y)$ no rotation needed.
local global

→ Mesh: 2 EBFE

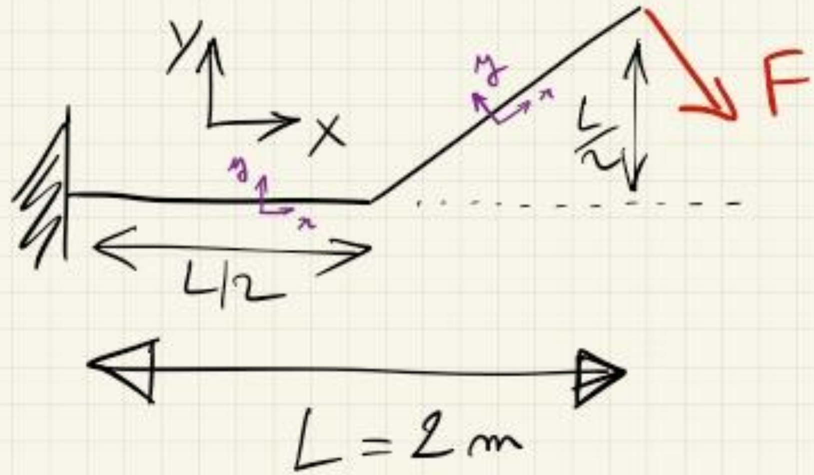
→ Make the assembling of the system $K\,U = F_{ext}$

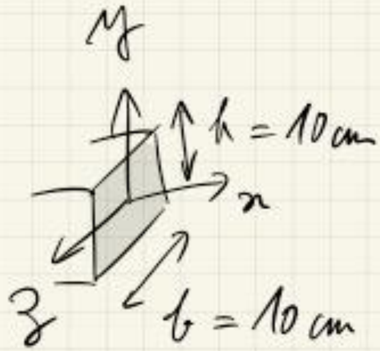
→ Find U and plot the deformed shape.

EB_ByHand

Example ②:

To do with MATLAB





$Q_e = P_6 \, q_e$

Rq: $K_e = P_6 \, k_e \, P_6^T$

↓ stiffness matrix within local frame

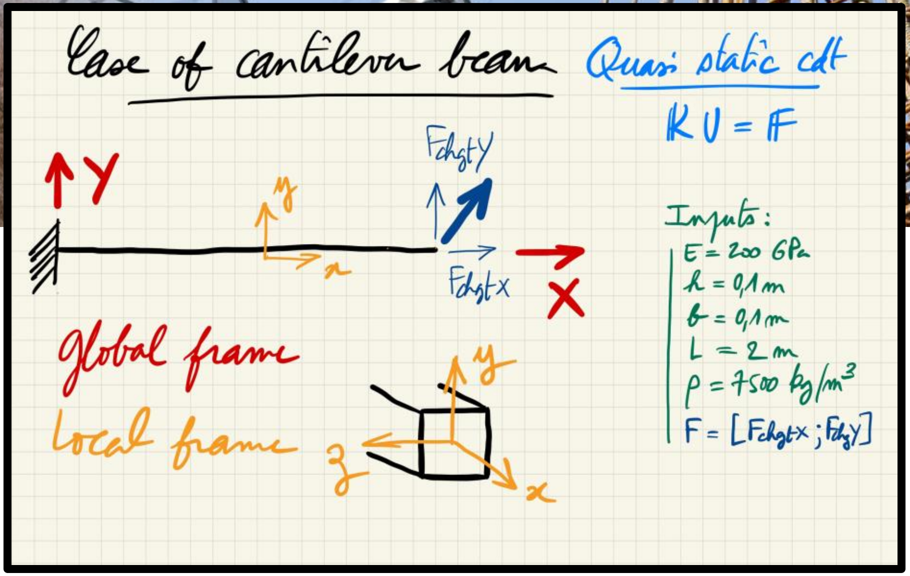
→  (x,y) # (X,Y) rotation needed.
local global

→ Mesh: 2 EBFE

→ Make the assembling of the system $K \, U = F_{ext}$

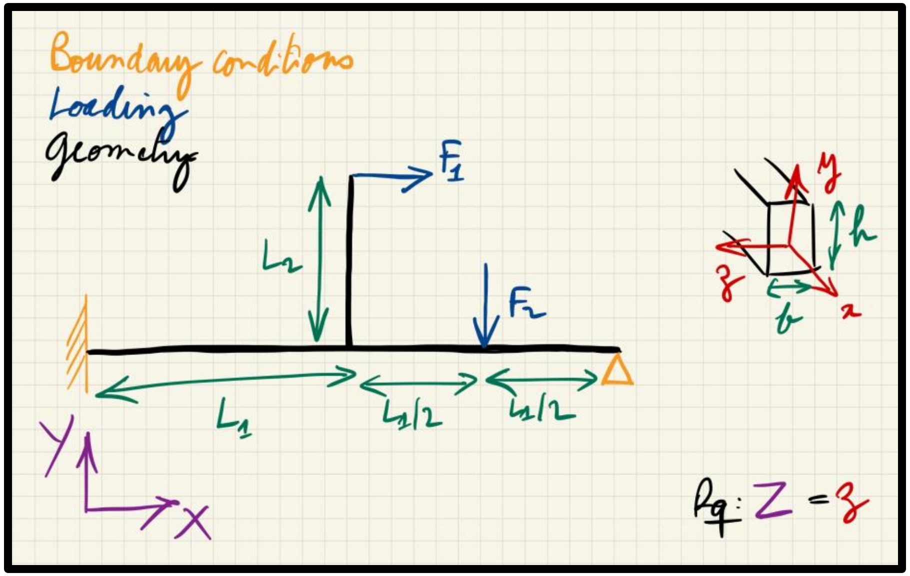
→ Find U and plot the deformed shape.

Cf. MOODLE
for source files



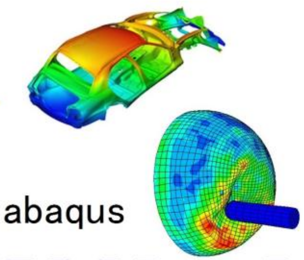
EB_CantileverBeam

EB_ComplexGeom



EB_ABAQUS

for meshing



SIMULIA abaqus

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